LAMINAR FLOW OVER A POROUS BED MODELED AS AN ARRAY OF CYLINDRICAL RODS

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Abstract. *In this work a porous bed is modeled as an array of cylindrical rods with the purpose of modeling the interface between a porous medium and a clear flow domain. The flow properties for this geometry are investigated. The governing equations are numerically solved using the finite volume method in a generalized coordinate system. The algebraic equation system obtained is solved by the SIP method. For pressure-velocity coupling, the SIMPLE method is applied. Results for field velocity across the array of cylindrical rods and the influence of porosity and permeability are investigated.*

Keywords: *porous media, numerical solution, laminar flow.*

1. Introduction

The study of fluid flow in media that contain an interface between clear and porous media has several applications in many fields of industry and environment. Moreover, in many practical situations the flow occurs in domains composed by a clear (unobstructed) region and a porous medium, in addition to flow around a solid obstacle. This type of flow received considerable attention from researchers in the last decades. The importance of this theme can best be appreciated by noting several of their application areas. Atmospheric boundary layer over forests and dispersion of pollutants in the soil are some examples of applications. Accordingly, the development of a numerical tool able to treat all these regions in one computational domain is of advantage for engineering design.

Beavers and Joseph (1967), were the first to study a boundary condition for a fluid flow in a region that contains an interface between a clear and porous medium. They developed an experiment and detected a non negligible velocity at the interface.

Neale and Nader, (1974), proposed a continuity condition for the velocity at the interface by the introduction of the Brinkman term on the momentum equation for the porous region.

Vafai and Thiyagaraja, (1987), made an analytical study for the fluid flow and heat transfer for three types of interface: *a*) interface between two layers of porous matrix; *b*) interface between a clear and porous medium; *c*) interface between a permeable and impermeable media. The continuity of the shear stress and the heat flux were accounted for in their study, with the employment of the Darcy Forchheimer-extended equation. Other analyses consider the same type of boundary conditions for fluid flow and heat transfer as in Vafai and Thiyagaraja, (1987), Poulikakos and Kazmierczak, (1987), Vafai and Kim, (1990b), Kim and Choi, (1996) and Ochoa-Tapia and Whitaker, (1997).

Vafai and Kim, (1990a) presented an exact solution for fluid flow at the interface between a clear and a porous medium including the inertia term and the boundary effects. In the study therein, the shear stress in the fluid and porous medium are taken as equal at the interface.

Ochoa-Tapia and Whitaker (1995a-b), proposed an interface condition where a jump in the shear stress at the interface region is assumed. In the study therein, the jump in the shear stress is inversely proportional to the medium permeability. They proposed a set of interface conditions that were used in Kuznetsov (1996, 1997, 1998a-b, 1999).

Ochoa-Tapia and Whitaker, (1998), also presented a different shear stress jump boundary condition at the interface where the inertia effects are considered.

Goyeau, Lhuillier and Gobin, (2002), presented a study on the momentum transport at the interface between a clear and porous medium using only one set of equations to describe the flow in the two media. They also used a model with two sets of equations (two domains), i.e., employing distinct equations to describe the clear and porous media. Homogeneous and non homogeneous porous regions were investigated using a shear stress jump boundary condition proposed by Ochoa-Tapia and Whitaker (1995a-b). For the homogeneous porous layer, both models are well adjusted. However, the model with only one set of equations using the non homogeneous porous medium show a better description of the momentum variation due to the heterogeneities near the interface.

Rocamora and de Lemos, (2000b), and de Lemos and Pedras, (2000a-b, 2001) developed a macroscopic model of two equations where a constant is introduced in the turbulent kinetic energy equation. The value of this constant is obtained through numerical experimentation applied to a porous medium formed by cylindrical rods with a spatially periodic array.

