EVALUATION OF A FLAP TYPE WAVE GENERATOR

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Abstract. This paper deals with the evaluation of a flat type wave generator installed in the test tank of the laboratory of the Department of Naval Architecture and Ocean Engineering of the University of São Paulo. The formulation of a wave maker theory results from a match between the stroke of an actuator and the mechanics of the wave propagation. This is summarized in the so-called transfer function between wave height and stroke of the flap. This model is based on velocity potential obtained for small amplitude waves. The performance of the wave generator is evaluated by performing tests in which frequency and wave height and stroke of the flat were recorded and then evaluated against that determined theoretically. An important parameter to design a wave generator is the motor power to drive the actuator. This power depends on the driving force necessary to counteract the hydrodynamic force and inertial term of the paddle and moment due to the flap weight. In this paper a theoretical expression for the driving force is shown and then compared with that obtained experimentally. All results have shown that the linear theory predicts the performance of wave generator properly for small amplitude and low frequency waves.

Keywords*. wavemaker, transfer function, velocity potential, driving force*

1. Introduction

Exploitation of offshore natural resources, oil in particular, has increased in the last three decades and so demand for installations of working platform has also risen. However, sea environment is sometimes hazardous and in order to assure a safe operation, a good prediction of the behaviour of the floating unit under the combined action of the current, wind and waves during its design stage is required. Unfortunately theoretical prediction is not an easy task since it relies on a highly nonlinear set of mathematical equations. Most frequently those equations require coefficients that are difficult to be obtained taking into account a theoretical approach only. Besides, the modelling of some interaction between vessel and seawater is a state-of-the-art concept mainly when the combined action of current, wave and wind is considered. Therefore in the field of marine engineering tests with scale model in tanks are quite common. Those tanks have current, wind and wave generators facilities in order to try to simulate the sea environment properly.

There are quite a large number of tanks around the world that differ in terms of geometry, dimensions and sort of facilities (Souza, 2002). In the case of Brazil a wave generator was installed in the tank of Laboratory of the Department of Naval Architecture and Marine Engineering of the University of São Paulo recently. This wave generator was designed by the staff of the Department that also managed its installations. The wave generator consists basically of the actuator and the power system to move it. Its design requires the transfer function between wave height and flap stroke and the total force to move the flap in order to select the power of the prime mover properly. The aim of this paper is to compare theoretical and experimental results for both the transfer function and the force.

The project of a wave maker requires a mathematical formulation for the water waves propagation. Linear and non linear theory for water waves are presented in Le Méhauté (1976), Dean and Dalrymple (1991), Rahman (1995), Chakrabarti (1994) and Battacharyya, (1978). The mathematical connection between desired waves and wave maker is determined through boundary conditions by imposing the same displacement for water and actuator. A comprehensive study concerning design of wave generator is presented by Dias (1981). There are a lot of different types of actuators (Battacharyya, 1978) but the flap is one of the most common types.

2. Description of the tank and wave maker

The main dimensions of the tank under consideration are shown in Table 1. In comparison to others tanks it is a very short one and a detailed discussion about the design criteria of the wave generator is shown in Souza, 2002. Fig.1 shows a pictorial view of the wave generator with flap type actuator that is driven by the electric-mechanic system. The prime mover is a servomotor that turns a ball screw system which transforms the rotary displacement into the translatory one. The latter motion is transferred to the flap through the rod. The wave frequency range considered for the wave generator was 0.5 Hz to 3.0 Hz for 1.5 m nominal water depth. The mechanical system was designed to generate waves with slope up to 5% at low frequency.

