MATHEMATICAL MODELING AND DEVELOPMENT OF A VAPOR COMPRESSION REFRIGERATION SYSTEM SIMULATION PROGRAM

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Abstract

Looking for a computational tool to be used to design refrigeration systems, this current mathematical model was developed and its respective simulation program, which, starting from preliminary geometric and thermal definitions, permits the simulation of simple vapor compression refrigeration system.

The mathematical model includes a calculation module of the thermal load, and simulation modules of each of the components of the refrigeration system which are: an evaporator, a condenser and an expansion device and which take into account the basic hypothesis of steady state.

The simulation program was developed using EES – Engineering Equation Solver, having as basic input data the characteristics of the compressor, cooling fluid, the thermal insulator for the refrigerated area and some geometrical aspects of the other components.

The use of the simulation program leads to the acquisition of the basic dimensions of the system's components allowing the realization of the sensitivity analysis verifying, for example, effects of variations in the diameters of the component's tubes of both the evaporator and the condenser on the length of these heat exchangers, the effect of the heat transfer areas reduction caused by the application of the most efficient thermal isolators, etc.

1. Introduction

Engineering simulations of heat transfer and fluid flow phenomena are frequently limited to specific situations or to specific processes. This fact limits the engineering capacity of a company. In this context there is a possibility of developing simple simulations programs to be used in engineering design and that can be useful to predict systems behavior. So we propose a simple simulation program implemented using the EES – Engineering Equation Solver that allows to obtain immediate simple solutions.

2. Matematical Model

The present mathematical model was elaborated considering the following hypotheses: the analyzed processes are steady, the refrigerant fluid is considered a pure substance, dry air is the external fluid which is being considered as a perfect gas and when applied the thermodynamic laws for control volume the variations of potential energy are neglected.

2.1. Conderser analysis

The condenser was modeled as being composed of two sections, refrigerant fluid is superheated in the initial section and in the other section the refrigerant fluid is found in the saturation state. Along the development of this mathematical model, the pressure variations inside the condenser tube has not been taken into consideration. The refrigeration fluid temperature at the condenser end was considered equal to its saturation temperature.

In the initial section, the heat transfer rate, $\dot{Q}_H$, kW, through the condenser tube is:

$$\dot{Q}_H = U_c \cdot A_c \cdot \frac{(T_e - T_s)}{\ln\left(\frac{T_e - T_{amb}}{T_s - T_{amb}}\right)} \tag{1}$$

where T_e is the inlet temperature, ^oC, T_s is the outlet temperature, ^oC, U_c is the global heat transfer coefficient in the initial section of the tube, $W/m^{2o}C$, and A_c it is the heat transfer area, m^2:

$$A_c = L_c \cdot \pi \cdot D_i \tag{2}$$

where L_a is the length of the initial section of condenser tube, m, and D_i is its internal diameter, m.

The global heat transfer coefficient in this condenser section can be evaluated by:

$$\frac{1}{U_c} = \frac{1}{h_i \cdot A_c} + \frac{1}{h_o} + \frac{e_t}{K_t \cdot A_c} \tag{3}$$

where h_i is the heat transfer coefficient inside the tube, $W/m^{2o}C$, h_o is the external heat transfer coefficient, W/m2ºC, K_t is the thermal conductivity of the tube material, W/mK, and e_t is the tube wall thickness, m.

The external heat transfer coefficient is given by:

$$h_o = h_{oc} + h_{or} \tag{4}$$

where h_{oc} is the natural convection heat transfer coefficient, $W/m^{2o}C$, and h_{or} is the radiation heat transfer coefficient, $W/m^{2o}C$.

The natural convection heat transfer coefficient is given by:

$$h_{oc} = \frac{Nu \cdot D}{K_{ar}} \tag{5}$$

In this expression k_{ar} is the dry air thermal conductivity, W/mK, and, according to Silvares, 1999, the Nusselt number can be expressed in the form:

$$Nu = 0{,}0188 \cdot Gr^{0{,}7556} \tag{6}$$

where Gr is the Grashof number.