The development carried out at LCFT/ITA contemplates both a *macroscopic* model, through numerical treatment with one unique set of governing equations (de Lemos and Pedras, (2000a, 2001), Rocamora and de Lemos, (2000a-d) and a *microscopic* model, used in the calibration of the parameters of the macroscopic closure (Pedras and de Lemos, (2001a-d)).

Recently, Silva and de Lemos (2001, 2002a-c, 2003), and de Lemos and Silva, (2002a-b) presented numerical solutions for laminar and turbulent flows in a channel partially filled with porous material taking account the shear stress jump at the interface (Ochoa-Tapia and Whitaker (1995a)).

Although the cited results show a macroscopic treatment of the flow at regions with interface between a clear and porous medium, till this moment, a detailed investigation of the fluid flow in such media is not common in the available literature.

The objective of this work is to investigate the fluid flow in a porous matrix formed by cylindrical rods.

2. Microscopic Model

2.1 Geometry

The flow under consideration is schematically shown in Figure 1 and consists in a channel filled with cylindrical rods. The fluid with constant property flows longitudinally from left to right permeating through the obstacles. Additionally, case of Figure 1 uses a wall condition to the *north* and *south* and spatial periodicity to the *east* and west.

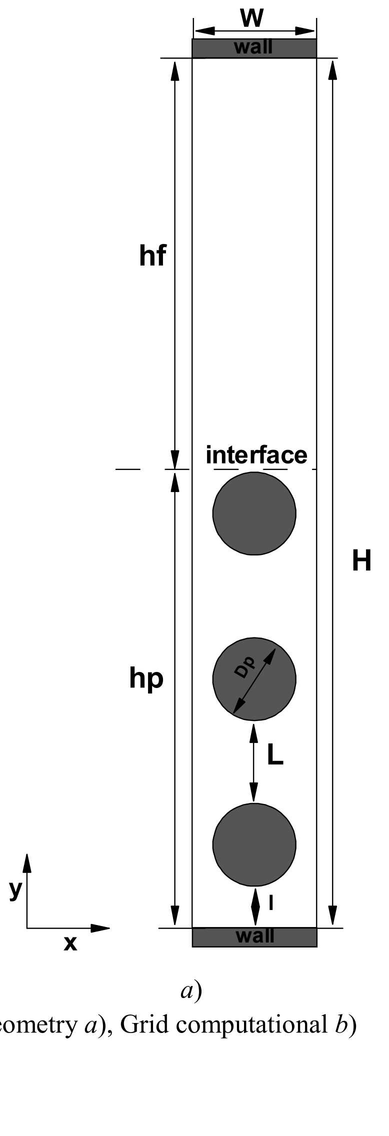
2.2 Governing equations

The microscopic continuity equation is given by.

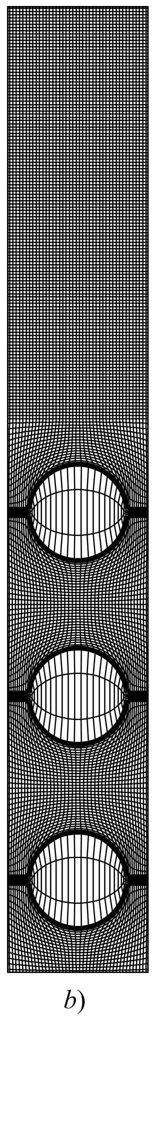
$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

The microscopic Naviek-Stokes equation for an incompressible fluid with constant properties can be written as,

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) \right] = -\nabla p + \mu \nabla^2 \mathbf{u} \tag{2}$$



a)



b)

Figure 1: Geometry *a*), Grid computational *b*)

2.3 Boundary Conditions

Equations (1) and (2) are solved under the following boundary and interface conditions:

$$u|_{y=0} = 0 \tag{3}$$

where h_f is the free flow depth, h_p is the porous depth, $H=h_f+h_p$ is the total depth, W is the length of the channel, μ is the fluid dynamic viscosity and D_p is the solid particle diameter.

$$u|_{y=H} = 0 \tag{4}$$

3. Numerical Model

The numerical method used for discretizing the equation system is the Control volume method of Patankar. In the implementation herein, a system of generalized coordinates was used although all simulation to be shown employed only Cartesian coordinates. Nevertheless, the use of a general system η-ξ for discretizing the equations was found to be adequate for future simulations.