Table 1 Main dimensions of the tank

Length	21.5 m
Depth	1.78m

Width	4.85 m

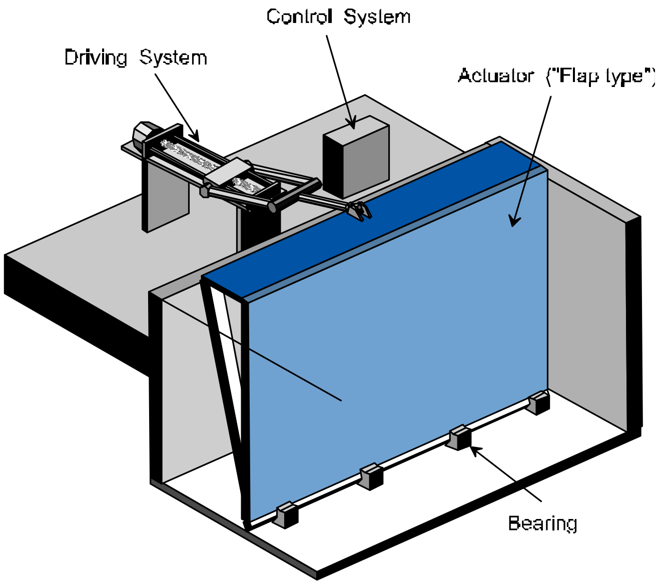

Figure 1 Pictorial view of the wave generator

3 Mathematical model

The simplest approach to establish a theory for surface waves is for two-dimensional waves, ie, the crests are straight, infinitely long, parallel and equally spaced and with constant heights. Further normal assumptions are that water is incompressible and irrotational fluid. Under such assumptions the so-called velocity potential (f) should be determined because it allows the derivation of all the desired wave characteristics. The simplest velocity potential is that obtained assuming small amplitude waves or very small wave slope since linear theory can be applied. In this paper a linear theory for a flap wave maker is considered. Once the velocity potential is known the transfer function and hydrodynamic reaction can be calculated. Composing the latter force with the inertial term and moment due to the weight of the flap the total force on the arm can be determined.

3.1. Velocity potential

Fig. 2 depicts the scheme of the wave generator and shows the co-ordinate system as well. The origin of the co-ordinate system coincides with the intersection of the vertical center line of the paddle at rest (zero stroke) with still water level. The horizontal axis x is positive in the direction of the wave propagation and z, vertical co-ordinate, is positive upward. It must be pointed out here that in this paper only the key point of the theory that is based on Dean and Dalrymple (1991) is presented.

The governing equation for the velocity potential is the bidimensional Laplace equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2} = 0 \qquad (1)$$

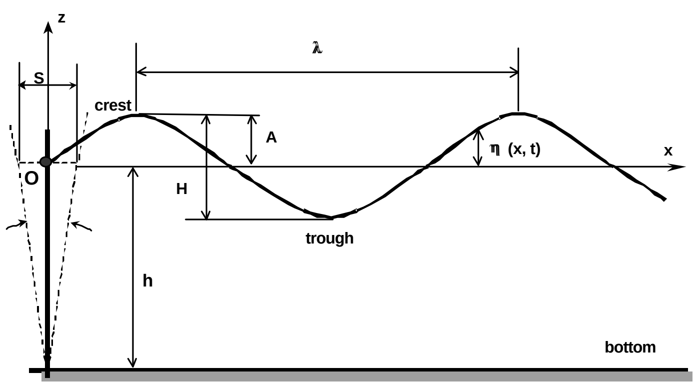

Figure 2 Simplified scheme of the wave generator

In the case of the tank Eq. (1) has to satisfy the following boundary conditions:

a) $\vec{u}.\vec{n}=0$ on $z=-h$ (2)

where $\vec{u}$ is the water velocity vector and $\vec{n}$ is the unit normal vector and h is the water depth. This boundary condition means that the bottom of the tank is impermeable and as it is horizontal the Eq. (2) results:

$$w=-\frac{\partial f}{\partial z}=0 \text{ on } z=-h$$

where w is the vertical velocity of the water.

b) $\vec{u}.\vec{n}=\frac{\partial h / \partial t}{\sqrt{\left(\partial h / \partial x\right)^2+1}}$ on $z=h(x,t)$ (3)

where h is the wave height and t is the time. This boundary condition indicates that the surface water velocity should coincide with the velocity of the wave height. Assuming that wave height is very small it is equivalent to:

$$-\left[\frac{\partial f}{\partial z}\right]_{z=0}=\frac{\partial h}{\partial t} \quad (4)$$

c) $-\frac{\partial f}{\partial t}+\frac{1}{2}\left[\left(-\frac{\partial f}{\partial x}\right)^2+\left(-\frac{\partial f}{\partial z}\right)^2\right]+gz=C(t)$ on $z=h(x,t)$ (5)

where g is the gravity acceleration.