The radiation heat transfer coefficient is given by:

$$h_{or} = \varepsilon \cdot \sigma \cdot \left(T_p^2 + T_{amb}^2\right) \cdot \left(T_p + T_{amb}\right) \tag{7}$$

where ε is the tube surface emissivity, σ is the Stefan-Boltzmann constant and T_p is the wall tube internal temperature, ºC.

The internal heat transfer coefficient in the initial section can be calculated using the Dittus-Boelter correlation:

$$Nu = 0{,}023 \cdot Re^{0{,}8} \cdot Pr^{0{,}3} \tag{8}$$

In the second section of the condenser, in which happens the flow of saturated fluid, considering the same hypotheses defined for the previous section, we will have:

$$\dot{Q}_H = U_c \cdot A_c \cdot (T_e - T_{amb}) \tag{9}$$

The global heat transfer coefficient, U_c of this condenser is given by:

$$\frac{1}{U_c} = \frac{1}{h_i \cdot A_c} + \frac{1}{h_o} + \frac{e_t}{K_t \cdot A_c} \tag{10}$$

In the second section, the condenser tube internal heat transfer coefficient is determined using the correlation proposed by Traviss and reported by Silvares, 1999.

$$Nu_{liq} = \frac{Re_{liq}^{0,9} \cdot Pr_{liq} \cdot F_1^{B}}{F_2} \tag{11}$$

where Re_{liq} is the liquid phase Reynolds number and Pr_{liq} is the liquid phase Prandtl number.

The adimensional parameter F1 is given by:

$$F_1 = 0,15 \cdot \left(x_{tt}^{-1} + 2,85 \cdot x_{tt}^{0,524}\right) \tag{12}$$

where x_{tt} is the Martinelli parameter:

$$x_{tt} = \left(\frac{\mu_{liq}}{\mu_{vap}}\right)^{0,1} \cdot \left(\frac{1-x}{x}\right)^{0,9} \cdot \left(\frac{\rho_{vap}}{\rho_{liq}}\right)^{0,5} \tag{13}$$

where μ_{liq} is the saturated liquid dynamic viscosity, kg/ms, μ_{vap} is the saturated liquid dynamic viscosity, kg/ms, ρ_{vap} is the refrigerant vapor phase density, kg/m^3, ρ_{liq} is the refrigerant liquid phase density, kg/m^3.

The exponent B is given by:

- $B = 1$ for $F_1 \leq 1$
- $B = 1,15$ for $F_1 > 1$

The adimensional parameter F_2 is given by:

$$F_2 = 0,707 \cdot Pr_{liq} \cdot Re_{liq}^{0,5} \qquad \text{for } Re_{liq} < 50 \tag{14}$$

$$F_2 = 5 \cdot Pr_{liq} + 5 \cdot \ln\left(1 + Pr_{liq} \cdot \left(0,09636 \cdot Re_{liq}^{0,585} - 1\right)\right) \qquad \text{for } 50 < Re_{liq} < 1125 \tag{15}$$

$$F_2 = 5 \cdot Pr_{liq} + 5 \cdot \ln\left(1 + Pr_{liq}\right) + 2,5 \cdot \ln\left(0,00313 \cdot Re_{liq}^{0,812}\right) \qquad \text{for } Re_{liq} > 1125 \tag{16}$$

The used correlation admits that the two-phase flow inside the tube is annular. This hypothesis is plausible for qualities between 0,1 and 0,9. In the two-phase flow section, the preponderant thermal resistance is the external that is composed by the natural convection and by the radiation heat transfer and, therefore, the mistakes introduced by this hypothesis are of reduced effect.

In the section where the flow has quality varying from 0,0 to 0,1 a reduced amount of vapor will exist, not characterizing the formation of the annular area. The correlation of Dittus – Boelter was used, utilizing the liquid phase Reynolds number for the internal heat transfer coefficient determination.