Since the entire derivation herein if set up for solving two-dimensional flows, both cases employ the spatially periodic boundary condition along the x coordinate. This if done in order to simulate fully developed flow for which analytical solutions are available for comparison. The spatially periodic condition is implemented by running the solution repetitively, until outlet profiles in $x=W$ match those at the inlet (x=0).

Figure 2 shows a general control volume in a two-dimensional configuration. The faces of the volume are formed by lines of constant coordinates η-ξ.

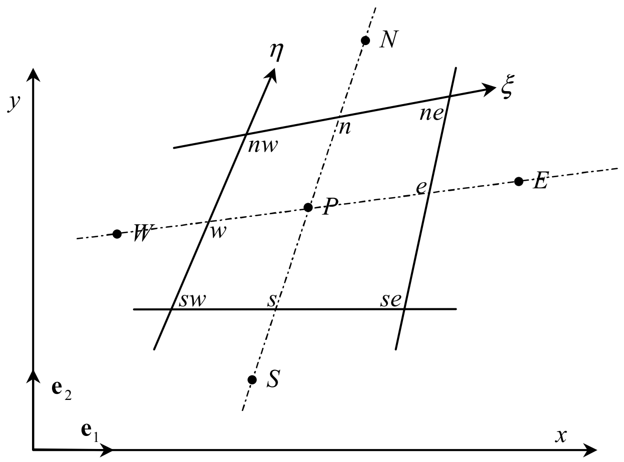


Figure 2: Notation for control volume discretization.

For steady-state, a general form of the discrete equations for a general variable $\boldsymbol{\varphi}$ becomes,

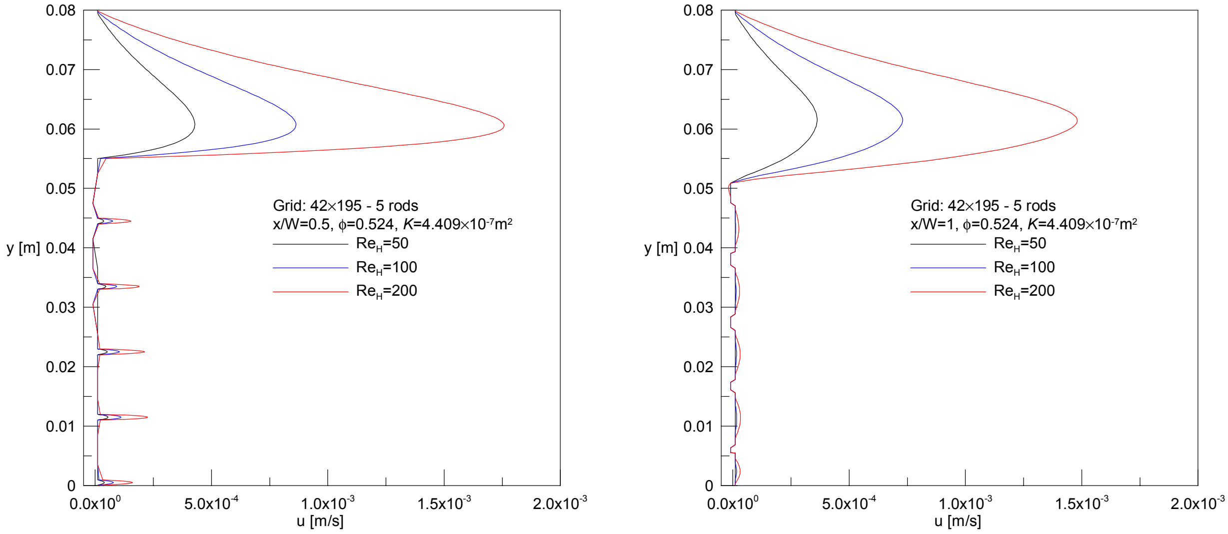
$$I_e + I_w + I_n + I_s = S_{\boldsymbol{\varphi}} \tag{5}$$

