DAMAGE IDENTIFICATION IN SHELL STRUCTURES USING NATURAL FREQUENCIES

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Abstract. *This paper presents structural damage identification method applied to shell structures. In this case, the location and extend structural damage at plate and shell structures can be correctly determined with a limited set of measurement, and no knowledge of modal shapes of the damaged structure is required. The hierarchical finite element based on the p-version concept for the analysis of shells is used to calculate the characteristic equations of original and damaged structures, then, a set of equations is generated. The Gauss–Newton least-squares technique is utilized to determine structural damage from the set of equations. Finally, numerical examples are used to demonstrate the effectiveness of method..*

***Keywords**. damage identification, natural frequencies, finite element method, p-version, shell.*

1. Introduction

Structural damage can be detected by a number of non-destructive techniques, such as acoustic emission, ultrasonic, thermograph and modal testing. As any changes of stiffness, whether local or distributed, result in changes of modal parameters, such as natural frequencies, mode shapes, etc., the location and the severity of damage in structure can be determined from the changes of modal characteristics. Furthermore, since the natural frequencies can be effectively determined by measuring at only one point of the structure and are independent of the position chosen, the method based on the measurement of natural frequencies is potentially very attractive.

Modal parameters are used as the basis for damage detection in a number of studies (Chen *et al*., 1996, Pandey *et al*., 1995 and Topole *et al*., 1995.). However, a common feature of these studies is that the mode shapes for damaged structure are required. Hence, these methods are to a large extent limited to structures in which the mode shapes are relatively easy to measure, such as pin-joint trusses. In order to avoid the difficulty of identifying mode shapes, several methods only using natural frequencies have been proposed. Cawley *et al*. (1979) presented a method for estimating the location of defects in structures from measurements of changes in natural frequencies. This method can be used to detect and locate damage in structures, however it cannot correctly quantify damage in general cases. Lallement (1988) proposed techniques for localizing modeling error or structural damage based on the sensitivities of eigensolutions. These localization techniques are capable of representing model-structure much larger distances in the case of spectra with separated frequencies. Link (1990) utilized the sensitivity matrix of the eigenvalues to calculate initial adjustment parameters that are used for correcting the initial model. The iteration procedure is initiated by solving the eigenvalue problem again using the corrected model. Assuming that the mode shapes for the damaged structure can be obtained analytically, and using the sensitivity relations, they obtained a system of equations related to natural frequencies. As a continuum model is used, this method cannot be used for determining a specific position of a damaged member. Moreover, To *et al*. (1991) present a procedure for determining the revised modal parameters through a non-linear sensitivity analysis. The procedure is based on the assumption about the eigenvectors of the modified structure and it employs the stationary property of the Rayleigh quotient to determine the modified eigenvalues and eigenvectors.

Bicanic *et al*. (1997) developed a formulation for damage prediction using only the changes of natural frequencies. It is shown that the method described cannot only predict the location of damage but can also determine the amount of damage from a limited number of natural frequencies. Although the use of the proposed technique is illustrated for framed structures, the same concept can be extended for various types of structures. Firstly, a characteristic equation related to relative change in stiffness is developed. Secondly, two generalized equations related to the element scalar damage indicators and the changes in mode shapes are derived. Thirdly, two computational procedures, the direct iteration (DI) technique and the Gauss–Newton least-squares (GNLS) technique, are developed to solve for the element scalar damage indicators as primary unknowns.

Paschoalini (2001) presented a subparametric hierarchical finite element based on the *p*-version concept for the analysis of plates and shells. The first level of approximation for the solution is obtained through the isoparametric quadrilateral quadratic nine-node Lagrangian shell finite element, based on the degeneration of three-dimensional solid element and the Reissner-Mindlin's formulation, with consistent numerical integration. For other approximation levels, successive hierarchical refinements are used, objectifying to remove the characteristic of excessive rigidity of the isoparametric element in the analysis of thin plates and shells.

The purpose of this work is a damage prediction using only the changes of natural frequencies for plate and shell structures considering the formulation developed for Bicanic *et al*. (1997) and the finite element presented for

Paschoalini (2001). Numerical examples are used to demonstrate the effectiveness of the method in the analysis of plates and shells.