In the section where the flow has quality varying from 0,9 to 1,0 a reduced amount of liquid will exist, not characterizing the formation of the annular area. The Dittus – Boelter correlation was used, utilizing the vapor phase Reynolds number for determination of the internal heat transfer coefficient.

2.2. Evaporator analysis

In this model the evaporator is treated as a horizontal tube with constant diameter, it is not being considered the variation of the pressure of the refrigerant fluid along its length. It will be considered, also, that at the outlet of this equipment the refrigerant fluid quality will be igual to 1,0.

The length of the component evaporator tube will be determined using the following expression:

$$L_e = \frac{\dot{Q}_p}{U_e \cdot (Ti - T_{es}) \cdot D_i \cdot \pi} \tag{17}$$

where T_{es} is the evaporator outlet temperature and T_i is the refrigerated ambient temperature, ºC, and U_e is the evaporator global heat transfer coefficient, W/m^{2o}C, $\dot{Q}_p$ is the thermal load, W.

According to Demétrio, 1998, the global heat transfer coefficient for cylindrical walls is given by:

$$\frac{1}{U_e} = \frac{1}{h_i} + \left(\left(\frac{D_i}{2K_t}\right) \cdot \ln\left(\frac{D}{D_i}\right)\right) + \frac{D_i}{h_e D} \tag{18}$$

where h_i is the heat transfer coefficient in the evaporator tube internal face, W/m^2°C, h_e is the heat transfer coefficient in the external face, W/m^2°C, K_t is the evaporator tube material thermal conductivity, D_i is the internal and D is the external tube diameter, m.

According to Silvares, 1999, the natural convection heat transfer coefficient in the evaporator tube external face is has values from 7,69 to 8,16 W/m^2°C.

The evaporator internal face heat transfer coefficient varies strongly along its length. Looking for to maintain the simplicity of the present model, it is suggested to use the value of 8,0 kW/m^2°C, compatible with the graphic representation of this magnitude in function of the refrigerant fluid quality presented by Stoecker and Jabardo, 1994.

The thermal load, $\dot{Q}_p$, W, is determined being considered as the addition of the heat transfer rates, $\dot{Q}_1 \, \dot{Q}_2, \ldots \dot{Q}_6$, W, through walls of the refrigerator considered as planes, vertical or horizontal plates, made of two materials: the external steel plate and a insulating thermal layer.

The heat transfer rates in the vertical walls are given by:

$$Q_n = U_{vn} \cdot A_{vn} \cdot (T_{amb} - T_i) \tag{19}$$

where T_i is the temperature of the present air in the refrigerated place, °C, U_{vn} is the global heat transfer coefficient through the vertical walls, A_{vn} is the vertical walls heat transfer area.

According to Demétrio, 1998, the global heat transfer coefficient for plane walls, composed by multiple layers is given by:

$$Uv_{n1} = \left[\left(\frac{1}{h_{in}}\right) + \left(\frac{e_a}{k_a}\right) + \left(\frac{e_i}{k_i}\right) + \left(\frac{1}{h_{en}}\right)\right]^{-1} \tag{20}$$

where h_{in} and h_{en} are the vertical walls internal and external faces heat transfer coefficient, e_a it is the steel plate thickness, m, k_a is the steel plate thermal conductivity, kW/m°C, e_i is the thermal insulating material thickness, m, and k_i its conductivity, W/m°C.

According to Demétrio, 1998, for laminar natural convection heat transfer processes, without phase change, simplified equations can be used when the fluid is air and the solid surface is a vertical plane plate.

$$h_{ven} = 1{,}42 \cdot \left(\frac{T_{amb} - T_{ea}}{L}\right)^{0,25} \qquad \text{for: } 10^4 < Gr\,Pr < 10^9 \tag{21}$$

where T_{ea} is the temperature of the external steel plate face, °C, and L is the wall vertical dimension, m.

A similar expression is used to evaluate the internal heat transfer coefficient:

$$h_{vin} = 1{,}42 \cdot \left(\frac{T_{ii} - T_i}{L}\right)^{0,25} \qquad \text{for: } 10^4 < Gr\,Pr < 10^9 \tag{22}$$

where T_{ii} is the insulation internal face temperature, °C.