This is the Bernoulli equation on the free surface assuming gauge atmospheric pressure. Expanding this equation at $z=0$ by Taylor series and taking into account linear terms only and bearing in mind that small wave amplitude is under consideration the above boundary conditions can be expressed as:

$$h=\frac{1}{g}\left[\frac{\partial f}{\partial t}\right]_{z=0} \quad (6)$$

d) $x=\frac{S(z)}{2}sin(wt)$ on the paddle. (7)

where S and w are the wavemaker stroke and frequency respectively. The above boundary condition imposes on the water the same displacement of the paddle.

A general solution to the Laplace equation that satisfies all given boundary conditions is not easy to be determined. In this paper the following equation proposed by Dean and Dalrymple (1991), assuming the flap bearing is at the bottom of the tank, is adopted.

$$f = A_p \cosh[k_p(h+z)]\sin(k_p x-wt)+\sum_{n=1}^{\infty}C_n e^{-k_s(n)x}\cos[(k_s(n)(h+z)]\cos wt \quad (8)$$

$$A_p=-\frac{-\int_{-h}^{0}\frac{S(z)}{2}w\cosh[k_p(h+z)]dz}{k_p\int_{-h}^{0}\cosh^2[k_p(h+z)]dz} \quad (9)$$

$$C_m = \frac{\int_{-h}^{0} \frac{S(z)}{2} w \cos[k_s(m)(h+z)]\,dz}{k_s(m) \int_{-h}^{0} \cos^2[k_s(m)(h+z)]\,dz} \tag{10}$$

The parameters k_p and k_s, the so-called wave numbers for progressive and standing waves, respectively, are determined carrying out the following equations:

$$w^2 = gk_p \tanh(k_p h) \tag{11}$$

$$w^2 = -gk_s \tan(k_s h) \tag{12}$$

In opposition to equation (11) that has one solution only, there are an infinite number of solutions to equation (12) and each solution is denoted as $k_s(n)$.

The first term of equation (8) represents a progressive waves while the series are related to standing waves which decay away from the wavemaker. In practical terms C_n is negligible two or three water depths away from the actuators. This gives us a clue about positioning the wave probe if the main interest are the progressive waves.

In this paper the functional form of $S(z)$ is assumed as:

$$S(z) = S_0\left(1 + \frac{z}{h}\right) \tag{13}$$

where S_0 is the total stroke of the flap in the still level of the water.

Taking into account this assumption the parameters A_p and C_m yield:

$$A_p = \frac{S_0 w}{2} \frac{1}{k_p^2} \frac{\sinh(k_p h) + \frac{1}{hk_p}\left[1 - \cosh(k_p h)\right]}{\frac{\sinh(2k_p h)}{4k_p} + \frac{h}{2}}$$

$$C_m = \frac{S_0 w}{2k_s^2(m)} \frac{\frac{\sin[k_s(m)h]}{k_s(m)} + \frac{1}{hk_s(m)}\left[\cos[k_s(m)h] - 1\right]}{\frac{\sin\left[2k_s(m)h\right]}{4k_s(m)} + \frac{h}{2}}$$

3.2 Transfer function

The transfer function can be found by evaluating the wave height far from the wavemaker where there are progressive waves only. Substituting the velocity potential in Eq. (6) yields:

$$h = -\frac{A_p}{g} w \cosh(k_p h)\cos(k_p x - wt) \tag{14}$$

The desired waves are given by:

$$h = \frac{H}{2}\cos(k_p x - wt) \tag{15}$$

where H is the wave height.

