  

# PRESSURE DROP FOR FLOW OF POWER LAW FLUIDS INSIDE ELLIPTICAL DUCTS

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***Abstract.*** *The hydrodynamic analysis for non-Newtonian flow inside straight ducts with elliptical cross-section was performed in this work for low Reynolds number. Power law fluids were considered since the most known non-Newtonian ones falls within this model. Simplified hypotheses for the fluid flow were established by adopting an adequate coordinate system, allowing to obtain a close form solution for the momentum equation. The flow field and physical parameter of interest such as average velocity and Fanning friction factor were then determined and presented as function of the behavior index and elliptical tube aspect ratio.*

***Keywords****. power law, non-Newtonian, pressure drop, elliptical duct, fluid flow, friction factor*

## 1. Introduction

Situations that involve processing non-Newtonian fluids characterize research field that has many applications mainly in chemical, pharmaceutical, food processing and petrochemical industries. Thus, rheological research that deals with the relation between shear stress and gradient of the velocity field are of fundamental importance for fluid characterization and its flow behavior. However, to obtain the solution for momentum and energy equations it is necessary to get several physical parameters of interest including those ones related to the design of thermo-hydraulics equipment processing fluid of such nature. In this way, the research concerned to the internal flow of non-Newtonian fluids has many technological applications. Along last decades, several efforts have been developed in this research field, among several we can cite Hartnett and Kostic (1989), and the works of Richardson (1979), Cho and Hartnett (1982), Cotta and Özisik (1986), Pinho and Whitelaw (1990), Billingham and Ferguson (1993), Quaresma and Macêdo (1998). It is worth noticing that the most of that work deal with the flow between parallel plates, or ducts with circular or square cross-section. Few works are dedicated to study the flow inside ducts with more complex cross-section, among some we cite the works of Etemad, Mujundur and Nassef (1996), Wachs, Clermont and Normandin (1999), and Chaves *et al.* (2001). Thus, giving continuity to this research field, this work deals with the hydrodynamic analysis involved in the flow at low Reynolds number of power law fluids inside straight ducts with elliptical cross-section. Due to flow characteristics, mentioned above, it is hypothesized that secondary flow is negligible, Tanner (2000), and that using an adequate non-orthogonal coordinate system it was possible to impose simplifying conditions in the whole flow as well to obtain a closed form solution for the velocity field. Parameters such as average velocity and Fanning friction factor were obtained as a function of the behavior index the cross-section aspect ratio.

## 2 Analysis

Non-Newtonian power law fluids present the following relation between the shear stress and the deformation rate:

$$\tau = K[\frac{dw(x,y)}{d\eta}]^{n} \quad , \tag{1}$$

where n is the behavior index of the flow, K is the consistency index and $\eta$ is an auxiliary coordinate axis orthogonal to the flow plane. That plane is a surface generated by the equation $w(x,y) = constant$. Equation (1) can be written as follows:

$$\tau = K\left|\frac{dw}{d\eta}\right|^{n-1}[\frac{dw}{d\eta}] = \mu_a \frac{dw}{d\eta} \quad , \tag{2a}$$

$$\mu_a = K\left|\frac{dw}{d\eta}\right|^{n-1} , \tag{2b}$$

where $\mu_a$ is called apparent viscosity. For pseudoplastic, $n < 1$, the apparent viscosity decreases with the increase of the shear stress. For dilatant fluids, $n > 1$, apparent viscosity increases as the shear stress increases.

To determine the velocity profile for the power law fluid flow inside straight ducts with elliptical cross-section it is assumed that the flow is laminar and hydrodynamically developed. To establish the correspondent momentum equation consider the following coordinate system:

$$x = v\, cos(u) , \tag{3a}$$

$$y = \rho_{asp}\, v\, sin(u) , \tag{3b}$$

$$\rho_{asp} = \beta/\alpha \quad , \quad \{\ \beta < \alpha\ \} \tag{3c}$$

where the ratio between the elliptical semi-axis, $\beta/\alpha$, defines the aspect ratio, $\rho_{asp}$ of the elliptical duct cross-section. When the aspect ratio $\rho_{asp} = 1$, the coordinate system transformation given by Equation (3a-c) generates the cylindrical coordinate system. In the general case, when the aspect ratio $\rho_{asp} < 1$, the coordinate system $(u,v)$ is not orthogonal, and the curves generated for $v = constant$ looks like similar ellipsis, concentric at $(0,0)$ as shown in Figure 1.