where I_e, I_w, I_n e I_s are the fluxes of $\boldsymbol{\varphi}$ at faces *east*, *west*, *north* and *south* of the control volume of Figure 2, respectively, and S_φ is a source term. The fully discretization of the transport equation is reported in Pedras and de Lemos, (2001b). Here, all computations were carried out until the residue of the algebraic equations was brought down to 10^{-9}, where the residue was defined as the difference between the right and left sides of the discretized equations.

Figure 1*b*) shows the used computational grid, having 42 grid nodes in the longitudinal x-direction and 259 nodes along the cross-stream y coordinate. The spatially periodic boundary condition was applied along the main flow direction in order to simulate fully developed flow.

4. Results and discussion

The Figure 3*a*) and 3b) show the effect of the Reynolds number in the velocity fields with ϕ=0.524, K=4.409×10^{-7}m^2, for a computational grid having 42 grid nodes, in the axial direction , x and 195 grid nodes in the transversal direction , y , at the positions x/W=0.5 and x/W=1, respectively. One notes that, the higher the Reynolds number, the higher the global mass flux.



a) *b*)

Figure 3: Effect of Reynolds number for H=0.08m: *a*) section at x/W=0.5; b) section at x/W=1.

Figure 4*a*) and 4*b*) show the influence of the porosity,ϕ and the permeability, K, in the velocity fields. It is clearly seen from the Figure 4*a*) and 4*b*), the higher the porosity-permeability, the higher the mass flux at the porous region and, consequently, the lower the flux at the clear region, for both x/W=0.5 and x/W=1 positions, respectively.

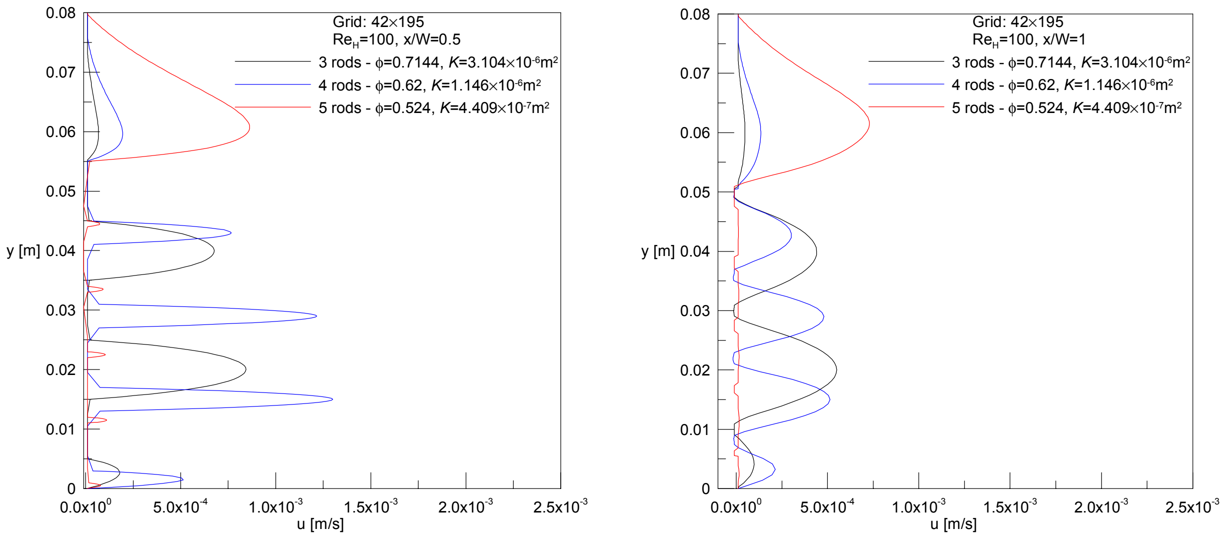
The permeability, K, is calculated using the modified Ergun equation for cylindrical rods [Kuwahara *et. al.* (1998)]:

$$K = \frac{\phi^3 D_p^2}{144(1-\phi)^2} \tag{6}$$

where ϕ is the porosity and D_p is the cylindrical rod diameter.

