

A Boundary Element Study on 3D Soil Vibration Isolation Using Open or Filled Trenches as Barriers

Ronaldo Carrion
Euclides de Mesquita Neto
Department of Computational Mechanics
Faculty of Mechanical Engineering
State University at Campinas
DMC/FEM/UNICAMP
C.P. 6122, CEP 13083-970, Campinas, SP, Brasil
e-mail: euclides@fem.unicamp.br

Abstract. *In the present paper a 3D Boundary Element (BE) implementation is employed to analyze soil vibration isolation strategies. In this stationary Dynamic Soil-Structure Interaction (DSSI) problem, the soil is externally excited and generates a propagating wave field. This resulting wave field induces vibrations on structures placed in the surrounding soil. The idea of this article is to exemplify some vibration isolation strategies using barriers, such as open or filled trenches, that may be modeled by a 3-dimensional BE implementation. The perfotmed implementation will present the vibration reduction as well as the resulting propagation wave pattern for distinct barrier properties.*
The BE formulation is based on the 3D full space stationary fundamental elastic solution. Barriers are modeled as bounded domains and the soil is described as an unbounded region. These bounded and unbounded domains are assembled by the sub-region concept. The regularization of the strong singular kernels, in both domains, is accomplished by means of rigid body concept. For the unbounded soil the idea of enclosing elements is applied to allow for the use of the rigid body concept. The proposed methodology is first tested for accuracy by solving an elastic wave propagation problem with known solution. In the sequence the method is applied to simple vibration isolation problem using trenches.

Keywords. *Boundary element method, Soil-structure interaction, Vibration isolation, Elastodynamic*

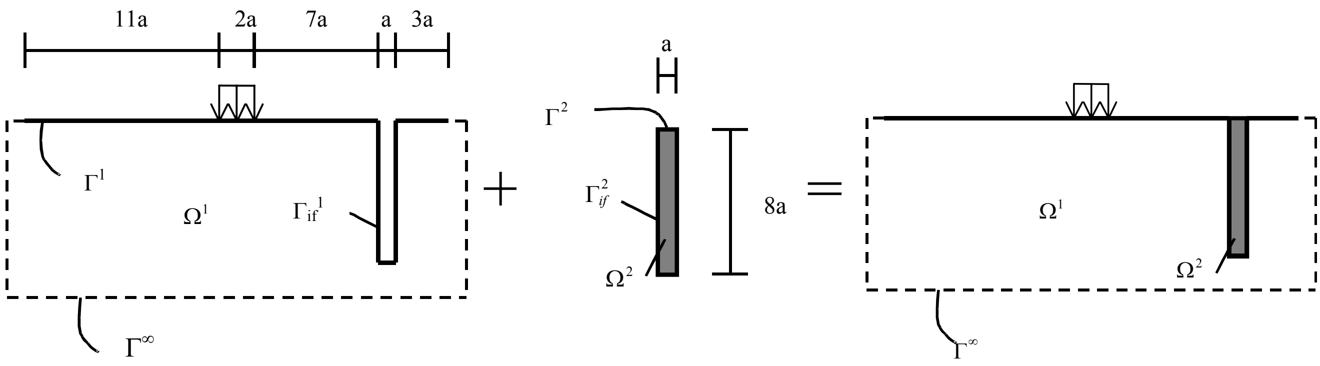
1. Introduction

This article reports a Boundary Element (BE) based methodology to study the isolation of vibrations generated by external sources in contact with the soil and transmitted by the ground in the form of waves. As described by Beskos, Dasgupta and Vardoulakis (1986), the analytical treatment of vibration isolation by wave barriers in the framework of linear elastodynamics is based on the theory of wave diffraction. Barkan (1962) was the first author to report some field investigations for studying the effectiveness of wave barriers. However it is apparent that the analytical treatment of 3D elastic wave diffraction problems is confined to very simple geometries and idealized conditions and that realistic problems involving complex geometries can only be solved numerically.

The present work employs the frequency domain direct version of the Boundary Element Method (BEM) for the solution of vibration isolation problems by barriers in a 3D context. Waves generated by a stationary soil excitation are reduced in amplitude by means of open or filled trenches protecting nearby structures. In fact the proposed methodology can deal with trenches of arbitrary shape but, in the present article, it is applied to rectangular barriers which are very usual in engineering practice. The soil medium is assumed to be linear, elastic, homogenous and isotropic. Constant quadrilateral boundary elements are employed. These elements have been chosen because they are simple to implement and because they deliver good results for the displacement components. It is known that in special case of vibration isolation problems, the free surface response away from the trench is more important than the deformation of the trench itself or the stress concentration around it. Under these circumstances an implementation with constant elements can produce results of acceptable accuracy (Dasgupta, Beskos and Vardoulakis 1986).