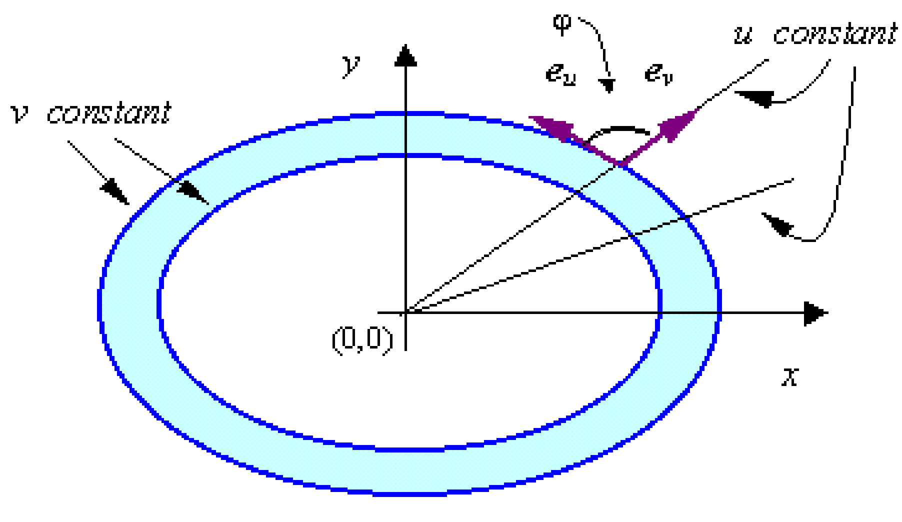


Figure 1. Non-orthogonal coordinate system $(u,v)$.

Metric coefficients $h_u(u,v)$ and $h_v(u,v)$, and the unitary base vectors $\boldsymbol{e}_u$ and $\boldsymbol{e}_v$ for the new coordinate system are given by

$$\frac{d\mathbf{r}}{du} = h_u(u,v)\, e_u = -v\, sin(u)\, \hat{\boldsymbol{i}} + \rho_{asp}\, v\, cos(u)\, \hat{\boldsymbol{j}} , \tag{4a}$$

$$\frac{d\mathbf{r}}{dv} = h_v(u,v)\, e_v = cos(u)\, \hat{\boldsymbol{i}} + \rho_{asp}\, sin(u)\, \hat{\boldsymbol{j}} , \tag{4b}$$

$$h_u(u,v) = \left|\frac{d\mathbf{r}}{du}\right| = v\sqrt{sin^2(u) + \rho_{asp}{}^2 cos^2(u)} \ , \tag{5a}$$

$$h_v(u,v) = \left|\frac{d\mathbf{r}}{dv}\right| = \sqrt{cos^2(u) + \rho_{asp}{}^2 sin^2(u)} \ , \tag{5b}$$

$$\boldsymbol{e}_u = v\,[-sin(u)\,\hat{\boldsymbol{i}} + \rho_{asp} cos(u)\,\hat{\boldsymbol{j}}] \,/\, h_u(u,v) \ , \tag{6a}$$

$$\boldsymbol{e}_v = [cos(u)\,\hat{\boldsymbol{i}} + \rho_{asp}\, sin(u)\,\hat{\boldsymbol{j}}] \,/\, h_v(u,v) \ . \tag{6b}$$

Due to the geometrical characteristics of ellipsis cross-section it is assumed that fluid velocity changes with the $u$ axis is negligible (Ahmeda, Normandin and Clermont, 1995). Therefore, $w = w(v)$. Defining an axis $\eta$ orthogonal to the curve $w = w(v)$ and pointing towards the wall of the duct, then the constitutive equation for the shear stress, Equation (1), can be written as

$$\tau = K\,[\frac{dw(v)}{d\eta}]^n \ . \tag{7}$$

As the coordinate system $(u,v)$ is non-orthogonal, the component orthogonal to the fluid flow plane does not coincide with the unitary base vector $\boldsymbol{e}_v$. Thus, a differential $d\eta$ must be determined not forgetting the non-orthogonality of the coordinate system. In order to do so, a generic vector element $d\boldsymbol{r}$ is projected into a unitary base vector $\boldsymbol{e}_\eta$ orthogonal do fluid flow plane. By doing so, we have

