COMPUTATION OF TURBULENT NATURAL CONVECTION IN A COMPOSITE CAVITY

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Abstract. *Turbulent natural convection in a two-dimensional vertical composite square cavity, isothermally heated at the left side and cooled from the opposing side, is numerically analyzed using the finite volume method in a generalized coordinate system. Governing equations are written in terms of primitive variables and are recast into a general form. The composite square cavity is formed by three distinct regions, namely, clear, porous and solid region. Accordingly, the development of a numerical tool able to treat all these regions as one computational domain is of advantage for engineering design of thermal systems.*

Keywords. *Natural Convection, Turbulence Modeling, Porous Media, Heat Transfer.*

1. Introduction

The analysis of buoyancy driven flows in an enclosed cavity provides useful information for evaluating the robustness and performance of numerical methods dealing with viscous flow problems. The importance of the enclosure natural-convection phenomena can best be appreciated by noting several of their application areas. The optimal design of furnaces and solar collectors - contributing to energy losses minimization - nuclear reactor safety and insulation, ventilation rooms and crystal growth in liquids are some examples of applications of heat removal or addition by free convection mechanism.

Natural convection occurs in enclosures as a result of gradients in density which, in turn, is due to variations in temperature or mass concentration. Natural convection in an infinite horizontal layer of fluid, heated from below, has received extensive attention since the beginning of the 20th century when Bérnard (1901) observed hexagonal roll cells upon the onset of convection in molten spermaceti with a free upper surface. The work of Rayleigh (1926) was the first to compute a critical value, Ra_c, for the onset of convection. The accepted theoretical value of this dimensionless group is 1708 for infinite rigid upper and lower surfaces. The study of natural convection in enclosures still attracts the attention of researchers and a significant number of experimental and theoretical works have been carried out mainly from the 80's.

During the conference on Numerical Methods in Thermal Problems, which took place in Swansea, Jones (1979) proposed that buoyancy-driven flow in a square cavity would be a suitable vehicle for testing and validating computer codes. Following discussions at Swansea, contributions for the solution of the problem were invited. A total of 37 contributions from 30 groups in nine different countries were received. The compilation and discussion of the main contributions yielded the classical benchmark of de Vahl Davis (1983).

The first to introduce a turbulence model in their calculations were Markatos & Pericleous (1984). They performed steady 2-D simulations for Ra up to 10^{16} and presented a complete set of results. Ozoe et al (1985) used the same turbulence model adopted by them for 2-D calculations up to $Ra = 10^{11}$.

Henkes et al. (1991)), performed 2-D calculations using various versions of the k-ε turbulence model. These versions included the standard as well as the Low-Reynolds number k-ε models. A comparison with experimental results for Nu showed the superiority of the Low-Reynolds number k-ε closures.

Fusegi et al (1991), presented 3-D calculations for laminar flow for Ra up to 10^{10}. Their graphs revealed the 3-D character of the flow. Comparisons were made with 2-D simulations and differences were reported for the heat transfer correlation between Nu and Ra.

A recent paper by Barakos et al. (1994), reworked the problem for laminar and turbulent flows for a wide range of Ra. Turbulence was modeled with the standard k-ε closure and the effect of the assumed wall functions on heat transfer were investigated.

Thermal convection in porous media has been studied extensively in recent years. Underground spread of pollutants, grain storage, food processing are just some applications of this theme. The monographs of Nield & Bejan (1992) and Ingham & Pop (1998) fully document natural convection in porous media.

The case of free convection in a rectangular cavity heated on a side and cooled at the opposing side is an important problem in thermal convection in porous media. Walker & Homsy (1978), Bejan (1979), Prasad & Kulacki (1984), Beckermann et al. (1986), Gross et al (1986), Manole & Lage (1992) have contributed with some important results to this problem.

The recent work of Baytas & Pop (1999), concerned a numerical study of the steady free convection flow in rectangular and oblique cavities filled with homogeneous porous media using a nonlinear axis transformation. The Darcy momentum and energy equations are solved numerically using the (ADI) method.