The heat transfer rate and the global heat transfer coefficient in the superior and inferior horizontal walls, are calculated using, the same expressions used for the vertical walls.

Natural convection heat transfer coefficient for the horizontal heated or cooled plates can be evaluated using traditional correlations also reported by Demétrio, 1998:

$$h_{se} = 0{,}59 \cdot \left(\frac{T_{amb} - T_{se}}{L}\right)^{0,25} \qquad \text{for: } 10^4 < Gr\,Pr < 10^9 \tag{23}$$

where T_{se} is the superior wall external steel plate face temperature, °C.

$$h_{si} = 0{,}59 \cdot \left(\frac{T_{si} - T_i}{L}\right)^{0,25} \qquad \text{for: } 10^4 < Gr\,Pr < 10^9 \tag{24}$$

where T_{si} is the superior wall internal insulating face temperature, °C.

$$h_{ie} = 1{,}32 \cdot \left(\frac{T_{amb} - T_{ie}}{L}\right)^{0,25} \qquad \text{for: } 10^4 < Gr\,Pr < 10^9 \tag{25}$$

where T_{ie} is the inferior wall external steel plate face temperature, ^oC.

$$h_{ii} = 1{,}32 \cdot \left(\frac{T_{iii} - T_i}{L} \right)^{0{,}25} \qquad \text{for: } 10^4 < Gr\,Pr < 10^9 \tag{26}$$

where T_{iii} is the inferior wall internal insulating face temperature

2.3. Capillary tube analysis

The adopted hypotheses for the development of the capillary tube model are: the expansion process is adiabatic, constant mass flow rate independent of the pressure inside the expansion tube, constant internal tube diameter and there won't be enthalpy variation along the tube.

The capillary tube length can be determined using the expression:

$$\left((P_1 - P_2) - f \cdot \left(\frac{\Delta L_{tc}}{D_i} \right) \cdot \left(\frac{V^2}{2 \cdot \nu} \right) \right) \cdot A_{stc} = \dot{m} \cdot (V_2 - V_1) \tag{27}$$

where P_1 is the capillary tube inlet pressure, kPa, P_2 is the capillary tube outlet pressure, kPa, f is the friction factor, ΔL_{tc} is the length variation, m, D_i is the capillary tube internal diameter, m, V is the refrigerant fluid medium speed, m/s, ν is the specific volume, m^3/kg, A_{stc} is the capillary tube internal transverse section area, m^2, V_2 is the refrigerant fluid outlet medium speed, m/s and V_1 is the inlet refrigerant fluid medium speed, m/s.

According to Stoecker and Jones, 1985, for low Reynolds number turbulent flow, such which the identified in capillary tubes, the friction factor can be expressed by:

$$f = \frac{0{,}33}{Re^{0{,}25}} \tag{28}$$

When two phase flow occurs, the refrigerant dynamic viscosity and its specific volume depends on its quality. The following expressions are used:

$$h = h_l \cdot (1 - x) + h_v \cdot x \tag{29}$$

$$\mu = \mu_l \cdot (1 - x) + \mu_v \cdot x \tag{30}$$

$$\nu = \nu_l \cdot (1 - x) + \nu_v \cdot x \tag{31}$$

where h_l is the saturated liquid enthalpy, kJ/kg, h_v is the saturated vapor enthalpy, kJ/kg, μ_l is the saturated liquid dynamic viscosity, Pa.s, μ_v is the saturated vapor dynamic viscosity, Pa.s, ν_l is the saturated liquid specific volume, m^3/kg, and ν_v is the saturated vapor specific volume, m^3/kg.

The inlet and outlet speeds can be determined using the equation:

$$\dot{m} = \frac{V_1 \cdot A_{stc}}{\nu_1} = \frac{V_2 \cdot A_{stc}}{\nu_2} \tag{32}$$

The application of this model allows the determination of the length of the capillary tube for temperature intervals. As those intervals are reduced, the method presents more precise results.