$$d\eta = d\boldsymbol{r} \cdot \boldsymbol{e}_\eta = [h_u(u,v)\,du\,\boldsymbol{e}_u + h_v(u,v)\,dv\,\boldsymbol{e}_v] \cdot \boldsymbol{e}_\eta \ , \tag{8a}$$

$$\boldsymbol{e}_\eta = \boldsymbol{e}_u \times \frac{(\boldsymbol{e}_v \times \boldsymbol{e}_u)}{sin(\varphi)} \ , \tag{8b}$$

$$sin(\varphi) = |\,\boldsymbol{e}_v \times \boldsymbol{e}_u\,| = \frac{\rho_{asp}\, v}{h_u(u,v)\, h_v(u,v)} \ , \tag{8c}$$

where $\varphi$ is the angle between the unitary base vectors $\boldsymbol{e}_u$ and $\boldsymbol{e}_v$. As $\boldsymbol{e}_u \perp \boldsymbol{e}_\eta$, the differential $d\eta$ is then determined and given by

$$d\eta = h_v(u,v)\,dv\ (\boldsymbol{e}_v \cdot \boldsymbol{e}_n) , \tag{9a}$$

$$(\boldsymbol{e}_v \cdot \boldsymbol{e}_n) = \left|\boldsymbol{e}_v \times \boldsymbol{e}_u\right| = sin(\varphi) , \tag{9b}$$

$$d\eta = h_v(u,v)\, sin(\varphi)\, dv = \rho_{asp}\, v\, dv \,/\, h_u(u,v) \ . \tag{9c}$$

Therefore, an equation for the shear stress in the new coordinate system is written as

$$\tau = K\,[-\frac{h_u}{\rho_{asp}\, v}\,\frac{dw(v)}{dv}]^n \ . \tag{10}$$

Pressure forces and shear stress forces acting on an elliptical ring element (shown in Figure 2) with thickness $d\eta$ and length $dz$, can be formulated as follow

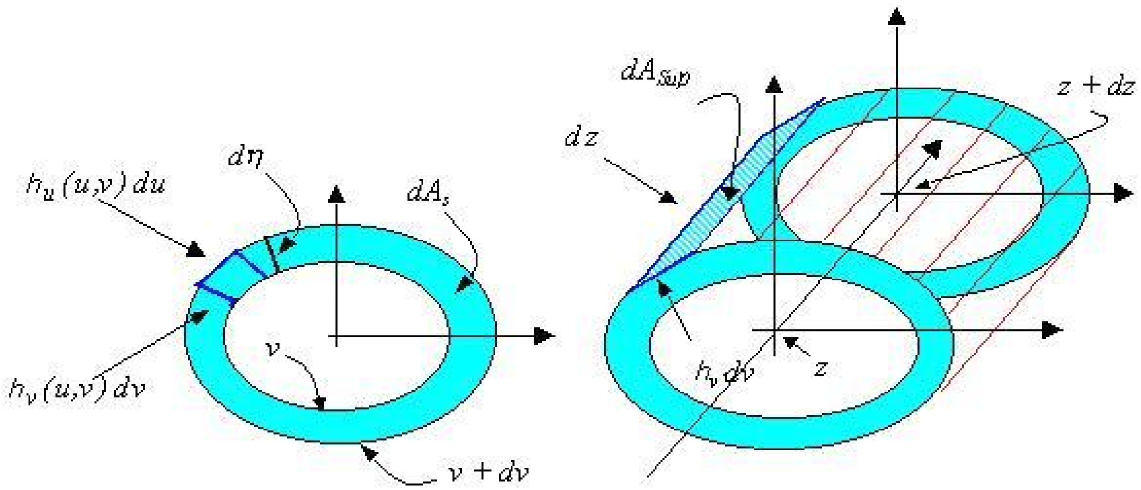


Figure 2. – Visualization of elliptical and axial differential elements having areas $dA_S$ and $dA_{Sup}$ in order to determine pressure and shear stress forces.

$$dF_P(z) = P(z)\,dA_s\ , \qquad dA_s = \int_0^{2\pi}[\,h_u(u,v)\,h_v(u,v)\,sin(\varphi)\,du\,]\,d\,v\ , \tag{11a}$$

$$dF_P(z+dz) = P(z+dz)\,dA_s = [P(z) + \frac{dP(z)}{dz}dz\,]\,dA_s\ , \tag{11b}$$

$$dF_{Visc}(v) = \int_{A_{sup}} \tau(u,v)\,d\,A_{sup} = \int_0^{2\pi}[\tau(u,v)\,h_u(u,v)\,du\,]\,dz \tag{12a}$$

$$dF_{Visc}(v+dv) = dF_{Visc}(v) + \frac{d}{dv}\int_0^{2\pi}[\tau(u,v)\,h_u(u,v)\,du\ dz]\,dv\ . \tag{12b}$$