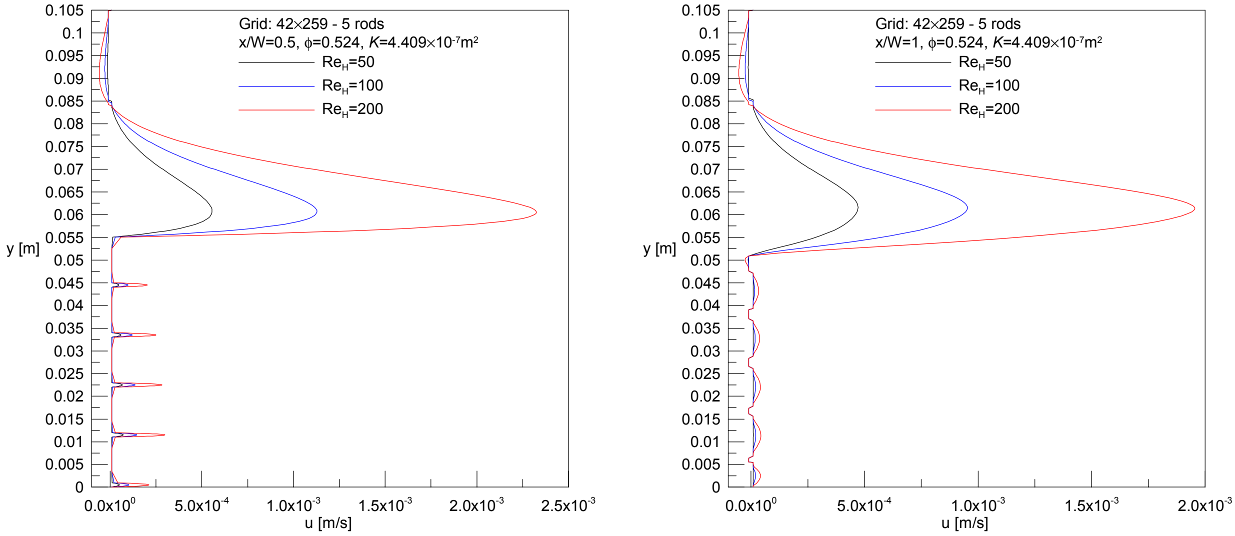
The Figure 5*a*) and 5*b*) show the effect of the Reynolds number in the velocity profiles, for H=0.105m, Re_H=100 using a mesh having 42 grid nodes in the longitudinal direction, x and 195 grid nodes in the transversal direction, y, respectively at the x/W=0.5 and x/W=1 positions. One verifies that, the higher the Reynolds number, the higher the global mass flux.

Figure 6*a*) and 6*b*) show the effect of the porosity and permeability in the velocity profiles. It is clearly seen from the Figure 6*a*) and 6*b*), as expected, the higher the ϕ-K, the higher the mass flux at the porous region and consequently, the lower the mass flux at the clear region. It is important to emphasize that the porosity, ϕ and the permeability K are correlate by Eq (6).