3. The simulation program

A simulation program was developed using the well-known computational program E.E.S - Engineering Equation Solver. This simulation program allows the choice and evaluation of design alternatives. Allows the behavior analysis produced by the alteration of dimensional parameters and it can be used to obtain a more precise values when designing small size refrigeration systems, optimizing the process and reducing costs.

The computational program EES solves group of equations including algebraic, differential and integral equations. This software has a library of mathematical functions and thermophysical properties.

4. Results

Some situations were analyzed and stand out the importance of the accomplished study. Such situations allowed the evaluation of the influence of the main parameters in the small size refrigerators designing, helping thus in the determination of the most appropriated parameters values.

4.1 Condenser analysis

Through this model can be analyzed the correlation between the internal diameter and the length of the condenser or evaporator tube, according to Figures 1 and 2, enabling thus a wider variability in the selection of the tube with relation to the cost and the final dimensions of the product.

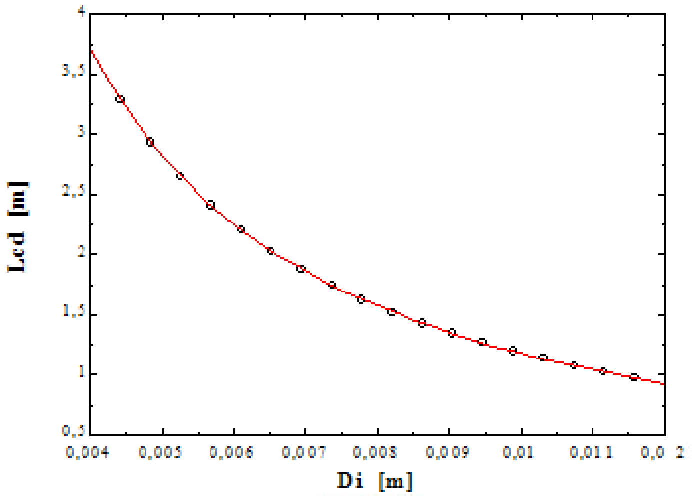

Figure 1. Diameter versus length of the condenser tube.

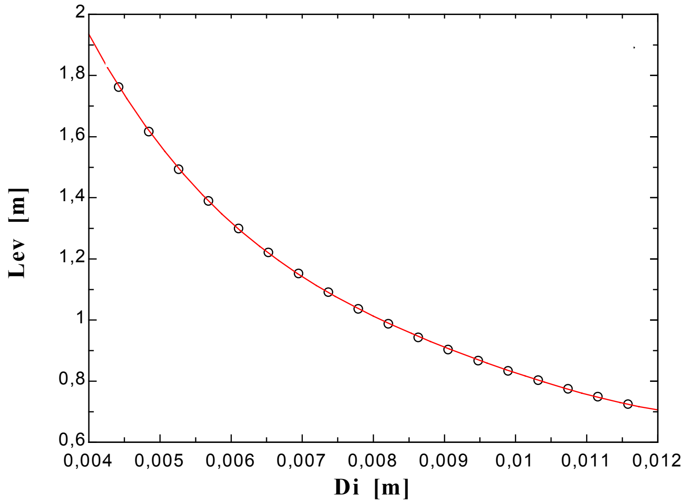

Figure 2. Diameter versus length of the evaporator tube

Analyzing the influence of the diameter of the capillary tube in relation to its length, graphs were built as a result of the use of the simulation program considering variation intervals of temperature equal 5ºC, Figure 3, and considering variation intervals of temperature equal 1ºC, Figure 4, due to what was observed a significant reduction in the length of the capillary tube, it is noticed then that the number of intervals is of fundamental importance for the application of the proposed model. Therefore, the use of the method of applied calculation in the construction of the graphs presented in the Figures 4 and 5 is advisable.