The full space stationary fundamental solution (Eringen and Suhubi, 1975) was chosen as the auxiliary state used to transform the Cauchy/Navier differential domain equations into the boundary integral equations, on which the BEM is based. The use of the full space fundamental solution implies that the soil free surface must be artificially created by a proper BE discretization. This is not a great disadvantage, since in this wave propagation and isolation problem the displacement of the free soil surface must be determined in order to assess the efficiency of the isolation strategy.

In the BE implementation two further methodological choices have been made. In this article the soil is an unbounded domain and the barrier is treated as a bounded region. These two sub-domains are coupled by equilibrium and kinematic compatibility equations. These two sub-regions may be seen in figure 1. The second choice in the BEM implementation is the use of rigid body argument to evaluate the strong singularity of traction kernels for the bounded and for the unbounded domain. In the unbounded domain the idea of the rigid body argument must be used in conjunction with the so-called enclosing elements (Carrion, 2002, Ahmad and Banerjee, 1988, Araújo et. alli 1997).



a) Sub-region 1: the unbounded soil Ω^1 b) Sub-region 2: the barrier Ω^2 c) The coupled problem

Figure 1: Lateral view of the unbounded soil and the bounded barrier

2. Boundary Element Formulation

The frequency domain differential equations of motion for a 3D linear elastic, homogeneous and isotropic continuum body, known as Cauchy/Navier equations, can be expressed in terms of the displacement components u_i as:

$$(\lambda + \mu)u_{j,ji} + \mu u_{i,jj} + b_i + \rho\omega^2 u_i = 0 \tag{1}$$

where λ and μ are Lamè constants, ρ is the mass density, ω is the circular frequency, b_i are the components of body force. Commas indicate spatial differentiation and summation over repeated indices is assumed.

The auxiliary state used in the BE standard formulation is assumed to be the stationary Full-Space Green's Functions or Fundamental Solution with frequency (ω) dependent displacement and traction kernels given by $u_{ij}^*(\omega)$ and $t_{ij}^*(\omega)$. With the aid of this auxiliary state the differential Eq. (1) can be transformed into the boundary integral Eq. (2) (Eringen and Suhubi 1975):

$$c_{ij}(\xi) \; u_j(\xi,\omega) = \int_\Gamma u_{ij}^*(\xi,x,\omega) \; t_j(x,\omega) \; d\Gamma(x) - \int_\Gamma t_{ij}^*(\xi,x,\omega) \; u_j(x,\omega) \; d\Gamma(x) \tag{2}$$

In equation (2) no body forces are considered, $b_i=0$. The components u_j and t_j represent, respectively, the displacements and tractions at the boundaries of problem being solved. The collocation point is ξ and the field point is x. The elements of tensor c_{ij} are called integration free terms. The solution of Eq. (2) is accomplished numerically. For this purpose, the boundary Γ of the body is discretized into a series of boundary elements over which displacements and tractions are assumed to be constant (Dominguez, 1993). Thus a system of linear algebraic equations is obtained and can be written in matrix form as:

$$\left[H_{ij}(\xi,x,\omega)\right]\left\{u_j(x,\omega)\right\} = \left[G_{ij}(\xi,x,\omega)\right]\left\{t_j(x,\omega)\right\} \tag{3}$$

where $H_{ij}(\xi,x,\omega)$ and $G_{ij}(\xi,x,\omega)$ are the influence matrices resulting from the numerical integration over the area of each Boundary Element of the fundamental solutions t_{ij}^* and u_{ij}^* multiplied by the interpolation functions and the proper Jacobian.