Substituting the expression for $sin(\varphi)$, into the metric coefficients $h_u$ and $h_v$, and into the shear stress equation $\tau$, the net result for the pressure forces acting on elliptical ring element is

$$d[F_P(z+dz) - F_P(z)] = -\int_0^{2\pi}[\,\frac{dP}{dz}\,\frac{v\,\rho_{asp}}{h_u\ h_v}\,h_u\ h_v\ d\,u\,]\,d\,v\,d\,z\ , \tag{13a}$$

$$d[F_P(z+dz) - F_P(z)] = 2\pi\,\rho_{asp}\,v\,[-\frac{dP}{dz}]\,dz\ , \tag{13b}$$

and for the shear stress forces,

$$d\,[F_{visc}(v+dv) - F_{visc}(v)] = \frac{d}{d\,v}\int_0^{2\pi}\{\kappa[-\frac{h_u}{\rho_{asp}\ v}\frac{d\,w}{dv}]^{\,n}\,h_u\,du\}\,d\,v\,d\,z\ , \tag{14a}$$

$$d\,[F_{visc}(v+dv) - F_{visc}(v)] \;=\; \kappa\, \rho_{asp}\, [-\frac{dw}{dv}]^{n}\; I(n,\rho)\; dv\, dz \;, \tag{14b}$$

with

$$I(n,\rho_{asp}) \;=\; \int_0^{2\pi} [1 + q\; sin^2(u)]^{\frac{n+1}{2}}\; du \;\;,\;\; q \;=\; \frac{1-\rho_{asp}^{\;2}}{\rho_{asp}^{\;2}} > 0\;. \tag{15}$$

From the equilibrium between the viscous and pressure forces, we have

$$2\pi \frac{dP}{dz} \rho_{asp}\; v \;=\; \kappa \rho [-\frac{dw}{dv}]^{n}\; I(n, \rho_{asp})\;. \tag{16}$$

By doing some changes in the above equation and also considering that the maximum velocity occurs at the centerline of the flow, we obtain the following relation for the flow field inside the duct elliptical cross-section

$$w(v) \;=\; w_{max}\, \{1 - [\,(\frac{v}{\alpha})^2\,]^{\frac{n+1}{2n}}\,\} \;=\; w_{max}\, \{1 - [(\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2})^2\,]^{\frac{n+1}{2n}}\,\} \tag{17a}$$

$$w_{\max} \;=\; [\frac{n}{n+1}][-\frac{dP}{dz}\frac{2\pi}{\kappa I(n,\rho_{asp})}]^{\frac{1}{n}}\; \alpha^{\frac{n+1}{n}} \tag{17b}$$

The average velocity is obtained as follows

$$w_m \;=\; \frac{1}{A_s} \int_{A_s} w(x,y)\, dA \;=\; [\frac{n+1}{3n+1}] w_{\max}\;. \tag{18}$$

Thus, the velocity distribution can also be written in terms of the average velocity, yielding

$$w(x,y) \;=\; [\frac{3n+1}{n+1}]\, \{1 - [\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2}\,]^{\frac{n+1}{2n}}\,\}\; w_m\;. \tag{19}$$

**3. Friction coefficient**

Fanning friction coefficient is defined as

$$f \;=\; \frac{\tau_{p,m}}{\rho\; w_m\;/\;2}\;. \tag{20}$$

The average shear stress at duct wall $\tau_{p,m}$, appearing in the definition of the Fanning friction coefficient, equation above, is defined and determined by

$$\tau_{p,m} \;=\; \frac{dF_{visc}\big|_{v=\alpha}}{P\;dz} \;=\; \frac{(\int_0^{2\pi} \kappa\, [-\frac{h_u}{\rho_{asp}\; v}\frac{dw}{dv}]^{n}\Big|_{v=\alpha} h_u\; du)\; dz}{P\;dz}\;. \tag{21}$$

From the results obtained for the metric coefficient $h_u$ and from the velocity profile $w(v)$, Equations (5a and 17a), the Fanning friction factor, *f,* is then obtained,

$$f = \frac{2K}{\rho\, w_m^{2-n}} \left[\frac{3n+1}{n}\right]^n \frac{\rho_{asp}\; I(n,\rho_{asp})}{\alpha^{n-1}\; P}, \tag{22}$$

with $I(n,\rho_{asp})$ given by Equation (15).