4.2 Capillary analysis

According to Stoecker and Jones, 1985, it is advisable the use of capillary tubes with lengths varying of 1 to 6 m, what does not occur with the length found in the calculations made during the analysis of the Graph 4, then the diameter of the capillary tube was reduced from 0,8mm to 0,6mm, propitiating thus reduction in the length, according to Figure 5.

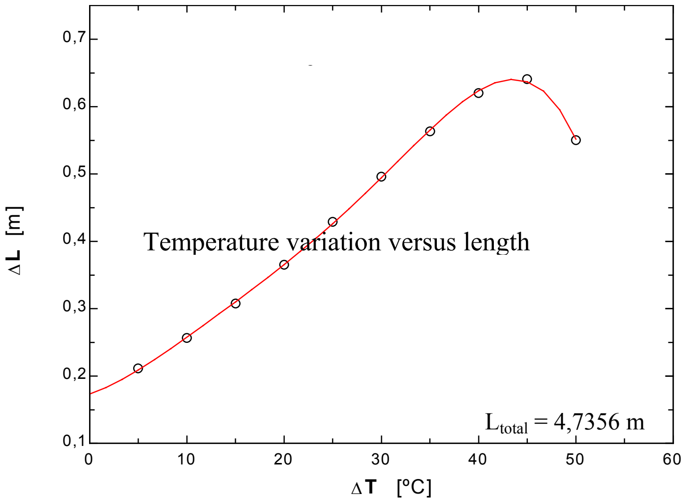

Figure 3. Temperature variation versus length of the capillary tube to each section with temperatures varying in 5ºC and using diameter of 0,8mm.

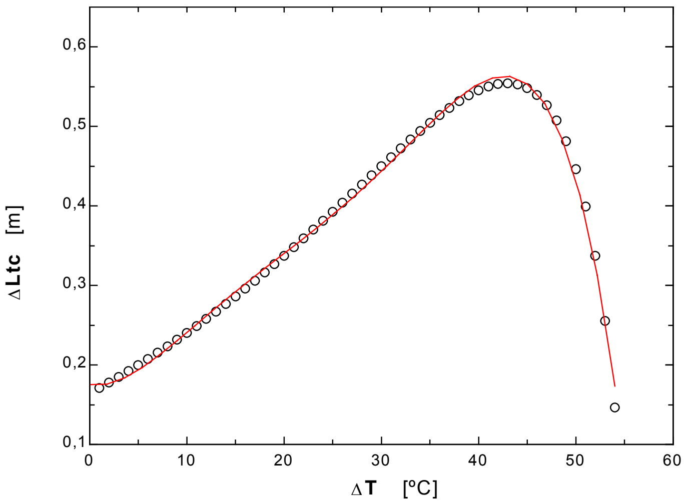

Figure 4. Temperature variation versus length of the capillary tube to each section with temperatures varying in 1ºC and using diameter of 0,8mm

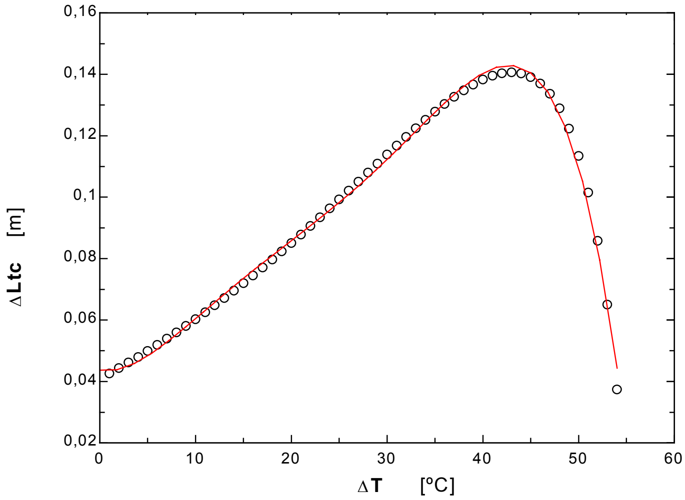

Figure 5. Temperature variation versus length variation of the capillary tube to each section, with temperatures varying in 1ºC and using diameter of 0,6mm.