After the boundary conditions are applied, the system can be solved to obtain all the unknown boundary values and, consequently, an approximate solution to the boundary value problem is obtained. Once the solutions at the boundary are obtained, Eq. (2) can be used to find the displacements u_j at any domain point ξ. The stresses at the domain can be obtained from a traction boundary integral equation (Brebbia and Dominguez, 1989) given by:

$$\sigma_{ij}(\xi,\omega) = \int_\Gamma D_{kij}(\xi,x,\omega) \; t_k(x,\omega) \; d\Gamma(x) - \int_\Gamma E_{kij}(\xi,x,\omega) \; u_k(x,\omega) \; d\Gamma(x) \tag{4}$$

where the kernels $D_{kij}(\xi, x, \omega)$ and $E_{kij}(\xi, x, \omega)$ can be found in the work of Gaul and Fiedler (1993).

2.1. Subregions Concept

As depicted in figure 1, the boundary Γ, of the unbounded soil Ω^1 is subdivided in three distinct parts, $\Gamma=(\Gamma^1 \cup \Gamma^1_{if} \cup \Gamma^{\infty})$. The soil free surface is Γ^1, the boundary interfacing with the barrier is Γ^1_{if} and the enclosing surface for the unbounded soil is designated as Γ^{∞}. The boundary of the barrier is divided in two regions ($\Gamma^2 \cup \Gamma^2_{if}$). The discrete matrix representation of these two regions may be divided according the position of the boundary nodes. Using the subscript (if) for the interface nodes and no indices for the other nodes the algebraic system shown in equations (3) may be written for the soil and the barrier respectively as (Brebbia and Dominguez 1989):

$$\begin{bmatrix} H^1 & H^1_{if} \end{bmatrix} \begin{Bmatrix} u^1 \\ u^1_{if} \end{Bmatrix} = \begin{bmatrix} G^1 & G^1_{if} \end{bmatrix} \begin{Bmatrix} t^1 \\ t^1_{if} \end{Bmatrix} \tag{5}$$

and

$$\begin{bmatrix} H^2 & H^2_{if} \end{bmatrix} \begin{Bmatrix} u^2 \\ u^2_{if} \end{Bmatrix} = \begin{bmatrix} G^2 & G^2_{if} \end{bmatrix} \begin{Bmatrix} t^2 \\ t^2_{if} \end{Bmatrix} \tag{6}$$

Using kinematic displacement compatibility and force equilibrium at the nodes of the common interfaces, respectively

$$u^1_{if} = u^2_{if} = u_{if} \qquad \text{and} \qquad t^1_{if} = -t^2_{if} = t_{if} \tag{7}$$

a final set of equations for the coupled region $\Omega^1 \cup \Omega^2$ can be obtained:

$$\begin{bmatrix} H^1 & H^1_{if} & 0 \\ 0 & H^2_{if} & H^2 \end{bmatrix} \begin{Bmatrix} u^1 \\ u_{if} \\ u^2 \end{Bmatrix} = \begin{bmatrix} G^1 & G^1_{if} & 0 \\ 0 & -G^2_{if} & G^2 \end{bmatrix} \begin{Bmatrix} t^1 \\ t_{if} \\ t^2 \end{Bmatrix} \tag{8}$$

Considering that for the present case the surface tractions t^1 and t^2 are the prescribed boundary variables and that the surface displacements u^1 and u^2 and the interface variables u_{if} and t_{if} are the unknown quantities, the equation system (8) above can be reshaped to yield:

$$\begin{bmatrix} H^1 & H^1_{if} & 0 & -G^1_{ig} \\ 0 & H^2_{if} & H^2 & G^2_{if} \end{bmatrix} \begin{Bmatrix} u^1 \\ u_{if} \\ u^2 \\ t_{if} \end{Bmatrix} = \begin{bmatrix} G^1 & 0 \\ 0 & G^2 \end{bmatrix} \begin{Bmatrix} t^1 \\ t^2 \end{Bmatrix} \tag{9}$$

It should be remarked that the steps described by equations (5) to (8) are rather formal, because the system shown in equation (9) can be directly assembled.

3. Numerical Implementation

When the field and the integration points do not coincide ($\xi \neq x$), the boundary integrals given in equation (2) are regular and integration is accomplished by using standard Gauss Quadrature. However, when $\xi = x$, these integrals become singular due to the $O(1/r)$ and $O(1/r^2)$ singularity order of the tensors u^*_{ij} and t^*_{ij}, respectively. The strategies to deal with the singular integrations are discussed below.