The above equation is written again, now introducing the Reynolds number, *Re*, and the hydraulic diameter, $D_h$,

$$f\,Re = 16\; \frac{I(n,\rho_{asp})}{2\pi} \left[\frac{D_h}{2\alpha}\right]^{n+1}, \tag{23}$$

where the corresponding hydraulic diameter, $D_h$, and the Generalized Reynolds number (Skelland, 1967), *Re*, is given by

$$D_h = \frac{4 \times Cross - section\ area}{Cross - section\ perimeter} \tag{24a}$$

$$Re = \frac{\rho\, w_m^{2-n}\, D_h^{\,n}}{8^{\,n-1}\, K \left[\frac{3n+1}{4n}\right]^n}. \tag{24b}$$

When $\rho_{asp} = 1$, the factor $I(n,\rho_{asp}) = 2\pi$. Therefore, for circular ducts we have $f\,Re = 16$, independently of the behavior index, n.

## 4. Results and remarks

To determine the Fanning friction coefficient, the integral for the factor $I(n,\rho_{asp})$, given by Equation (15), was calculated by using Gauss quadrature method. The product $f\,Re$ was determined through Equation (23) for various ellipsis aspect ratio ($\rho_{asp}$ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1) and for some behavior index ($n$ = 0.1, 0.2, 0.3, 0.5, 0.7, 1, 2, 3, 4 and 5). Results obtained for the product $f\,Re$ are presented numerically in Table (1) and are visualized graphically on Fig. (3). It can be figure out that the friction factor for pseudoplastic fluids (n < 1) changes little with the aspect ratio. For dilatant fluids (n > 1) the friction factor exhibits a strong dependency to the ellipsis aspect ratio when it tends toward zero ($\rho_{asp} \to 0$). In such cases, the apparent viscosity of dilatant fluids increases significantly, due to the higher fluid velocity gradients near the wall.

In order to validate the results, values in the literature were found for the particular case of unitary the behavior index, n = 1 (Shah and London, 1978) and it can be observed in Table 2 that there is an excellent agreement with the results presented here. It is interesting to notice that for the particular case of unitary aspect ratio, $\rho_{asp} = 1$, the Equation (19) gives the same solution for the flow in circular tubes, Skelland (1967)

$$w(x,y) = \left[\frac{3n+1}{n+1}\right]\left[1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right] w_m, \tag{25}$$

where R is the circular radius. In Figures 4 to 6 present graphically the velocity profiles for fluid flow with behavior index, n = 0.1, n = 1 and n = 5, respectively, inside ducts with aspect ratio, $\rho_{asp} = 0.5$.

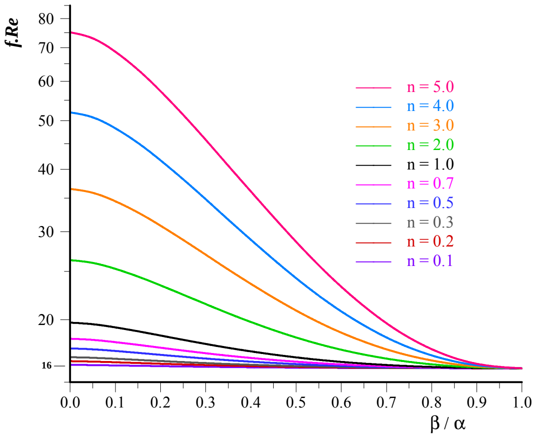


Figure 3 – Fanning friction factor $fRe$ as function of the aspect ratio, $\rho_{asp}$, and behavior index, n.

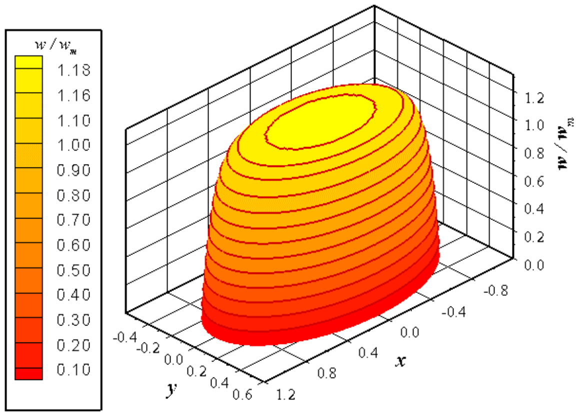


Figure 4 – Velocity profile for flow of pseudoplastic fluid with behavior index, n = 0.1, inside a duct with aspect ratio, $\rho_{asp}$ = 0.5.