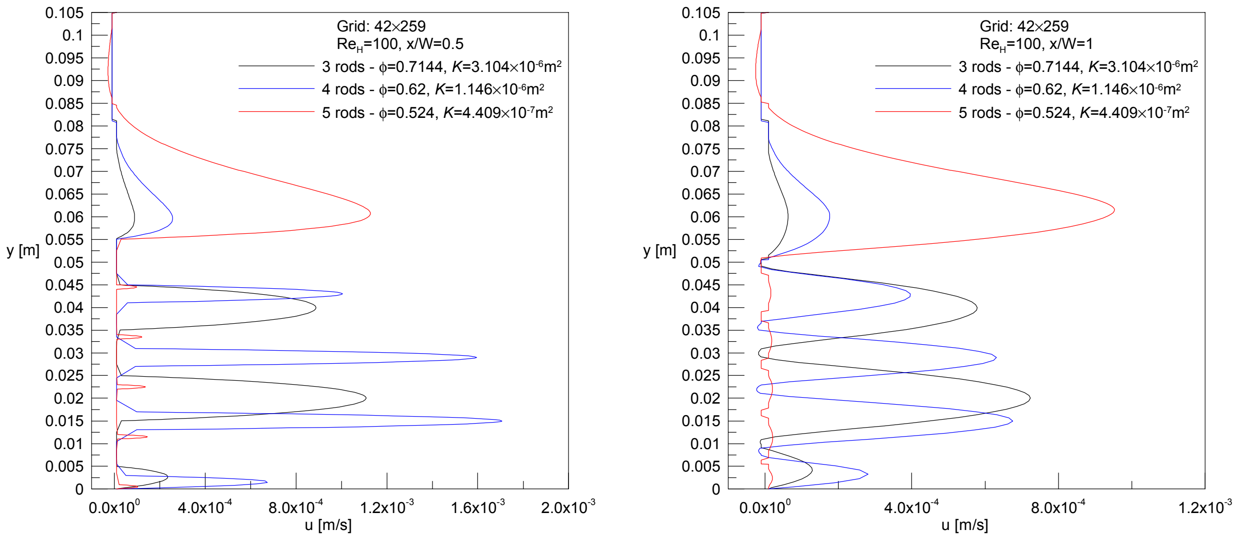
a) *b*)

Figure 4: Effect of porosity, ϕ, in the velocity fields for H=0.08m: *a*) section at x/W=0.5; b) section at x/W=1.



a) *b*)

Figure 5: Effect of Reynolds number for H=0.105m: *a*) section at x/W=0.5; b) section at x/W=1.



a) *b*)

Figure 6: Effect of the porosity, ϕ, in the velocity fields for H=0.08m: *a*) section at x/W=0.5; b) section at x/W=1

Table 1 and 2 show the pressure drop along the channel for H=0.08m and H=0.105m, respectively. One note that, the higher the Reynolds number or the porosity, the higher the pressure drop along the channel. The results of Table 1 for the pressure drop were obtained as follows:

$$\overline{\Delta p} = \frac{1}{A_t}\int_{A_t}(p_{\text{inlet}} - p_{\text{outlet}})dA \tag{7}$$