For the u^*_{ij} kernel, which presents weak singularity of order $O(1/r)$, a particular treatment is applied to make possible the use of standard Gauss Quadrature (Dominguez 1993). The idea is to divide the quadrilateral element in triangular sub-elements. Then, each triangular sub-element is treated as a quadrilateral one with two corners collapsed at the collocation point. With this methodology the Jacobian of the transformation presents the order $O(r)$ at the collocation point. This fact cancels the singularity at the mentioned point, which has also the order $O(1/r)$. This approach has been introduced by Lachat (1975) and has been further developed by Telles (1987).

A possible methodology to evaluate the strong singular kernel t_{ij}^* is based on the rigid body motion. Nevertheless, this argument is only applicable to static problems and bounded domains. In the sequence the procedure to apply the rigid body concept to the dynamics of unbounded domains is described.

3.1 Regularization of the dynamic kernel

As the dynamic and static kernels present the same order of singularity, it is possible to regularize the singular integral by subtracting and adding the static kernel from the dynamic one. This strategy can be represented as follows. Calling the indices for the static and dynamic problems, respectively, *sta* and *dyn*, adding and subtracting the static kernels to the boundary equation (2) governing the dynamic problem leads to:

$$\begin{aligned} c_{ij}(\xi)\ u_j(\xi,\omega) &= \int_\Gamma u_{ij}^*(\xi,x,\omega)\ t_j(x,\omega)\ d\Gamma(x) - \\ &\int_\Gamma \left\{ t_{ij}^*(\xi,x,\omega)_{dyn} - t_{ij}^*(\xi,x)_{sta} \right\}\ u_j(x,\omega)\ d\Gamma(x) + \int_\Gamma t_{ij}^*(\xi,x)_{sta}\ u_j(x,\omega)\ d\Gamma(x) \end{aligned} \tag{10}$$

The integrals in Eq. (10) containing the difference between the dynamic and static kernels are no longer singular and can be evaluated by standard Gauss Quadrature. The integration of the static singular kernel is evaluated based on the rigid body motion argument.

3.2 Treatment of unbounded domains

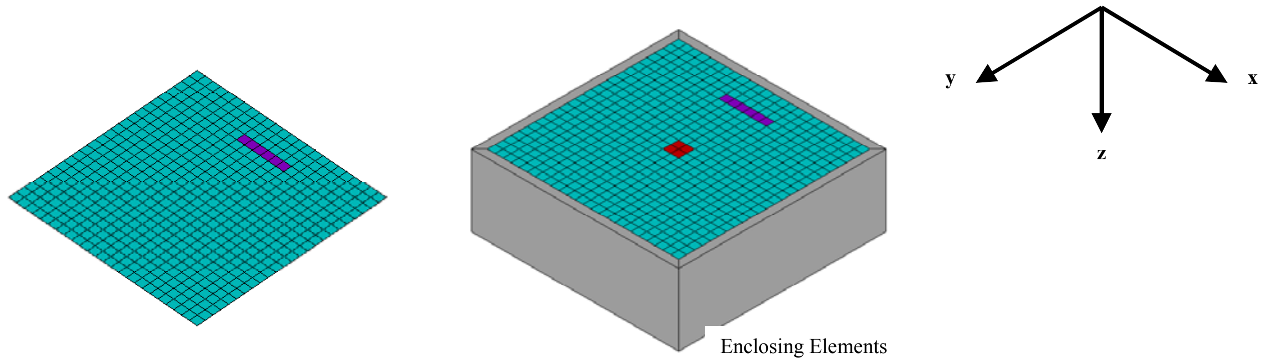
Now the rigid body argument is extended to deal with the unbounded half-space problem by using the so-called Enclosing Elements. The soil free surface is created by a BE discretization as shown in figures 2a and 3. The free surface is truncated at some distance from the source. This truncation induces some errors in the solution at points very close to the outer boundary, but no significant errors are induced away from the mesh border, where the excitation and the barrier are modeled (fig. 3). In the sequence, a series of Enclosing Elements are introduced to discretize the boundary Γ^∞, as shown in figure 2b. Now it is possible to apply the rigid body concept on the region containing the soil free surface and the enclosing elements to determine the contribution of the static solution singular kernels in the unbounded soil domain Ω^1, present in equation (10). This idea has been used by Ahmad and Banerjee (1988) in stationary problems and has been extended by Araújo, Nishikava and Mansur (1997) to transient solutions. Carrion (2002) has also studied the enclosing elements in his Ph.D. thesis.

4. Numerical Examples

In this section two analyses are performed. The first one is to validate the proposed BE approach and the second deals with two practical vibration isolation problems.