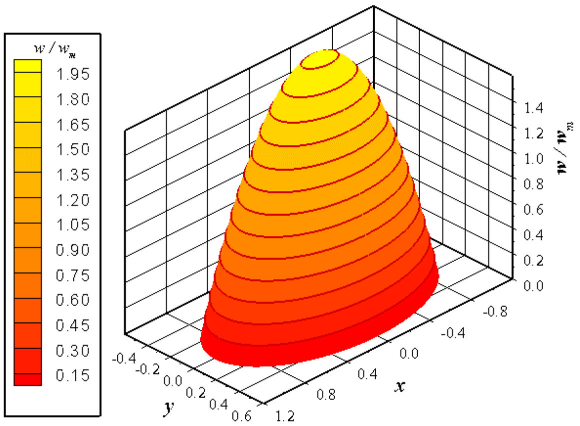


Figure 5 - Velocity profile for flow of Newtonian fluid with behavior index, n = 1, inside a duct with aspect ratio, $\rho_{asp}$ = 0.5.

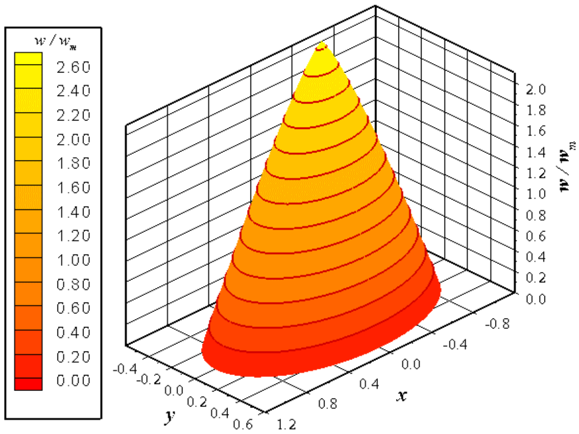


Figure 6 - Velocity profile for flow of dilatant fluid with behavior index, n = 5, inside a duct with aspect ratio, $\rho_{asp}$ = 0.5.

## 5. Final remarks

A closed form solution for the flow of non-Newtonian power law fluids inside straight ducts with elliptical cross-section was obtained in this work. The hydrodynamic parameters for the developed flow were determined. A model was established in which the planes of constant velocities in the flow approximate to the boundary of a family of similar ellipsis concentric to the duct. In this way, a new non-orthogonal coordinate system $(u,v)$ was proposed, that is able to generate elliptical curves when the variable $v$ is fixed. In this new system of coordinates the fluid flow velocity can expressed as function of only one independent variable, $V = V(v)$, simplifying substantially the formulae of the problem.

Through the viscous and pressure forces equilibrium was determined the velocity flowfield and hydrodynamic parameters such as the average flow velocity and the Fanning friction factor were determined through the viscous and pressure forces equilibrium, allowing the analysis of the flow of pseudoplastic and dilatant non-Newtonian fluids as a function of the behavior index and the aspect ratio.

## 6. Acknowledgement

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Table 1 - Parameter $f\,Re$ as function of aspect ratio, $\rho_{asp}$, and behavior index, n.