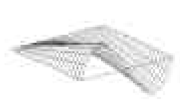
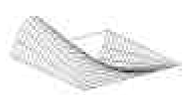
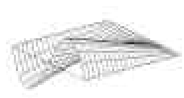
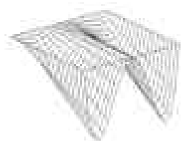
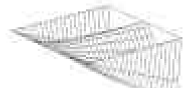
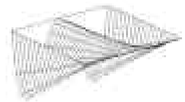
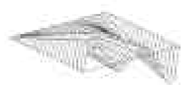
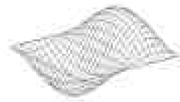
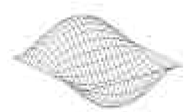
2. Hierarchical finite element

The displacement field of the shell element is interpolated through $N_i(\xi,\eta)$ polynomial quadratic shape functions of the lagrangian family given by:

$$\vec{\Delta}(x,h,z)=\sum_{i=1}^{n} N_i(x,h)\vec{d}_i + z\cdot\sum_{i=1}^{n} N_i(x,h)\cdot\frac{t_i}{2}\vec{v}_{1i}\cdot a_i - z\cdot\sum_{i=1}^{n} N_i(x,h)\cdot\frac{t_i}{2}\vec{v}_{2i}\cdot b_i \tag{1}$$

The quadratic expansion specified by the Eq. (1) can be refined by introducing the hierarchical shape functions $M_{pk}(\xi\eta)$ of superior order for two (Babuska *et al.*, 1981). The functions $M_{pk}(\xi\eta)$ are polynomials of degree p associated to each one on the edges of the element (k = 1, 2, 3 and 4) or polynomials of degree p, of the type bubble, associated to the element (k = 5, 6, 7,...). In this work, the quadratic expansion are refined by introducing hierarchical shape functions of third, fourth and fifth degrees. The shape functions presented by Szabo *et al.* (1991), based on integrals of Legendre polynomials, are used in this work. The hierarchical shape functions are listed in Tab. (1).

Table 1. Hierarchical shape functions of third (p=3), fourth (p=4) and fifth (p=5) degrees.

	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
p = 3						
p = 4						
p = 5						

Therefore, the displacement $\vec{\Delta}$, given by Eq. (1) for the isoparametric element, can be rewritten as

$$\vec{\Delta}(x,h,z)=\sum_{i=1}^{9} N_i(x,h)\,\vec{d}_i + z\cdot\sum_{i=1}^{9} N_i(x,h)\cdot\frac{t_i}{2}\vec{v}_{1i}\cdot a_i - z\cdot\sum_{i=1}^{9} N_i(x,h)\cdot\frac{t_i}{2}\vec{v}_{2i}\cdot b_i + \sum_{p=3}^{5}\sum_{k=1}^{6} M_{pk}(x,h)\cdot\vec{d}_{pk} \tag{2}$$

for the parametric hierarchical element. In this equation $\vec{\delta}_{pk}$, of components a_{pk}, b_{pk} and c_{pk} according to the axes X, Y and Z of the reference global system, is the constituted vector of the hierarchical parameters. The functions $M_{pk}(\xi\eta)$ when inserted in the Eq. (1) don't modify the level of approach of the element, but, $\vec{\delta}_{pk}$ stops having the meaning nodal variable physics. The components of the $\vec{\delta}_{pk}$ are dependent parameters of the nodal variables $\vec{d}_i$, a_i and b_i.

The Eq. (2) can be written in the following matrix form:

$$\{u\}=[N]\cdot\{a\} \tag{3}$$

where $\{u\}$ is a matrix constituted of the displacements $u(\xi\eta,\zeta)$, $v(\xi\eta,\zeta)$ and $w(\xi\eta,\zeta)$, $[N]$ is a matrix constituted of the shape functions $N_i(\xi\eta)$ and $M_{pk}(\xi\eta)$, and $\{a\}$ is a matrix constituted of the nodal displacements u_i, v_i, w_i, a_i and b_i, and hierarchical parameters a_{pk}, b_{pk} and c_{pk}.

According to the theory of plates and shells (Timoshenko *et al.*, 1959), considering a (x',y',z') local reference system, a point of the element presents the following strain components:

$$\begin{Bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \gamma_{x'y'} \\ \gamma_{y'z'} \\ \gamma_{x'z'} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x'} & 0 & 0 \\ 0 & \frac{\partial}{\partial y'} & 0 \\ \frac{\partial}{\partial y'} & \frac{\partial}{\partial x'} & 0 \\ 0 & \frac{\partial}{\partial z'} & \frac{\partial}{\partial y'} \\ \frac{\partial}{\partial z'} & 0 & \frac{\partial}{\partial x'} \end{bmatrix} \cdot \begin{Bmatrix} u' \\ v' \\ w' \end{Bmatrix} \tag{4}$$

or

$$\{\varepsilon'\} = [L] \cdot \{u'\} \tag{5}$$

where $\{u'\}$ corresponds to the displacements related to the local reference system and $[L]$ is a linear differential operator. The displacements $\{u'\}$ can be related to the global displacements $\{u\}$ by the following expression:

$$\{u'\} = [\theta]^{\mathrm{T}} \cdot \{u\} \tag{6}$$

where $[\theta]$ is a matrix constituted of the director cosines of the local reference system with relation to the global reference system. Thus

$$\{\varepsilon'\} = [L] \cdot [\theta]^{\mathrm{T}} \cdot \{u\} = [L] \cdot [\theta]^{\mathrm{T}} \cdot [N] \cdot \{a\} = [B] \cdot \{a\} \tag{7}$$

where $[B]$ is a matrix that relates strains to nodal displacements and rotations.