4.1 Validation of the formulation

First, the BE implementation for the unbounded domain is validated. The soil free surface is discretized with constant elements of dimension *a*x*a*, as can be seen in figures 2a and 3. The mesh showed in figure 2a has 632 constant boundary elements at the soil surface. The embedded barrier is *6a* long, *8a* deep and *1a* wide and (fig. 1b) is discretized with 68 elements. Besides that, 9 enclosing elements are used to create a boundary around the soil, (figure 2b). A unitary vertical harmonic traction load t_z is applied on the four central elements of the discretized soil surface (see fig. 3). The remaining of the surface is considered to be traction-free. The load is excited at the dimensionless frequency $A_o=\omega a/c_s=1$, where c_s is the shear wave velocity of the elastic soil. The vertical with to those reported by Romanini and Mesquita (1999), using a Green's function approach. Figure 4 shows the comparison between both approaches. An analysis reveals that there is a fairly good agreement between these approaches. The discrepancies are probably due to the difference in the solution methodology.



a) Discretization of the soil free surface and barrier

b) Discretization using the Enclosing Elements

c) Coordinate system

Figure 2: The truncated free soil surface discretization (a) and the enclosing elements (b)

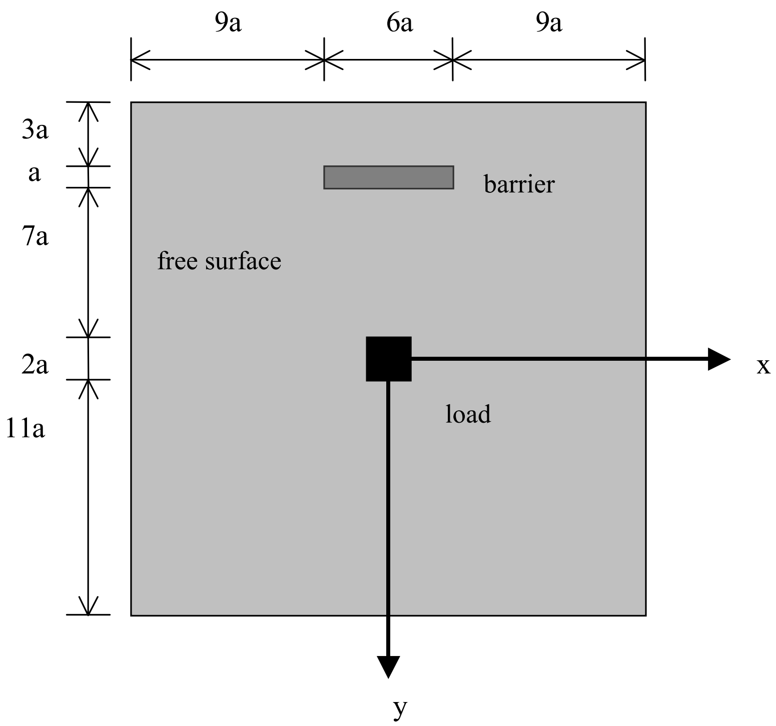
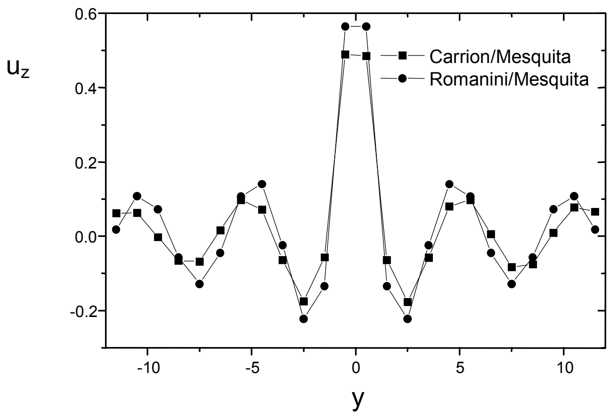
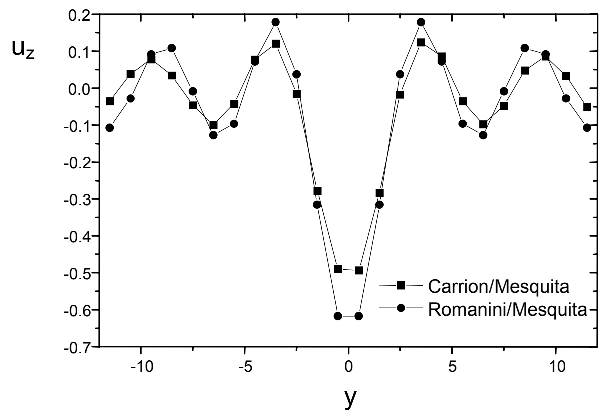


Figure 3: Scheme for the dimensions and position of the soil free surface, loaded area and barrier.