Combining Eq. (14) and Eq.(15) and substituting for A_p gives:

$$\frac{H}{S} = 4\frac{\sinh(k_p h)}{k_p h} \frac{k_p h \sinh(k_p h) - \cosh(k_p h) + 1}{\sinh(2k_p h) + 2k_p h} \tag{16}$$

where S is the stroke of flap at the water level.

This is theoretical transfer function that is compared with that obtained experimentally in the following.

3.3 Driving force

The driving force is due to hydrodynamic reaction and inertial term and the moment due to the weight of the flap.. The hydrodynamic reaction on the flap is the total moment induced by the water. Assuming that the flap is a flat plate and the back side of the flap generates the wave as the front side does but in the opposite direction, the moment about the bearing of the flap can be calculated as:

$$M_h = \int_A (h+z)\,p\,dA = \int_{-h}^{0} (h+z)\,p_{front}\,L\,dz - \int_{-h}^{0} (h+z)\,p_{back}\,L\,dz \tag{17}$$

where A is the total area, p_{front} and p_{back} are the frontal and back side pressure of the flap respectively and L .is the flap width.

From Bernoulli equation yields:

$$p = r\left[\frac{\partial f}{\partial t}\right]_{x=0} - \frac{1}{2}\left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right] - r\,g\,z \tag{18}$$

If Eq. (18) is substituted in Eq.(19) the terms related to kinetic energy of the fluid and static pressure (second and third term of the right side of Eq. (18) respectively) are canceled. Therefore considering only the remaining term the moment is given by:

$$M_h = 2L\int_{-h}^{0} (h+z)\,r\left[\frac{\partial f}{\partial t}\right]_{x=0} dz \tag{19}$$

Performing the integral calculation taking into account Eq.(17) results:

$$M_h = a_1\cos(wt) + a_2\sin(wt) \tag{20}$$

$$a_1 = -\frac{2\,w\,L\,rA_p}{k_p}\left(h\sin(k_p h) + \frac{1-\cosh(k_p h)}{k_p}\right)$$

$$a_2 = -2w\,L\,r\sum_{m=1}^{m_f} C_m(m)\frac{1}{k_s(m)}\left(h\sin[k_s(m)h] + \frac{\cos[k_s(m)h]-1}{k_s(m)}\right)$$

The inertial term depends on the angular acceleration of the flap . Lets assume that the angular motion of the flap can be described by:

$$q = q_0 \sin wt$$

Therefore the inertial moment is:

$$M_i = I\frac{d^2q}{dt^2} = Iw^2\sin wt \tag{21}$$

where I is the moment of inertia of the flap about its longitudinal axis at the hinge of the flap.

The moment due to weight is given by:

$$M_w = W\,x_g \tag{22}$$

where W is the weight of the flap and x_g is the distance between gravity center of the flap and its longitudinal axis. In fact the flap is not a homogenous flat plate but asymmetrical one since an arrangement of girders forming the framework is at the back. Therefore the x_g is not null even for the plate in its vertical position. A simple way to describe x_g during the flap motion is:

$$x_g = x_{g0} + h_g q_0 sin(w\,t) \tag{23}$$

where x_{g0} is the distance of the gravity center flap for $q = 0$ and h_g is the height of the gravity center.

The total force necessary to drive the flap is then:

$$F = \frac{M_h + M_i + M_w}{B} \tag{24}$$

where B is the height of the flap.

4 Experimental results and analyses

As pointed out previously, the main purpose of this paper is to present and evaluate experimental results in comparison with those predicted theoretically. These analyses are performed in two ways: the first is through transfer function and the second by curves of the driven force against frequency.