Applying the "Virtual Work Principle" and "D'Alembert's Principle" the stiffness and mass matrices of the element is given by:

$$[K^e] = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} [B]^{\mathrm{T}} \cdot [D'] \cdot [B] \cdot |J(\xi,\eta)| \cdot d\xi \cdot d\eta \cdot d\zeta \tag{8}$$

$$[M^e] = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} \rho \cdot [N]^{\mathrm{T}} \cdot [N] \cdot |J(\xi,\eta)| \cdot d\xi \cdot d\eta \cdot d\zeta \tag{9}$$

where $[D']$ is a symmetric matrix constituted of the material elastic constants, ρ is a mass density for unit of volume and $|J(\xi,\eta)|$ is the determinant of the Jacobian matrix of the global-local coordinate transformation.

The characteristic equation, for a structural system with n degrees of freedom, can be expressed in the form

$$[K] \cdot [\Phi] = [M] \cdot [\Phi] \cdot [\Lambda] \tag{10}$$

where $[K]$ and $[M]$ are global stiffness and mass matrix $(n \times n)$, $[\Lambda]$ the diagonal matrix $(n \times n)$ that contains the n eigenvalues λ_i and $[\Phi] = [\{\phi_1\}, \cdots, \{\phi_i\}, \cdots, \{\phi_n\}]$ the matrix $(n \times n)$ that contains the n eigenvectors $\{\phi_i\}$.

Firstly, to obtain the eigenvalues and eigenvectors it is solved the isoparametric system

$$[K_{iso}] \cdot [\Phi_{iso}] = [M_{iso}] \cdot [\Phi_{iso}] \cdot [\Lambda_{iso}] \tag{11}$$

Considering n_{iso} the number of degrees of freedom of the isoparametric analysis, $[K_{iso}]$, $[M_{iso}]$, $[\Phi_{iso}]$ and $[\Lambda_{iso}]$ are sub matrices $(n_{iso} \times n_{iso})$. The solution obtained through the first analysis of the system can be refined by introducing the hierarchical shape functions of third degree:

$$\begin{bmatrix} [K_{iso}] & [K_{iso,h3}] \\ [K_{h3,iso}] & [K_{h3}] \end{bmatrix} \cdot \begin{bmatrix} [\Phi_{iso}] & [\Phi_{iso,h3}] \\ [\Phi_{h3,iso}] & [\Phi_{h3}] \end{bmatrix} = \begin{bmatrix} [K_{iso}] & [K_{iso,h3}] \\ [K_{h3,iso}] & [K_{h3}] \end{bmatrix} \cdot \begin{bmatrix} [\Phi_{iso}] & [\Phi_{iso,h3}] \\ [\Phi_{h3,iso}] & [\Phi_{h3}] \end{bmatrix} \cdot \begin{bmatrix} [\Lambda_{iso}] & [\Lambda_{iso,h3}] \\ [\Lambda_{h3,iso}] & [\Lambda_{h3}] \end{bmatrix} \tag{12}$$

where the matrices corresponding to the isoparametric system were already obtained previously in the initial analysis. Considering n_{h3} the total number of hierarchical variables introduced in the first analysis, $[K_{iso,h3}]$, $[M_{iso,h3}]$, $[\Phi_{iso,h3}]$ and $[\Lambda_{iso,h3}]$ are sub matrices $(n_{iso} \times n_{h3})$ corresponding to the joining of the isoparametric system with the hierarchical system related with the first analysis, $[K_{h3}]$, $[M_{h3}]$, $[\Phi_{h3}]$ and $[\Lambda_{h3}]$ are sub matrices $(n_{h3} \times n_{h3})$ corresponding to the hierarchical system of the first analysis. In the same way, the solution obtained through the second analysis of the system can be refined introducing hierarchical shape functions of fourth degree and after fifth degree. Thus, for the i^{th} analysis can be written

