

SEMI-EMPIRICAL MODEL OF THE EFFICIENCY CURVE OF A FRANCIS TURBINE

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Abstract. In this study was performed a mean global analysis, developing a model for the dissipation in the discharge and in the suction ducts to determining the net height of the Francis turbine 08 of the Itaipú Binacional. The efficiency of this turbine was modeled in a function either of the blade velocity ratio or of the flow rate of operation of the turbine. Based on the INDEX TEST, the measured efficiency was fitted by a function of the blade velocity ratio by using a least square method. Proceeding a comparison of the results obtained by the fitted second order equation with the equation of the model, what allowed to obtain the coefficient K_{blade}

Keywords. Semi-empirical method, Francis turbine, efficiency curve.

1. Introduction

Thermal systems simulation that can be applied to heating Plant, Thermal, Hydro and Nuclear Power Plant is gotten by calculating of all operation variables, as temperatures, pressures and mass flow rate.

Evans et al. (1968) indicate that the Chemical process industries and others programs to simulate operation variables.

Several simulation programs were been developed for the industries. Examples of these programs are: PACER (Purdue-Dartmouth), MAEBE (University of Tennessee), COPE (Esso Research and Engineering), CHEOPS (Shell oil), PEDLAN (Mobil Oil) and FLEXIBLE FLOW SHEET (M. W. Kellogg), and so on.

Stoecker (1971) developed a generic program for the simulation of the steady state systems, nevertheless this programs not generate the equation system, as specific routines.

Reynolds (1979) developed equations to compute physical and thermodynamic properties for working fluids. Stoecker (1980) indicate simulation and optimization method for the thermal systems.

Klein e Alvarado (1993) developed the program EES (Engineering Equation Solver) to solve a set of algebraic equations, calculating the thermodynamic properties, which are used in the Engineering equation solver. This program, also, not generate the equation system.

Tests realized by the Itaipú Binacional (INDEX TEXT) (1994) obtained experimental curves of the operating flow rate, efficiency, and power of the turbine, in function of the distributor open fraction.

Melo and Abreu (1996) developed a generic program for the simulation of thermal systems. In this program the equation system is generate automatically by de conservation laws and by the identifications of the characteristic curve of each component.

Melo (1999) developed a program for the design of a pumping system. Based on the node downstream and upstream of each component and its characteristic curve, routines generate, automatically, the equation system, that take in consideration the conservation laws. Routines, based on the Newton Raphson method, solve the equation system.

Melo (2002) developed a model of energy cogeneration for obtain an air refrigeration cycle. The discharge gas from a Diesel engine is expanded in the turbine that is coupled to an air compressor. The air is expanded in other turbine and compressed in other compressor. The coupling of the characteristic curves of the turbines and compressors was made. Special routines was developed to calculate the thermodynamic properties of the working fluids, to interpolating the points of the characteristic curves of turbines and compressors and for the solution method used in this paper. were obtained the optimization of air mass flow rate and gas combustion mass flow rate, components efficiency, compression and expansion relation, inlet temperature in the compressor, inlet temperature in the cooler and coefficient of performance of the air cycle.

The motivation for this work rose by necessity in deciding problems of tests of efficiency in Hydro Power Plant, requested for Companies. It is verified that the majority of the available references are sufficiently old and outdated. In these references, the models of specification of systems with flow machines are simplified. The coupling is not made between what the external system supplies or requires of the machine with what it is capable to converting hydro energy into mechanics or mechanics into hydro energy.