4.1 Transfer function

Figure (3) shows the transfer function obtained from both theoretical and experimental approaches for nominal 1.5 m water level. Details of the numerical values adopted in this paper are shown in the Appendix A. The theoretical transfer function was performed through Eq. (16). However as there are inevitable measurement errors all experimental results inside a band of $\pm 3\%$ around the theoretical curve were considered as acceptable. The experimental results were split into three groups according to the wave slope: the first group is up to 4%, the second one between 4% and 6% and third group higher than 6%. Figure (3) exhibits a good fit between theoretical and experimental results for waves of the first group up to 1.7 Hz. This result was expected since the lower the wave slope is the closer the wave dynamics is to the theory of this paper. The second wave group comes within the band up to 1.3 Hz and the third wave group always falls outside the band. However after 1.7 Hz discrepancies occur with the waves of the first group and experimental results are farther from the band than second or third group waves. An attempt to explain it is the spurious waves generated at the back side of the flap coming through the gap between the flap itself and the wall to the front side of the tank. The gap is 0.02 m wide but enough to allow the interference in the main waves specially in the case of high frequency waves in which the amplitude is small. However this phenomenon demands further analysis.

To complete the analysis of the transfer function results obtained for 0.85 m water level are shown in the Fig. (4). The transfer function defined by Eq. (16) is not linear and thus it seems cautious to check it under a different condition. The results are presented in Fig. (4) without splitting the wave slope in groups. It shows a reasonable agreement between theoretical and experimental results but up to 1.2 Hz.

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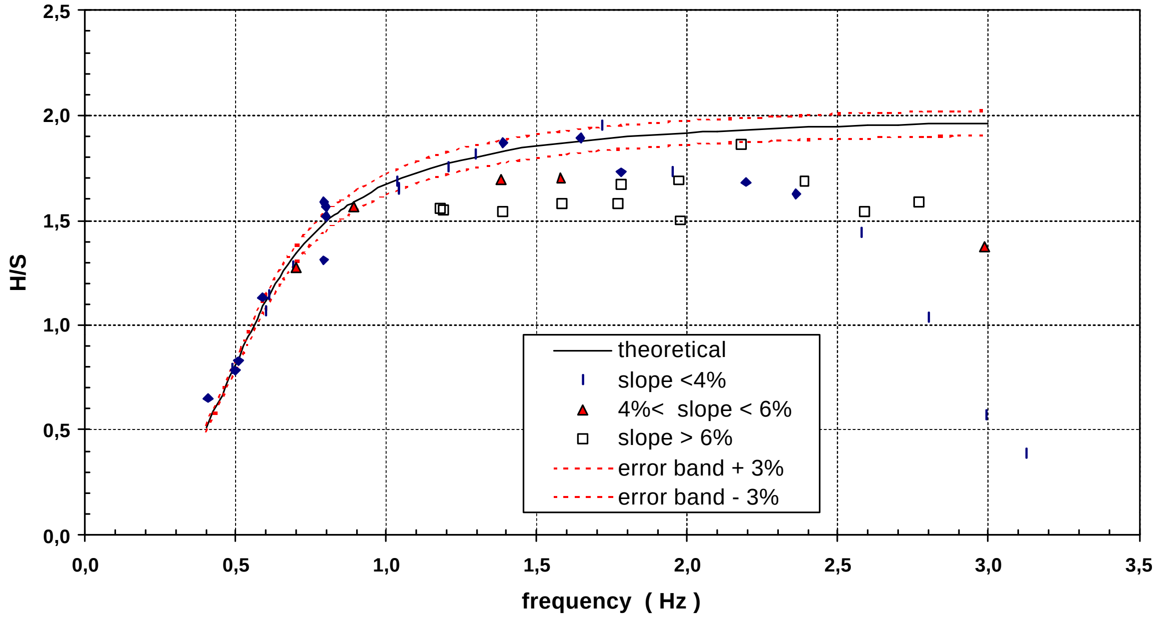

Figure 3 Theoretical and Experimental Transfer Functions (1.5m water depth)