$$\begin{bmatrix} [K_{iso}] & \cdots & [K_{iso,hi}] \\ \vdots & & \vdots \\ [K_{hi,iso}] & \cdots & [K_{hi}] \end{bmatrix} \cdot \begin{bmatrix} [\Phi_{iso}] & \cdots & [\Phi_{iso,hi}] \\ \vdots & & \vdots \\ [\Phi_{hi,iso}] & \cdots & [\Phi_{hi}] \end{bmatrix} = \begin{bmatrix} [M_{iso}] & \cdots & [M_{iso,hi}] \\ \vdots & & \vdots \\ [M_{hi,iso}] & \cdots & [M_{hi}] \end{bmatrix} \cdot \begin{bmatrix} [\Phi_{iso}] & \cdots & [\Phi_{iso,hi}] \\ \vdots & & \vdots \\ [\Phi_{hi,iso}] & \cdots & [\Phi_{hi}] \end{bmatrix} \cdot \begin{bmatrix} [\Lambda_{iso}] & \cdots & [\Lambda_{iso,hi}] \\ \vdots & & \vdots \\ [\Lambda_{hi,iso}] & \cdots & [\Lambda_{hi}] \end{bmatrix} \quad (13)$$

where the matrices corresponding to the i^{th} analysis were already obtained previously in $(i\text{-}1)^{th}$ analysis. Considering n_{hi} the total number of hierarchical variables introduced in i^{th} analysis, $[K_{iso,hi}]$, $[M_{iso,hi}]$, $[\Phi_{iso,hi}]$ and $[\Lambda_{iso,hi}]$ are sub matrices (n_{iso} x n_{hi}) corresponding to the joining of the isoparametric system with the hierarchical system related with the i^{th} analysis, $[K_{hi}]$, $[M_{hi}]$, $[\Phi_{hi}]$ and $[\Lambda_{hi}]$ are sub matrices (n_{hi} x n_{hi}) corresponding to the hierarchical system of the i^{th} analysis.

3. Damage prediction

Characteristic equations of the original and the damaged structure are considered

$$(K - l_i M)\, f_i = 0 \quad (14)$$
$$(K^* - l_i^* M^*) f_i^* = 0 \quad (15)$$

where K and M are global stiffness matrix and global mass matrix for the original structure; l_i and f_i are the i^{th} eigenvalue and the corresponding mode shape for the original structure, whereas quantities denoted with a superscript * refer to the respective terms for the damaged structure.

Structural damage is considered to be a result of a change in the stiffness matrix only, i.e. there is no change in the mass matrix

$$K^* = K + \Delta K \quad (16)$$
$$M^* = M \quad (17)$$

where ΔK is the change of the global stiffness matrix. Change of stiffness leads to a change in modal characteristics. Therefore, the changes in an eigenvalue Δl_i; and an eigenvector Δf_i can be expressed in the form

$$\Delta l_i = l_i^* - l_i \quad (18)$$
$$\Delta f_i = f_i^* - f_i \quad (19)$$

Moreover, it is assumed that the change of stiffness due to structural damage will not lead to any interchanges of eigenvalues. Using Eq. (16), (17), (18) and (19), the change in eigenvector can be expressed from Eq. (15):

$$\Delta f_i = -f_i - (K - l_i^* M)^{-1} \Delta K (f_i + \Delta f_i) \quad (20)$$

From the spectral decomposition, and assuming that the original eigenvectors are mass-normalized, the term $(K - l_i^* M)^{-1}$ can be computed as

$$(K - l_i^* M)^{-1} = \sum_{k=1}^{N} \frac{f_k f_k^T}{l_k - l_i^*} \quad (21)$$

Here, only eigenvalues that differ between the original structure and the damaged structure are considered in order to avoid the denominator of Eq. (21) vanishing.

Upon substitution of Eq. (21), Eq. (20) can be rewritten as

$$\Delta f_i = -f_i + \frac{f_i^T \Delta K (f_i + \Delta f_i) f_i}{\Delta K\, l_i} + \sum_{k=1, k\neq i}^{N} \frac{f_k^T \Delta K f_i + f_k^T \Delta K \Delta f_i}{l_i^* - l_k} f_k \quad (22)$$

Premutiplying Eq. (15) by f_i^T yields

$$f_i^T \Delta K (f_i + \Delta f_i) = \Delta l_i + \Delta l_i f_i^T M \Delta f_i \quad (23)$$

Neglecting the higher-order term, leads to

$$f_i^T \Delta K\left(f_i + \Delta f_i\right) = \Delta l_i \tag{24}$$

Upon substitution of Eq. (24), Eq. (22) becomes

$$\Delta f_i = \sum_{k=1,k\neq i}^{N} \frac{f_k^T \Delta K f_i + f_k^T \Delta K \Delta f_i}{l_i^* - l_k} f_k \tag{25}$$

It can be seen from Eq. (25) that the change of an eigenvector can be expressed as the linear combination of the original eigenvectors. When k is large enough, the terms with subscripts greater than k can be neglected. Therefore, NC, denoting the number of the original mode shapes available, can suitably replace N and the Eq. (25) is rewritten as

$$\Delta f_i = \sum_{k=1,k\neq i}^{NC} C_{ik} f_k \tag{26}$$

where the mode participation factor C_{ik} is defined as

$$C_{ik} = \frac{f_k^T \Delta K f_i + f_k^T \Delta K \Delta f_i}{l_i^* - l_k} \tag{27}$$