2. Formulation of the Problem

Based on the Bernoulli equation, in the continuity equation and in the Darcy equation, to evaluate the dissipation terms, the turbine net height can be determined by:

$$H_1 = H_b - \left[\frac{f_d L_d}{D_d^5} + \int_0^{L_s} \frac{f_s(x) dx}{(D_s(x))^5} \right] \frac{8(Q(S))^2}{\Pi^2 g} \quad (1)$$

where:

H_1 - Net Height

H_b - Brut Height

$Q(S)$ – Operating flow rate, in function of open fraction of distributor.

f_d - Friction factor in the discharge duct

f_s - Friction factor in the suction duct

L_s - Total length of suction duct

L_d - Total length of discharge duct

D_d - Inner diameter of discharge duct

x - Coordinate measured in the symmetry axe in the suction duct

$D_s(x)$ - Inner diameter of the suction duct, in function of x coordinate

S - Open fraction of distributor

The friction factor is determined iteratively, by the Newton Raphson method, using the Colebrook equation, that has the following form:

$$y = 1,74 - 0,868589 \ln \left(2 \frac{e}{D} + \frac{4,675 \pi D \nu y}{Q} \right) \quad (2)$$

and

$$f = \frac{1}{y^2}$$

where:

$\frac{e}{D}$ - Relative roughness of the pipe

Q - Operating flow rate

D - Inner Diameter

ν - Cinematic viscosity

From equation (2), one deduce that:

$$g(y) = y - 1,74 + 0,868589 \ln \left(\frac{2e}{D} + \frac{4,675 \pi D \nu y}{Q} \right) = 0 \quad (3)$$

$$g'(y) = 1 + 0,868589 \frac{\frac{4,675 \pi D \nu}{Q}}{\frac{2e}{D} + \frac{4,675 \pi D \nu}{Q} y} \quad (4)$$

The algorithmic for determining the zero of the equation (3), using the Newton Raphson method, is the following:

$$f_0 = 0,02$$

$$y_1 = \frac{1}{\sqrt{f_0}}$$

Repeat

begin

$$y_0 = y_1$$

$$y_1 = y_0 - \frac{g(y_0)}{g'(y_0)}$$

$$\text{Until } \left| \frac{y_1 - y_0}{y_1} \right| \leq \text{Tolerance}$$

end

$$f = \frac{1}{y_1^2}$$

The Figure 1 shows the suction pipe region of unit 08 of Itaipú Binacional.