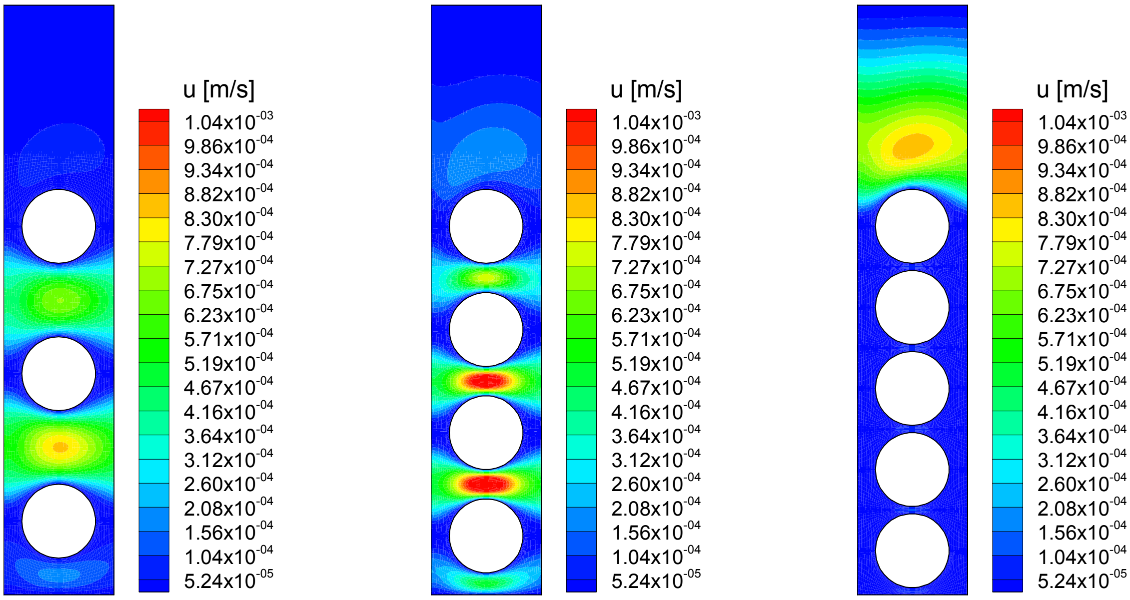
where A_t is the transversal channel area, ϕ is the porosity, p_{inlet} is the inlet pressure and p_{outlet} is the outlet pressure.

Table 1: Pressure drop for H=0.08m.

D_p=0.01m, h_p=0.055m, h_f=0.025m, H=0.08m, W=0.015m	$\overline{\Delta p}\,[\text{N/m}^2]\times 10^{-5}$
5 rods, ϕ=0.524, K=4.409×10^{-7}m², l=L=0.001m	
Re_H=50	1.33134
Re_H=100	2.6507
Re_H=200	5.4391
Re_H=100	
3 rods, ϕ=0.7144, K=3.104×10^{-6}m², l=0.005m, L=0.01m	0.8321
4 rods, ϕ=0.62, K=1.146×10^{-6}m², l=0.003m, L=0.004m	3.9501
5 rods, ϕ=0.524, K=4.409×10^{-7}m², l=L=0.001m	2.6507

Table 2: Pressure drop for H=0.105m.

D_p=0.01m, h_p=0.055m, h_f=0.05m, H=0.105m, W=0.015m	$\overline{\Delta p}\left[\mathrm{N/m^2}\right]\times10^{-5}$
5 rods, ϕ=0.524, K=4.409×10^{-7}m^2, l=L=0.001m	
Re_H=50	1.3024
Re_H=100	2.6455
Re_H=200	5.4936
Re_H=100	
3 rods, ϕ=0.7144, K=3.104×10^{-6}m^2, l=0.005m, L=0.01m	0.83239
4 rods, ϕ=0.62, K=1.146×10^{-6}m^2, l=0.003m, L=0.004m	3.9504
5 rods, ϕ=0.524, K=4.409×10^{-7}m^2, l=L=0.001m	2.6455



a) *b*) *c*)

Figure 7: Velocity Field for Re_H=100 e H=0.08m: *a*) 3 rods (ϕ=0.7144, K=3.104×10^{-6}m^2), *b*) 4 rods (ϕ=0.62, K=1.146×10^{-6}m^2), *c*) 5 rods (ϕ=0.524, K=4.409×10^{-7}m^2).

Figure 7 shows velocity fields in the axial direction, x, for 3, 4 and 5 rods, respectively. One note that the higher the number of cylindrical rods the lower the porosity and permeability, consequently, the lower the mass flux in the rods region and the higher the mass flux in the clear region.

5. Concluding Remarks

Numerical solutions for laminar flow in a composite channel were obtained for different values of Re_H and ϕ-K properties. The porous matrix is modeled as an array of cylindrical rods. Governing equations were discretized are results in the fluid region. Results herein may contribute to the analysis of important environmental and engineering flows where a detailed analysis is of great importance.

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