Rearranging Eq. (27), leads to

$$f_k^T \Delta K f_i + \sum_{l=1,l\neq i}^{NC} C_{il} f_k^T \Delta K f_l + l_k C_{ik} - l_i^* C_{ik} = 0 \tag{28}$$

Using Eq. (26) and (27), Eq. (23) can be now rewritten as

$$f_i^T \Delta K f_i + \sum_{l=1,l\neq i}^{NC} C_{il} f_i^T \Delta K f_l - \Delta l_i = 0 \tag{29}$$

where the higher-order term vanishes exactly due to the orthogonality of the original eigenvectors.

Furthermore, a scalar damage model is assumed, i.e. the change in the stiffness matrix can be expressed in the form

$$\Delta K = \sum_{j=1}^{NE} a_j K_j \tag{30}$$

where a_j is the scalar damage indicator for the j^{th} element; K_j is the contribution of element j to the global stiffness matrix; NE is a number of structural elements.

Therefore, Eq. (29) and (28) can be rewritten in the form

$$\sum_{j=1}^{NE} a_{iji} a_j + \sum_{j=1}^{NE} \sum_{l=1,l\neq i}^{NC} a_{kjl} C_{il} a_j - \left(l_i^* - l_i\right) = 0 \tag{31}$$

and, finally, as

$$\sum_{j=1}^{NE} a_{kji} a_j + \sum_{j=1}^{NE} \sum_{l=1,l\neq i}^{NC} a_{kjl} C_{il} a_j - \left(l_i^* - l_i\right) C_{ik} = 0 \tag{32}$$

where a_{iji}, a_{ijl}, a_{kji} and a_{kjl} are the eigenmode-stiffness sensitivity coefficients, which can be defined in a general form as

$$a_{kji} = f_k^T K_j f_i \tag{33}$$

Combining the two sets of Eq. (31) and Eq. (32) an enlarged set of a total of $NEQ=NL*NC$ equations, related to the element scalar damage indicators a_j and the mode participation factors C_{ik}, is written a system of non-linear equations. NL is the number of available natural frequencies. In this work the Gauss–Newton least-squares computational technique is used to solve for the element scalar damage indicators as well as the mode participation factors.

Once the mode participation factors C_{ik} are found, using Eq. (29) and (28) the eigenvectors for the damaged structure can be calculated as

$$f_i^* = f_i + \sum_{k=1,k\neq i}^{NC} C_{ik} f_k \tag{34}$$

where the pairing of the eigenvalues for the original structure and the damaged structure can be checked using the *MAC* factors (Modal Assurance Criterion), defined as

$$MAC(k,i) = \frac{\left|f_k^T f_i^*\right|^2}{\left|f_k^T f_k\right|\left|f_i^{*T} f_i^*\right|} \tag{35}$$

The highest *MAC(k, i)* factors indicate the most possible pairings of the original mode *k* and the damaged mode *i*.

4. Numerical examples

Next it is presented the results obtained with the hierarchical finite element in the damage prediction for plate and shell structures, considering the isoparametric analysis (***p*=2**) and the hierarchical analysis of third degree (***p*=3**), fourth degree (***p*=4**) and fifth degree (***p*=5**). The hierarchical formulation becomes possible to use different polynomials expansions along different edges and elements, but in this work the adaptative refinement was not explored. The hierarchical refinement was made using polynomials expansions of same degree along edges and elements:

- in the refinement of third degree (***p*=3**) of the isoparametric quadratic expansion were used polynomials of third degree on all edges of the elements;
- in the refinement of fourth degree (***p*=4**) of the isoparametric quadratic and third degree expansions were used polynomials of fourth degree on all edges of the elements;
- in the refinement of fifth degree (***p*=5**) of the isoparametric quadratic, third degree and fourth degree expansions were used polynomials of fifth degree on all edges of the elements and all elements.

Numerical examples are used to demonstrate the effectiveness of the method. In the examples, it is assumed that all natural frequencies for the damaged structure are both noise free and correctly paired with the natural frequencies for the original structure.

4.1. Square plate

A square plate modeled comprising 6 elements and 49 nodes is used to demonstrate the effectiveness of the method in the analysis of plate structure. The edges of square plate are simply supported, clamped, simply supported and free. The square plate have the edges of length $a = 1.0$ m, thickness $t = 0.001$ m, Yong's modulus $E = 2.068 \times 10^{11}$ N/m^2, density $r = 7.80 \times 10^3$ Kg/m^3 and Poisson's modulus $\nu = 0.3$. The geometry of the square plate and the element numbering are illustrated in Fig. (1).