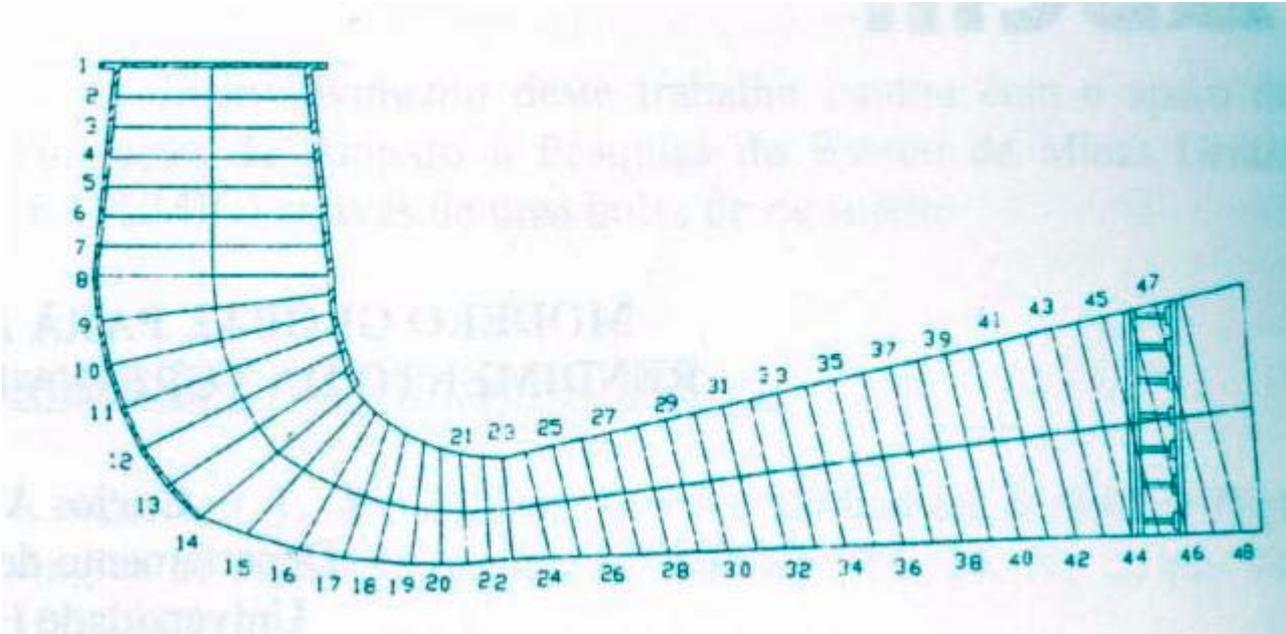


Figure 1. Suction pipe region

The integral of the equation (1) is numerically solved, using the Simpson formula, as:

$$\int_0^{L_s} \frac{f_S(x)dx}{(D_S(x))^5} = \frac{h}{3} [F_1 + F_{49} + 2(F_3 + F_5 + \dots + F_{47}) + 4(F_2 + F_4 + \dots + F_{48})]$$

$$h = \frac{L_s}{48} \text{ (The length } L_s \text{ was divided in 48 parts)}$$

$$F_i = \frac{f_i}{D_i^5}$$

Each friction factor f_i is determined by the Newton Raphson method, using the operation flow rate $Q(S)$ and the inner diameter D_i , that is function of the coordinate x in the symmetry axe of the suction piping.

β_1 – Angle formed with the tangent to blade, in the perpendicular plan to the axe machine and the oppose direction to the inlet tangential velocity of the blade

β_2 - Angle formed with the tangent to blade, in the perpendicular plan to the axe machine and the oppose direction to the outlet tangential velocity of the blade.

The turbine efficiency is obtained by:

$$\eta_t = \frac{u_1 c_{1u} - u_2 c_{2u}}{gH_L} \quad (5)$$

where:

u_1 is the tangential velocity of the rotor in the inlet of the blade, u_2 in outlet, c_{1u} is the inlet component of the fluid absolute velocity in the tangential direction and c_{2u} in outlet, both real, considering the dissipation on the rotor.

$$u_1 = \frac{n(rpm)\pi D_1}{60} \quad (6)$$

$$u_2 = \frac{n(rpm)\pi D_2}{60} \quad (7)$$

$$c_{1u} = u_1 - \frac{Q}{\pi D_1 b_1 \tan(\beta_1)} \quad (8)$$

$$\cos(\alpha_1) = \frac{\pi D_1 b_1 c_{1u}}{\sqrt{\pi^2 D_1^2 b_1^2 (c_{1u})^2 + Q^2}} \quad (9)$$

$$c_1 = \frac{c_{1u}}{\cos(\alpha_1)} \quad (10)$$

$$W_1 = \frac{u_1 - c_{1u}}{\cos(\beta_1)} \quad (11)$$

$$(c_{2u})_t = u_2 - \frac{Q}{\pi D_2 b_2 \tan(\beta_2)} \quad (12)$$

$$(W_2)_t = \frac{u_2 - (c_{2u})_t}{\cos(\beta_2)} \quad (13)$$

$$H_E = \frac{u_1 c_{1u} - u_2 (c_{2u})_t}{g} \quad (14)$$

$$\varepsilon = \frac{u_1^2 - u_2^2 + (W_2)_t^2 - W_1^2}{2gH_E} \quad (15)$$

The subscript t indicate the theoretical transformation

Based on the theoretical reaction degree concept ε , the coefficient K_{blade} , that take in consideration the dissipation on the rotor, the efficiency of the turbine can be written in function of the blade velocity ratio $\frac{u_1}{c_1}$, of the ratio $\frac{D_2}{D_1}$, of the theoretical reaction degree ε , of the outlet angle β_2 and of the incident angle α_1 as:

$$\eta_T = 2 \left\{ (1-\varepsilon) \cos \alpha_1 \left(\frac{u_1}{c_1} \right) - \left(\frac{D_2}{D_1} \right) \sqrt{1-\varepsilon} \left(\frac{u_1}{c_1} \right) \left[\left(\frac{D_2}{D_1} \right) \sqrt{1-\varepsilon} \left(\frac{u_1}{c_1} \right) - K_{blade} \cos \beta_2 \sqrt{1 + \left(\frac{u_1}{c_1} \right)^2 (1-\varepsilon) \left(\frac{D_2}{D_1} \right)^2 - 2(1-\varepsilon) \left(\frac{u_1}{c_1} \right) \cos \alpha} \right] \right\} \quad (16)$$

The power of the turbine is determined by:

$$P = \rho g Q H_L \eta_T \quad (17)$$

The efficiency obtained experimentally was fitted, by the least square method, in function of the blade velocity ratio (u_1 / c_1) of the following form:

$$\eta_T = A \left(\frac{u_1}{c_1} \right)^2 + B \left(\frac{u_1}{c_1} \right) + C \quad (18)$$

The operational parameter K_{blade} was obtained by comparing the equation (16) with the equation (18), resulting:

$$K_{blade} = \frac{C}{2 \cos(\beta_2) \left(\sqrt{1 + \left(\frac{u_1}{c_1} \right)^2 (1-\varepsilon) \left(\frac{D_2}{D_1} \right)^2 - 2(1-\varepsilon) \left(\frac{u_1}{c_1} \right) \cos \alpha} \right)_{average}} \quad (19)$$

3. Results and Discussions

The results of the model were based on the longitudinal cross section, that pass by the discharge and suction ducts, on the transversal cross section (perpendicular to the axis of the turbine), in the physical parameters and on the turbine rotation, disposed on Table 1.

Table 1. Parameters, used on the model

L_D	Length of the discharge duct = 158 m
D_D	Inner diameter of the discharge duct = 10,5m
H_b	Brut Height = 118,7 m
L_S	Length of the suction duct = 56 m
D_1	Inlet rotor diameter = 7,81 m
D_2	Outlet rotor diameter = 3,56 m
b_1	Inlet wide rotor = 2,206 m
b_2	Outlet wide rotor = 4,095 m
β_1	123 ⁰
β_2	142,5 ⁰
g	Gravity = 9,789 m / s ²
ρ	Water density = 995,1 Kg / m ³
e	Absolute Roughness = 4,572 x 10 ⁻⁵ m
n	Rotation of the turbine = 92,3 rpm

THE INDEX TEST indicate on the table 2 the flow rate, the net height and the efficiency of the turbine, in function of the distributor open fraction.

Table 2. Experimental data of the Itaipú Binacional

S(%)	Q(m ³ /s)	H ₁ (m)	η_{tb} %	P(MW)
46,30	399,55	117,47	89,89	394,37
51,70	443,64	117,31	92,03	461,74
58,8	507,01	116,99	94,66	543,55
64,40	553,31	116,70	95,82	601,20
67,75	576,11	116,57	96,13	628,31
70,50	596,33	116,43	96,05	650,23
73,30	614,76	116,37	95,85	669,08
75,80	632,73	116,25	95,45	686,00
80,80	664,35	116,09	94,89	716,67
90,60	716,14	115,69	93,06	758,98
99,40	752,45	115,38	91,29	783,36

On the table 3 are indicated the net height, in function of the distributor open ratio, or of operation flow rate, obtained by the present model.

On the table 4 are indicated the dissipation terms in m (J/m), in the discharge and suction ducts, determined by the present model.

On the Figure 2 are indicated the dissipation in the suction and the discharge ducts of the unit 08 of Itaipú Binacional and in the Figure 3 the net height curves.