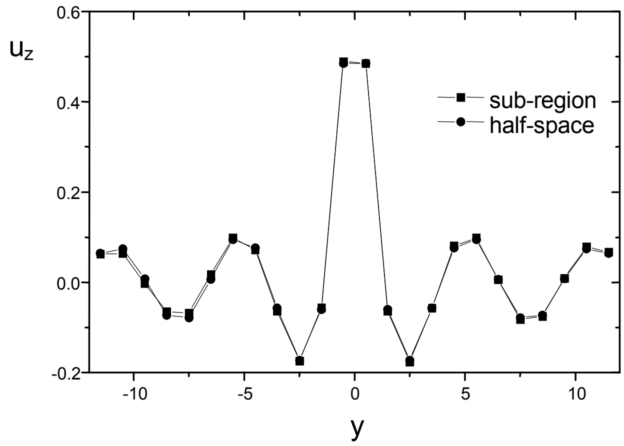
a) Real part $u_z(x=0.5, y, z=0)$



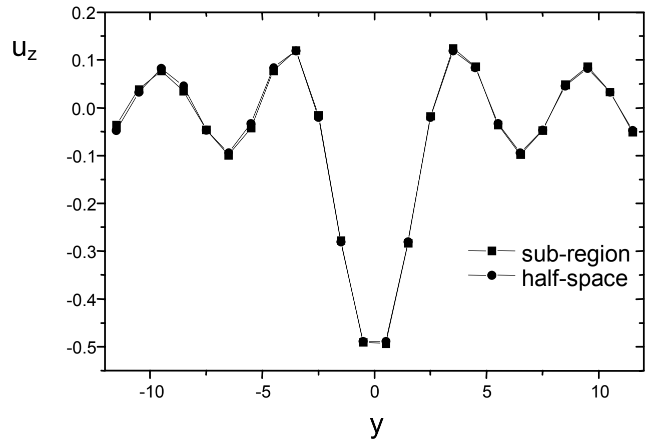
b) Imaginary part $u_z(x=0.5, y, z=0)$

Figure 4: Validation of the BE implementation: vertical free surface displacement due to a vertical excitation

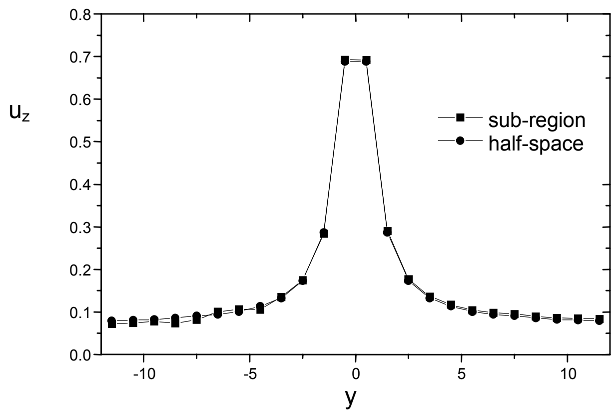
In the sequence, the sub-region implementation is validated. Let us first assume that the mechanical properties of the unbounded soil and the bounded barrier are the same. Under this assumption the behavior of the soil surface containing the barrier should be the same of the previous example, where only the homogeneous soil was considered.



a) Real Part of $u_z(x=0.5, y, z=0)$



b) Imaginary Part of $u_z(x=0.5, y, z=0)$



c) Absolute value of $u_z(x=0.5, y, z=0)$

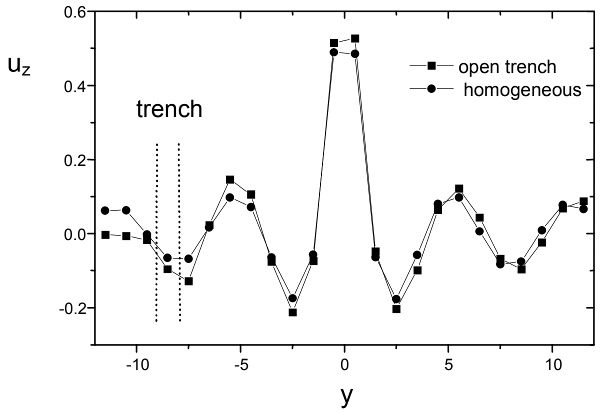
Figure 5: Vertical displacement of the soil surface u_z for the validation of the sub-region coupling.