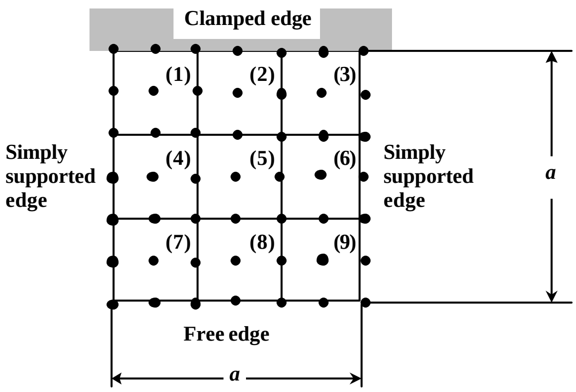


Figure 1. Square plate modeled, geometry, nodes and element numbering.

Two hypothetical damage scenario is induced by reducing Yong's modulus of different elements, with different magnitudes as summarized in Tab. (2). A finite element analysis was performed for both the original and the damaged case to calculate natural frequencies and mode shapes.

Table 2. Hypothetical damage scenarios by reducing Yong's modulus of different elements with different magnitudes.

Element	1	2	3	4	5	6	7	8	9
Scenario 1	-5%	0%	0%	0%	-5%	-3%	0%	-5%	0%
Scenario 2	0%	-2%	0%	-5%	0%	0%	-2%	0%	0%

In order to study the effectiveness of the method with respect to the required amount of modal information, fixing the number of damaged natural frequencies to same to the number of elements (*NL*=*NE*=9), two numbers of original eigenvector (*NC*=12 and *NC*=16) are selected for the identification processes for damage scenarios, as show in Fig. (2), (3), (4) and (5).

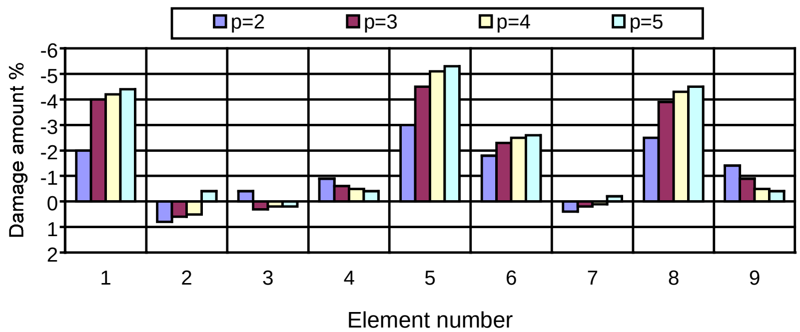


Figure 2. Predict damage for scenario 1, with *NL*=9 damaged frequencies and *NC*=12 original eigenvectors.

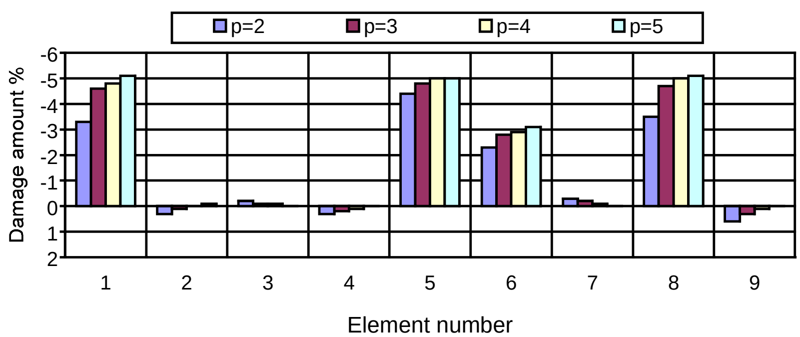


Figure 3. Predict damage for scenario 1, with *NL*=9 damaged frequencies and *NC*=16 original eigenvectors.

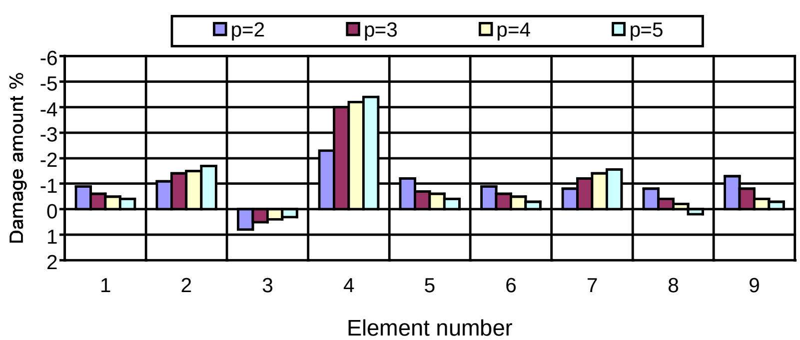


Figure 4. Predict damage for scenario 2, with *NL*=9 damaged frequencies and *NC*=12 original eigenvectors.