The coefficients fitted A, B, and C of the equation (18), are:

$$A = -21,4242 \text{ s}^2 / \text{m}^6 ; B = 35,1306 \text{ s} / \text{m}^3 ; C = -13,44199 \text{ m}^3 / \text{s}$$

The maximum relative error, between the experimental values and the calculated by equation (18) was of 0,35 %. The average values of the operational parameters are:

$$\varepsilon = 0,633581 ; \alpha_1 = 13,4754^0 ; \frac{u_1}{c_1} = 0,8228 ; K_{blade} = 4,6479$$

$$\left(\sqrt{1 + \left(\frac{u_1}{c_1} \right)^2 (1 - \varepsilon) \left(\frac{D_2}{D_1} \right)^2 - 2(1 - \varepsilon) \left(\frac{u_1}{c_1} \right) \cos \alpha} \right)_{average} = 0,6783$$

Table 3. Net height, obtained from the model

S(%)	Q(m ³ /s)	H _E (m)
46,3	399,55	113,809
51,7	443,64	113,652
58,8	507,01	113,426
64,4	553,31	113,129
67,7	576,11	113,129
70,5	596,33	113,038
73,3	614,76	112,0
75,8	632,73	112,880
80,8	664,35	112,755
90,6	716,14	112,7
99,4	752,45	112,565

Table 4. Dissipation in the discharge and the suction ducts

S(%)	Discharge	Suction
46,3	0,9348	0,2607
51,7	1,0529	0,3179
58,8	1,2446	0,4101
64,4	1,4229	0,4953
67,7	1,5359	0,5490
70,5	1,6337	0,5953
73,3	1,7306	0,6413
75,8	1,8185	0,6809
80,8	1,9661	0,7524
90,6	2,1618	0,8446
99,4	2,2118	0,8681