Figure 5 shows the vertical displacement $u_z(x=0.5,y, z=0)$ of the soil surface along the y-axis for x=0.5, z=0, due to the same excitation imposed in the previous validation example. Real as well as imaginary parts and the absolute value of the displacement field are depicted. Two results are shown in each plot. One result reports the homogeneous soil and the other the coupled soil and barrier, when both domains possess the same properties. An analysis shows that there is a very good agreement between both solutions. This corroborates to indicate that the present implementation of the sub-region technique should be correct.

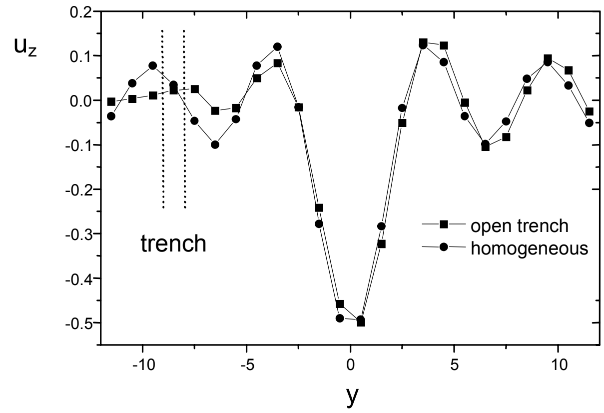
4.2 Vibration isolation problems

The effectiveness of an open or a filled trench as a barrier to ground-propagating waves generated by an external soil load is studied in two examples.

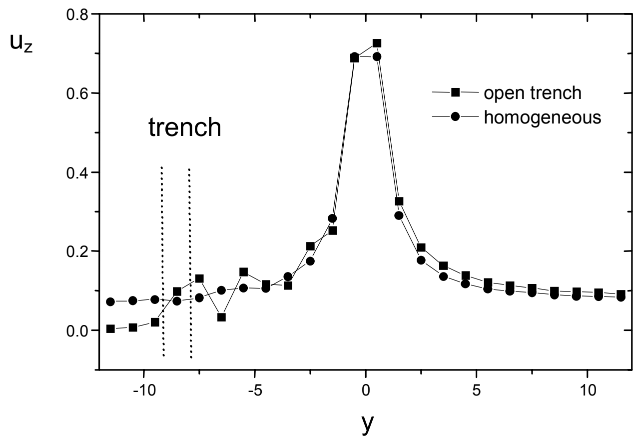
Example 1: In this case, the open trench is simulated by assigning the stiffness and density of the barrier a much smaller value than the counterpart of the surrounding soil: $G_{soil} = 100\ G_{trench}$ and $\rho_{soil} = 100\ \rho_{trench}$. All the other characteristics are the same of the above mentioned examples. Figures 6 show the real, imaginary and absolute value of the vertical displacements u_z(x=0.5, y, z=0) compared to the homogeneous case already mentioned. It is useful to point out that the trench nodal point is located in the coordinate *(x=0.5, y=8.5, z=0)*. An analysis of the figures, specially of the absolute value given in figure 6c shows that trenches may be effectively used as vibration isolation barriers. The trench causes a decrease of the vibration amplitude for the part of the soil protected by the barrier, left side in picture 6c. On the other hand, waves reflected at the barrier induce an increase of vibration amplitude at the unprotected side of the barrier, right side in figure 6c.



a) Real part of u_z(x=0.5, y, z=0)



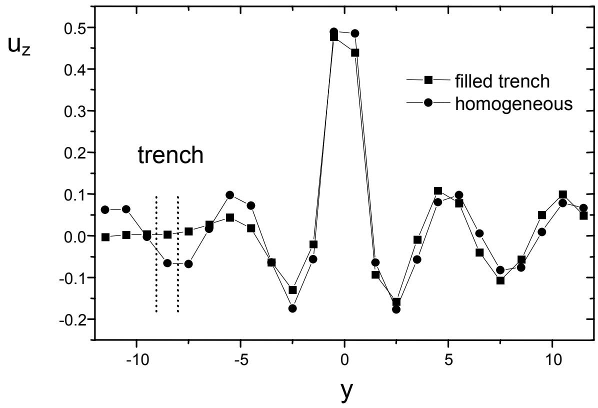
b) Imaginary part of u_z(x=0.5, y, z=0)