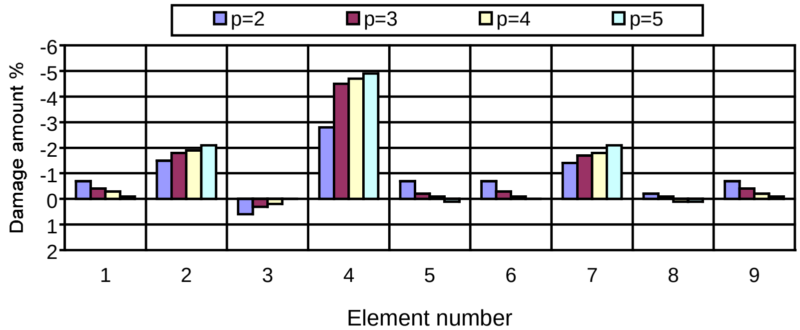


Figure 5. Predict damage for scenario 2, with *NL*=9 damaged frequencies and *NC*=16 original eigenvectors.

4.2. Cylindrical shell with clamped edge

A cylindrical shell modeled comprising 16 elements and 81 nodes is used to demonstrate the effectiveness of the method in the analysis of shell structure. One curved edge is considered built-in or clamped to an infinitely rigid foundation while other edges are free. The cylindrical shell have the edges of length $S = L = 0.3048$ m, radius of curvature $R = 0.6096$ m, thickness $t = 3.048 \times 10^{-3}$ m, Yong's modulus $E = 2.068 \times 10^{11}$ N/m^2, density $\rho = 7.80 \times 10^3$ Kg/m^3 and Poisson's modulus $\nu = 0.3$. The geometry of the cylindrical shell is illustrated in Fig. (6a) and the element numbering in Fig. (6b).

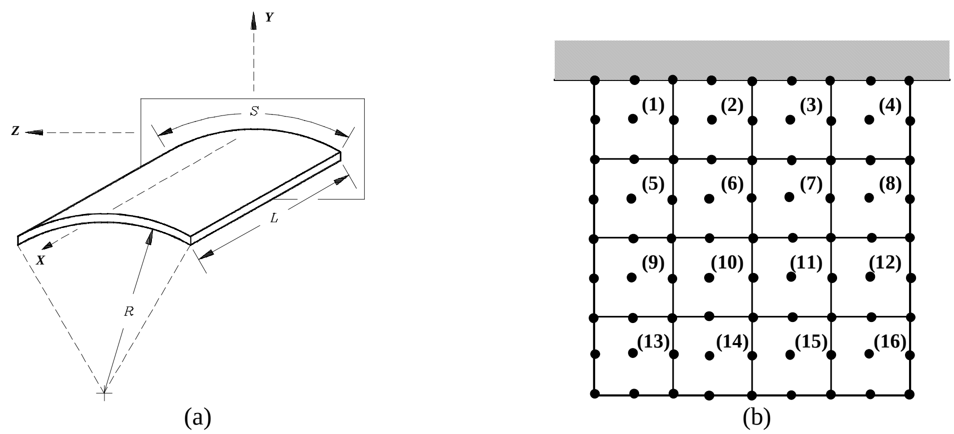


Figure 6. Cylindrical shell with clamped edge, geometry (a) and element numbering (b).

Two hypothetical damage scenarios is induced by reducing Yong's modulus of different elements, with different magnitudes as summarized in Tab. (3)

Table 3. Hypothetical damage scenarios by reducing Yong's modulus of different elements with different magnitudes.

Element	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Scenario 1	-5%	0%	-3%	0%	-5%	-3%	0%	-5%	0%	-2%	-5%	0%	0%	-3%	0%	-2%
Scenario 2	-2%	0%	0%	-5%	0%	0%	-3%	0%	-2%	-5%	0%	0%	-3%	0%	-2%	0%

In order to study the effectiveness of the method with respect to the required amount of modal information, fixing the number of damaged natural frequencies to same to the number of elements (*NL*=*NE*=16), two numbers of original eigenvector (*NC*=18 and *NC*=24) are selected for the identification processes for damage scenarios, as show in Fig. (7), (8), (9) and (10).