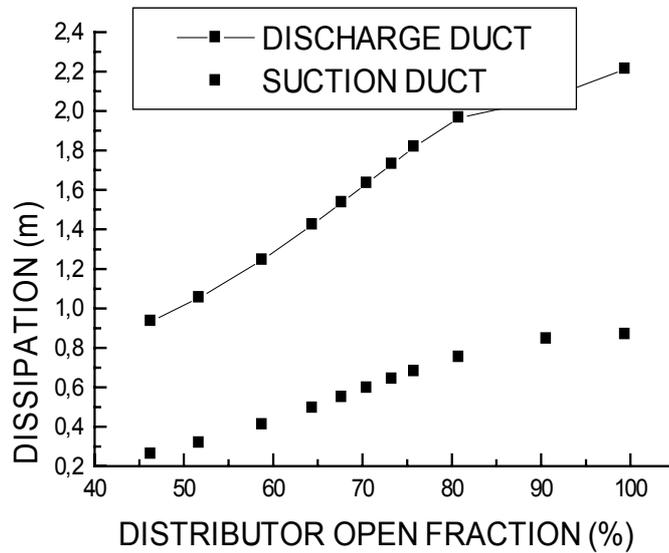


Figure 2. Dissipation in the discharge and in the suction ducts

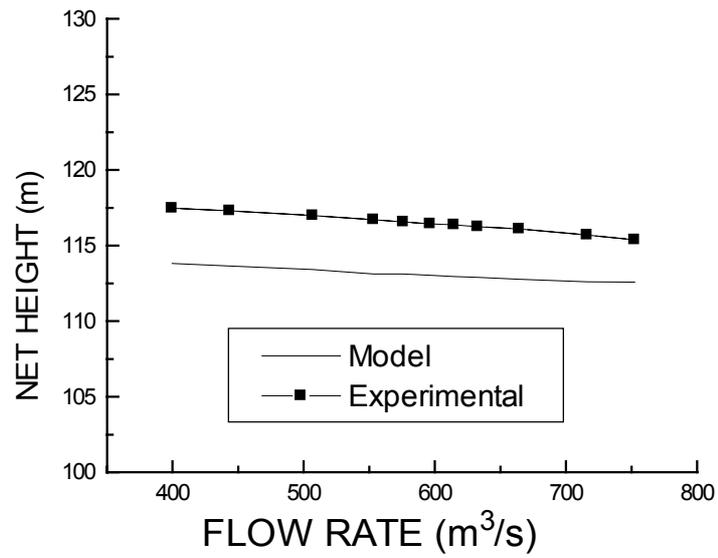


Figure 3. Net height of the turbine

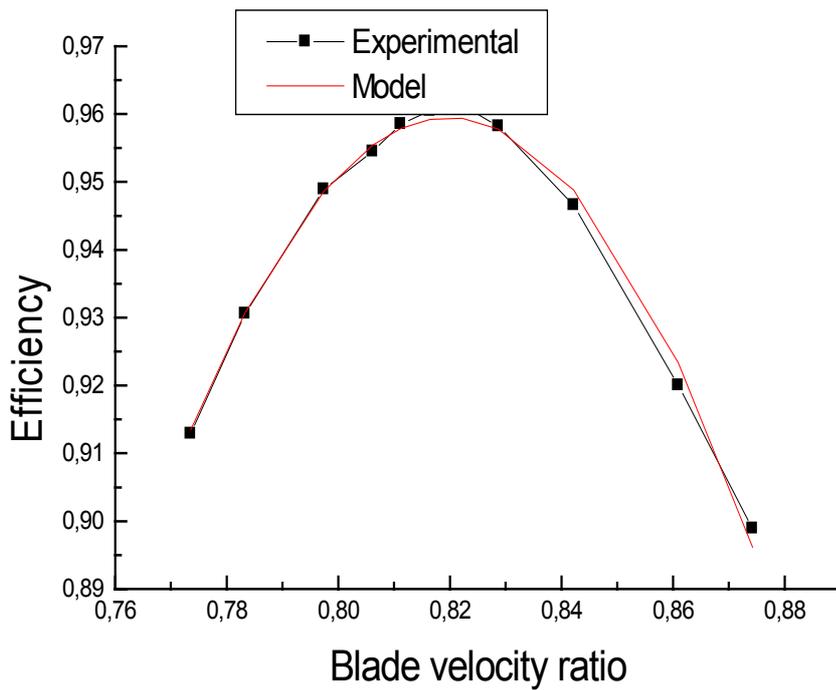


Figure 4. Efficiency of the turbine

In the Figure 4 are indicated the efficiency of the turbine, obtained by the model and experimentally. In the Figure 5 are indicated the power of the turbine, obtained by the model and experimentally.

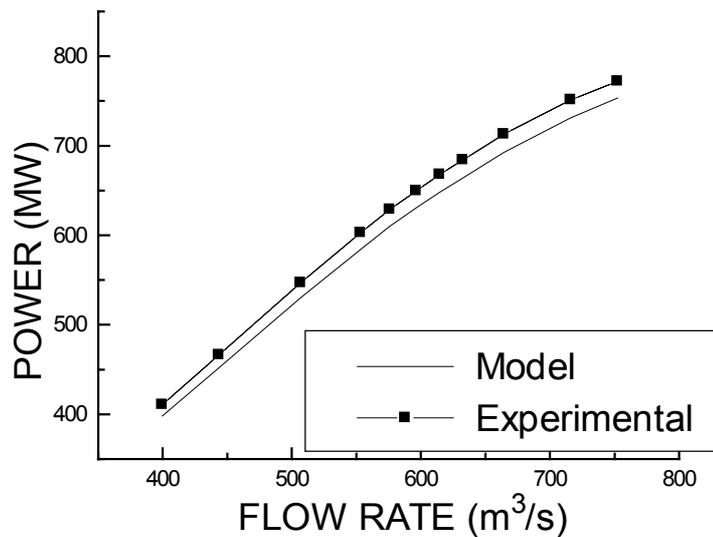


Figure 5. Power of the turbine

4. Conclusions

The values of the net height and power of the present model can be approached sufficient to the experimental ones for the reduction of the value of the absolute roughness of the simulated duct.

The values of the efficiency are in good agreement with the experimental values, as indicated in the Figure 4.

As indicated, the present model can be used to simulate the Francis turbine of the Hydro Power Plant.

5. References

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