| n / $\rho_{asp}$ | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 1.0 | 2.0 | 3.0 | 5.0 |
|---|---|---|---|---|---|---|---|---|---|
| 0.999 | 16.0000 | 16.0000 | 16.0000 | 16.0000 | 16.0000 | 16.0000 | 16.0000 | 16.0000 | 16.0000 |
| 0.90 | 16.0012 | 16.0027 | 16.0043 | 16.0083 | 16.0132 | 16.0221 | 16.0663 | 16.1327 | 16.3320 |
| 0.80 | 16.0054 | 16.0118 | 16.0191 | 16.0368 | 16.0583 | 16.0979 | 16.2933 | 16.5867 | 17.4734 |
| 0.75 | 16.0089 | 16.0194 | 16.0315 | 16.0605 | 16.0959 | 16.1610 | 16.4817 | 16.9635 | 18.4268 |
| 0.70 | 16.0135 | 16.0294 | 16.0478 | 16.0918 | 16.1454 | 16.2439 | 16.7287 | 17.4575 | 19.6851 |
| 2 / 3 | 16.0173 | 16.0376 | 16.0611 | 16.1173 | 16.1857 | 16.3113 | 16.9293 | 17.8586 | 20.7138 |
| 0.60 | 16.0267 | 16.0582 | 16.0943 | 16.1808 | 16.2861 | 16.4789 | 17.4258 | 18.8515 | 23.2868 |
| 0.50 | 16.0464 | 16.1009 | 16.1634 | 16.3125 | 16.4932 | 16.8233 | 18.4368 | 20.8730 | 28.6427 |
| 0.40 | 16.0741 | 16.1607 | 16.2598 | 16.4949 | 16.7785 | 17.2941 | 19.7981 | 23.5938 | 36.0969 |
| 1 / 3 | 16.0976 | 16.2114 | 16.3411 | 16.6475 | 17.0155 | 17.6815 | 20.8991 | 25.7924 | 42.3226 |
| 0.30 | 16.1110 | 16.2401 | 16.3869 | 16.7321 | 17.1477 | 17.8960 | 21.5005 | 26.9925 | 45.7961 |
| 0.25 | 16.1330 | 16.2872 | 16.4620 | 16.8721 | 17.3611 | 18.2400 | 22.4514 | 28.8879 | 51.3872 |
| 0.20 | 16.1573 | 16.3386 | 16.5435 | 17.0216 | 17.5885 | 18.6024 | 23.4337 | 30.8424 | 57.2851 |
| 1 / 6 | 16.1745 | 16.3748 | 16.6003 | 17.1246 | 17.7439 | 18.8473 | 24.0845 | 32.1344 | 61.2545 |
| 1 / 7 | 16.1871 | 16.4011 | 16.6414 | 17.1985 | 17.8544 | 19.0199 | 24.5357 | 33.0282 | 64.0319 |
| 1 / 8 | 16.1967 | 16.4209 | 16.6722 | 17.2532 | 17.9358 | 19.1458 | 24.8608 | 33.6712 | 66.0450 |
| 1 / 9 | 16.2043 | 16.4362 | 16.6958 | 17.2950 | 17.9974 | 19.2407 | 25.1029 | 34.1491 | 67.5489 |
| 0.10 | 16.2103 | 16.4484 | 16.7144 | 17.3276 | 18.0453 | 19.3139 | 25.2881 | 34.5140 | 68.7018 |
| 1 / 16 | 16.2307 | 16.4877 | 16.7736 | 17.4291 | 18.1923 | 19.5358 | 25.8381 | 35.5944 | 72.1326 |
| 0.05 | 16.2374 | 16.4997 | 16.7913 | 17.4585 | 18.2342 | 19.5977 | 25.9880 | 35.8873 | 73.0664 |
| 0.001 | 16.2570 | 16.5319 | 16.8362 | 17.5294 | 18.3321 | 19.7391 | 26.3184 | 36.5279 | 75.1068 |

Table 2 – Fanning friction factor comparison for Newtonian fluid, $f\,Re$.

| $\rho_{asp}$ | 0.999 | 0.90 | 0.80 | 0.75 | 0.70 | 2 / 3 | 0.60 | 0.50 | 0.40 |
|---|---|---|---|---|---|---|---|---|---|
| $f$ Re | 16.0000 | 16.0221 | 16.0979 | 16.1610 | 16.2439 | 16.3113 | 16.4789 | 16.8233 | 17.2941 |
| $f$ Re$^{(*)}$ | 16.000 | 16.022 | 16.098 | 16.161 | 16.244 | 16.311 | 16.479 | 16.823 | 17.294 |

Table 2 (continuation).

| $\rho_{asp}$ | 1 / 3 | 0.30 | 0.25 | 0.20 | 1 / 6 | 1 / 8 | 0.10 | 0.05 | 0.001 |
|---|---|---|---|---|---|---|---|---|---|
| $f$ Re | 17.6815 | 17.8960 | 18.2400 | 18.6024 | 18.8473 | 19.1458 | 19.3139 | 19.5977 | 19.7391 |
| $f$ Re$^{(*)}$ | 17.681 | 17.896 | 18.240 | 18.602 | 18.847 | 19.146 | 19.314 | 19.598 | 19.739 |

$^{(*)}$ Results from Shah and London (1978).

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