4.3 Thermal insulation analysis

Some situations were studied where the importance was evaluated of some factors that aided in the dimensioning of the isolation. For that the influence of the thickness and material was evaluated in function of the heat transfer rate in the walls.

Analyzing the variation of the thickness of the isolation versus the heat transfer rate in the walls, Figure (6), is noticed that the inclination of the curve tends to decrease after the thickness of 50 mm, decreasing the variation of the heat transfer rate in the walls, not being viable the increase of the thickness of the isolation, because the heat transfer rate will not decrease in the same proportion, turning the equipment not compact.

To analyze the variation of the thermal conductivity of the isolation versus the heat transfer rate in the walls, it was built a graph that embraces from materials with low thermal conductivity like Blanket-Al-silica, even materials with thermal conductivity more elevated as the rubber vulcanized, Figure (7). It can be observed that the heat transfer rate begins to have a great variation starting from the instant that the thermal conductivity surpasses the value of 0,04 W/mK. The material to be used can be selected through the graph, that it allows the comparison among the amount of heat that its equipment can change with the cost among the several material types, allowing the choice of one that provides a good heat transfer with low acquisition cost.

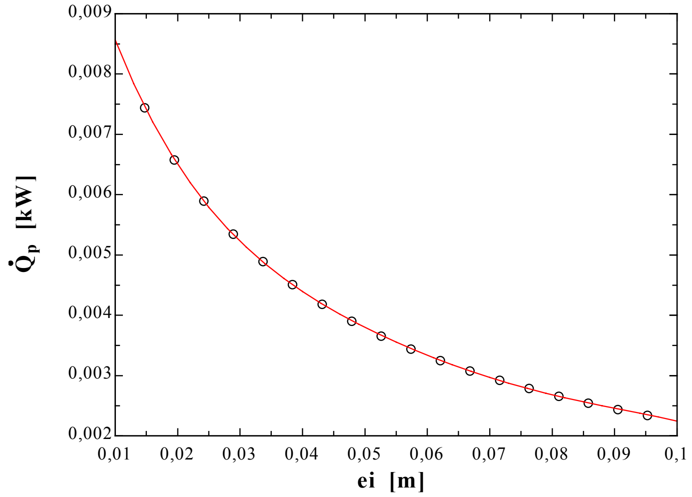

Figure 6. Heat transfer rate of the walls versus thickness of the insulating.

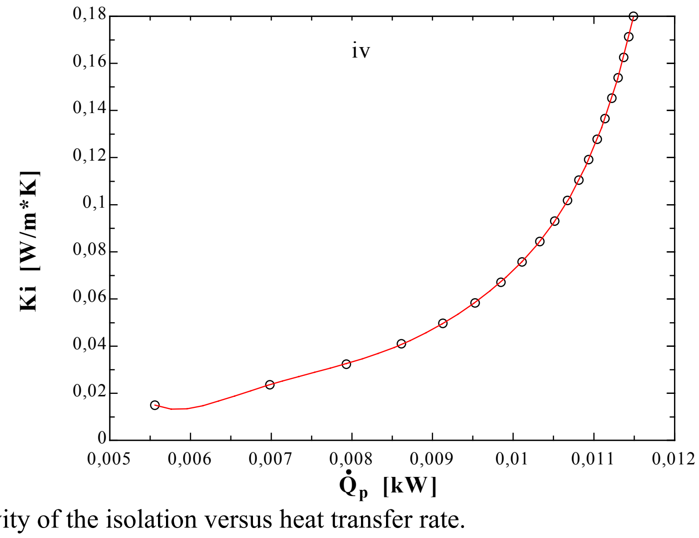

Figure 7. Thermal Conductivity of the isolation versus heat transfer rate.

5. Conclusion

The authors consider that the technology transfer is a very important tool to be used to stimulate the country development. In this context this computational programs that allows a small company produce better products can be of fundamental importance. So, the continuation of this development is suggested.

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