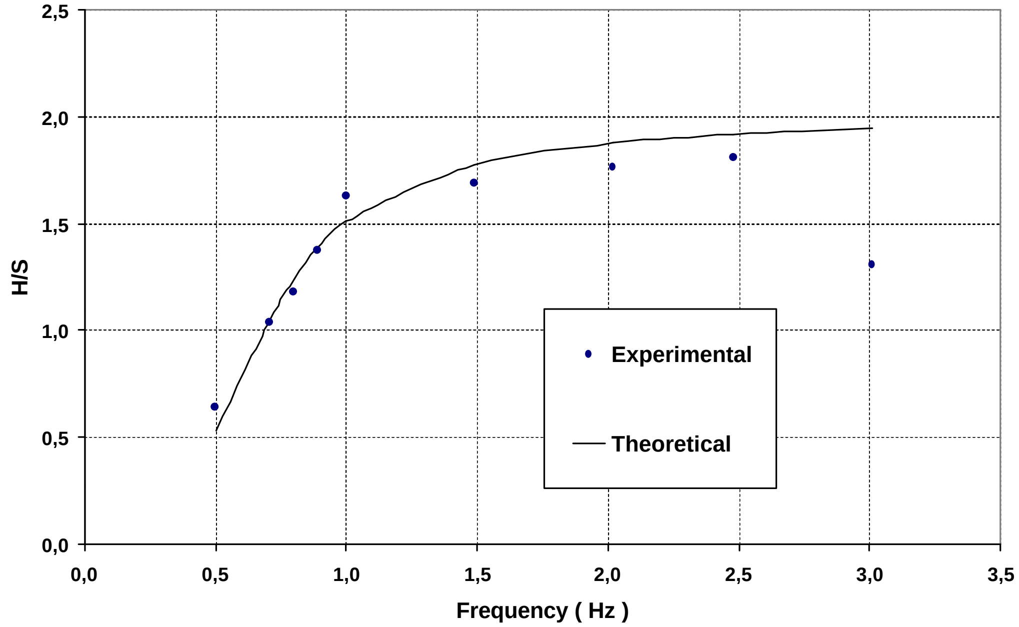

Fig. 4 Theoretical and Experimental Transfer Functions (1.0 m water depth)

4.2 Driving force

Once the theoretical driving forces have been calculated with respect to the frequency through Eq. (24) it is compared with data obtained from experimental measurements. Perhaps a few comments should be made here concerning the kinematics of the wave generator before analysing the results themselves. In fact the experiments have been performed imposing a sinusoid signal on the electric motor revolution. However due to the geometry of the mechanism of the wave generator the stroke motion of the flap is near but not precisely a sinusoid. Besides, as the gravity center are not null for $q = 0$, a static force is necessary to hold the flap in the vertical position which was taken as initial condition. During the experiments this initial force was set to zero. As a consequence a periodic signal with non zero average was recorded as shown in Fig. (5). The “force” considered in this paper is the difference between maximum and minimum forces recorded for each frequency. Here it must be pointed out that the main difference between theoretical and experimental approach is that former has assumed a sinusoid motion of the flap itself instead of the motor revolution.