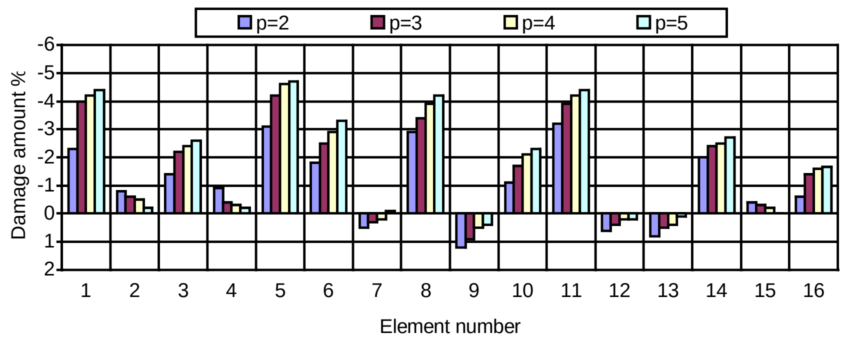


Figure 7. Predict damage for scenario 1, with *NL*=16 damaged frequencies and *NC*=18 original eigenvectors.

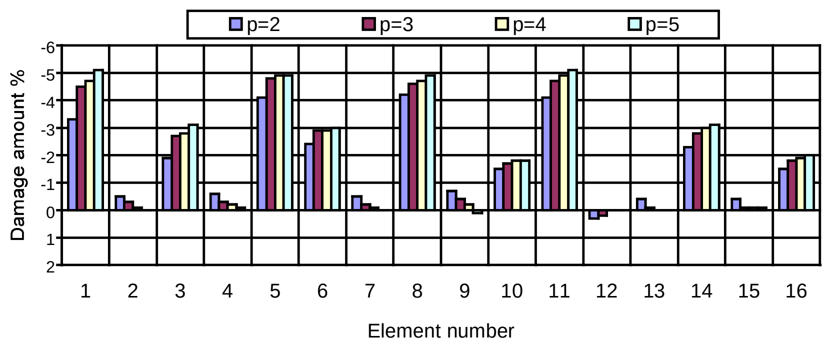


Figure 8. Predict damage for scenario 1, with *NL*=16 damaged frequencies and *NC*=24 original eigenvectors.

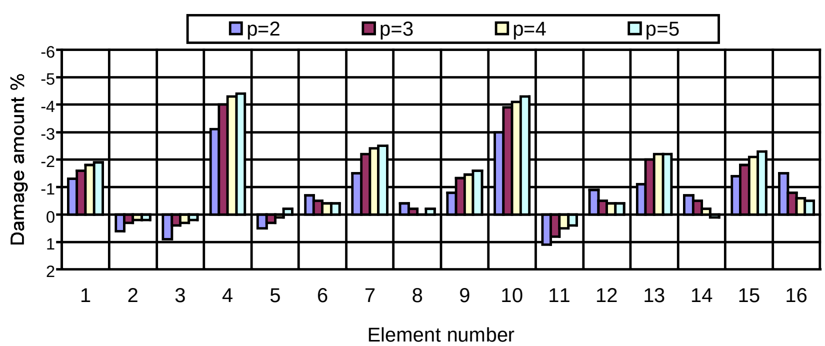


Figure 9. Predict damage for scenario 2, with *NL*=16 damaged frequencies and *NC*=18 original eigenvectors.

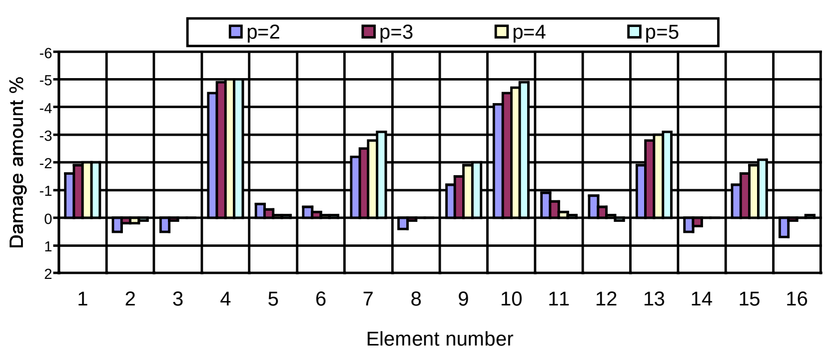


Figure 10. Predict damage for scenario 2, with *NL*=16 damaged frequencies and *NC*=24 original eigenvectors.

5. Conclusion

Considering the numerical examples in the damage prediction for plate and shell structures, it can be verified that the hierarchical analysis of third degree (***p*=3**), fourth degree (***p*=4**) and fifth degree (***p*=5**) showed accurate solutions compared with the isoparametric analysis (***p*=2**). The results indicate that the approach can be successful in not only predicting the location of damage but also in determining the extent of structural damage, not more than *NE* damage natural frequencies are required to determine both the location and the size of damage and no knowledge of mode shapes for the damaged structure is required.

6. References

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