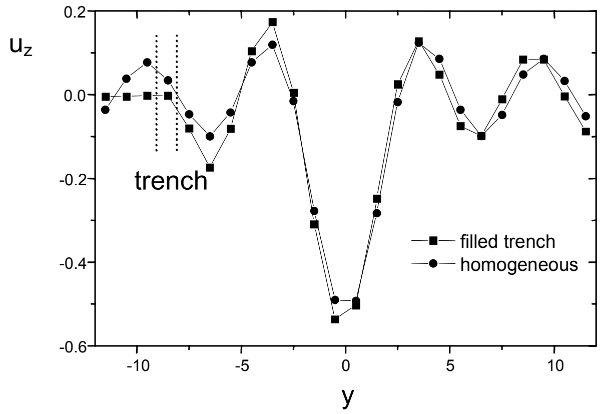
c) Absolute value of the displacement u_z(x=0.5, y, z=0)

Figure 6: Displacements on the half-space surface for an open trench.

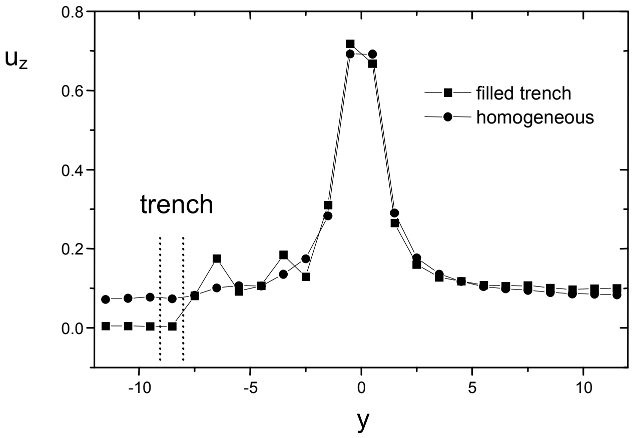
Example 2: In the second case, the inverse situation is considered. A filled trench is simulated by making both, the barrier stiffness and density, much larger the values given to the surrounding soil: $G_{soil} = 1/100\ G_{trench}$ and $\rho_{soil} = 1/100\ \rho_{trench}$. The soil properties are those of the validation examples. Figure 7 shows the normal displacements $u_z(x=0.5, y, z=0)$ in comparison with the homogeneous soil without barrier. The amplitude of the soil surface displacement is reduced on the protected side of the barrier, as expected. It seems that the filled barrier, in a first approach, is more effective than the open barrier. Further studies must be conducted to assess the role of the excitation frequency and barrier properties.



a) Real part of $u_z(x=0.5, y, z=0)$



b) Imaginary part of $u_z(x=0.5, y, z=0)$



c) Absolute value of $u_z(x=0.5, y, z=0)$

Figure 7: Displacements on the half-space surface for the case of filled trench.

5. Concluding Remarks

A numerical method based on the frequency domain direct version of the Boundary Element Method has been developed to solve isolation vibration problems by means of barriers. The sub-region technique and the rigid body motion argument, together with the idea of enclosing elements are the main concepts used in the presented BE scheme. Quadrilateral constant boundary elements have been used in the implementation due to its simplicity. The vibration isolation, by open or filled trenches acting as barriers for waves that were generated by a vertically harmonic soil load has been, exemplarily, considered.

In the present article only simple isolation examples, for a single excitation frequency A_o were shown. But the reported BE methodology represents a numerical scheme that allows the investigation of the influence of several parameters on the vibration isolation. The influence of barrier geometrical and mechanical properties compared to those of the soil, excitation frequency, distance from the excitation source and so on, may be investigated by the proposed scheme. It should also be mentioned that through the described sub-region concept, layered soil-profiles with arbitrary layer geometry may be included in the analysis. But it should also be mentioned that the strategy of assembling a global system matrix, like the one given in equation (9) requires large amounts of storage capacity and represents a limitation to perform the mentioned studies. Alternative schemes that do not require the assembling of the complete system matrices should be investigated in the future. They would expand the analysis capabilities of the proposed BE scheme.

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