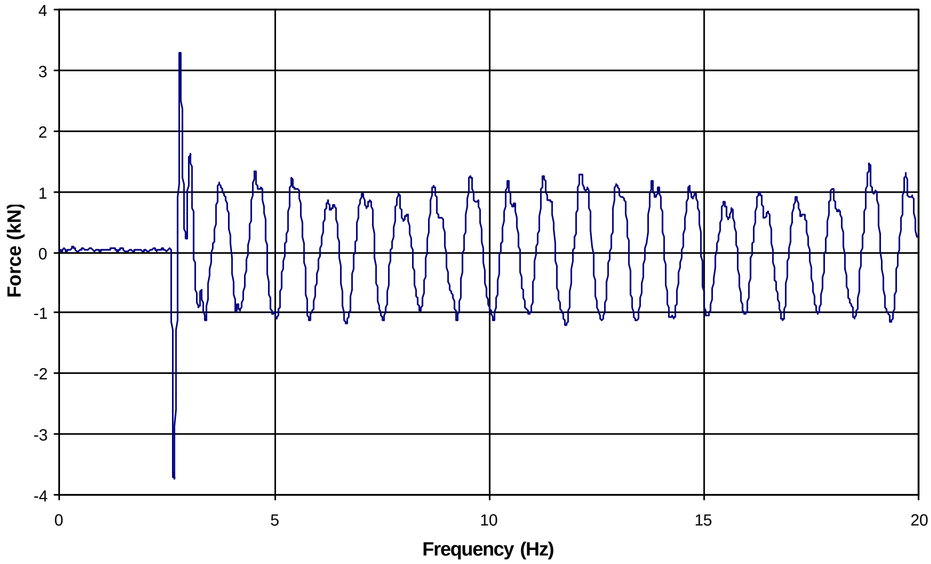

Figure 5 Recorded driving force signal

The results of the force displayed in Fig (6) should be analysed bearing in mind the above comments. This figure shows the ratio of the driving force to the stroke against frequency obtained from both theoretical and experimental approaches considering 1.5 m water level. In broad terms the theoretical curve of the force rises with respect to frequency but exhibits a local maximum at the beginning. The experimental results seem to capture the shape of theoretical curve and there is a good adjustment between them in the range of 0.5 Hz to 2.5 Hz mainly. If the result for 0.8 Hz is omitted the difference between theoretical and experimental results is less than 10%. This result is promising since the friction of the mechanism and viscosity of the water were ignored and only the linear term of velocity potential was taken into account to predict the driving force. It must be emphasized here that the experimental results shown in Fig.(6) have been presented without distinguishing the slope of the waves. For frequencies higher than 2.5 Hz the experimental results distance themselves from the theoretical curve although the rise with respect to frequency is maintained. So far the total forces have been analysed but it could be useful to check the relative importance of each term that affects the total driving force. Specially, the force due to hydrodynamic effect deserves to be verified since it demands more complex calculations than other terms. In order to do so two more curves are added to Fig. (6) both obtained theoretically. One of them is the force due to hydrodynamic effect and the second curve is related to the inertial and weight moment terms. Those curves disclose that the hydrodynamic component is clearly the most important term that needs to be counteracted by the driving system

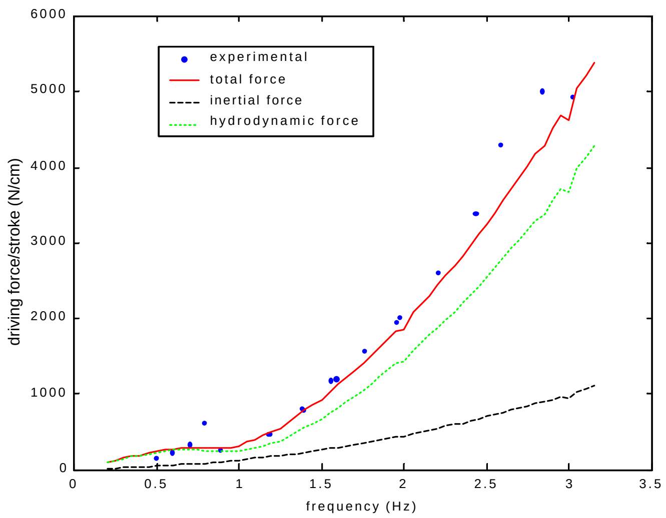

Fig. 6 Theoretical and Experimental driving forces with respect to frequency

5 Conclusions

The performance of a flap type wave generator was evaluated experimentally through the so-called transfer function having as reference an ideal curve obtained from linear velocity potential. The results have disclosed a very good fit between theoretical and experimental approaches but only for waves with slopes up to 4% and frequencies up to 1.7 Hz. Further analysis is necessary to find out the reason of the discrepancies between the theory and the experiment for frequencies higher than 1.7 Hz.

The main force required for the driving force to counteract is that derived from hydrodynamic effect. This force was modeled taking into account linear velocity potential and adding it to terms due to inertia and weight moment the total driving force was obtained. The results have shown again that linear theory can predict driving forces with reasonable accuracy and it is very useful to determine the power to be installed in the wave generator.

6. References

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Appendix A

DATA RELATED TO THE FLAP

Weight (N)	3754,0
Moment of inertia (kg.m^2)	768,0
Inital horizontal gravity center coordinatet (m)	0.05
Veritical gravity center coordinate(m)	1.30
Height (m)	2.05
Width (m)	4.80