Traditionally, modeling of macroscopic transport for incompressible flows in porous media has been based on the volume-average methodology for either heat Hsu & Cheng (1990) or mass transfer Bear (1972),Whitaker (1966),Whitaker (1967). If time fluctuations of the flow properties are also considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: *a*) application of time-average operator followed by volume-averaging Masuoka & Takatsu (1996), Kuwahara & Nakayama (1998) and Nakayama & Kuwahara (1999), or *b*) use of volume-averaging before time-averaging is applied Lee & Howell (1987), Antohe & Lage (1997)-Getachewa et al (2000). However, both sets of macroscopic mass transport equations are equivalent when examined under the recently established *double decomposition* concept Pedras & de Lemos (2000), Pedras & de Lemos (2001a), Pedras & de Lemos (2001b) and Pedras & de Lemos (2001c). This methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature Rocamora & de Lemos (2000) and de Lemos & Rocamora (2002). A general classification of all proposed models for turbulent flow and heat transfer in porous media has been recently published de Lemos & Pedras (2001).

Following this path, the work of Braga & de-Lemos (2002a), presented results for laminar natural convection in a square cavity heated on the sides. Later, Braga & de-Lemos (2002b) extended their results for considering laminar natural convection in a horizontal annular cavity. Turbulent regime in horizontal cylindrical annuli, both for concentric and eccentric cases, was also calculated (Braga & de-Lemos (2002c). Further, the study of natural convection in cavities completely filled with porous material was reported in the work of Braga & de-Lemos (2002d). In that work the two geometries mentioned above, namely square and annular cavities were considered. The turbulent natural convection in enclosures with clear fluid and completely filled with porous material was investigated in Braga & de-Lemos (2002e). Finally, the modeling of turbulent natural convection in saturated rigid porous media was presented in de Lemos & Braga (2003).

Motivated by the foregoing work, this paper presents results for both laminar and turbulent flows in a composite cavity heated from the left side and cooled from the opposing side. The enclosure is formed by three distinct regions, namely, clear, porous and solid region. The turbulence model here adopted is the standard k-ε with wall function.

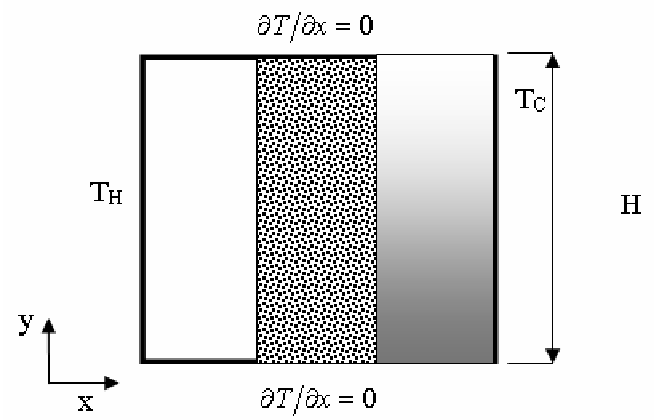

Figure 1 – Geometry under consideration.

2. The Problem Considered

The problem considered is shown schematically in Fig. (1), and refers to the two-dimensional flow of a Boussinesq fluid of Prandtl number 1 in a square cavity of side H=1m. The cavity is assumed to be of infinite depth along the z-axis and is isothermally heated from the left side and cooled from the opposing side. The composite square cavity is equally divided in three distinct regions, namely, clear, porous and solid region.

The no-slip condition is applied for velocity and the resulting flow is treated as steady. The controlling parameter is the Rayleigh number, $Ra = \frac{g\beta H^3 \Delta T}{\nu\alpha}$. Further, a relationship between the porosity, permeability and the particle diameter is given by $D_p = \sqrt{\frac{144K(1-\phi)^2}{\phi^3}}$.

3. Governing Equations

The equations used herein are derived in details in the work of Pedras & de Lemos (2001a), de Lemos & Rocamora (2002).and de Lemos & Braga (2003).

Basically, for porous media analysis, a macroscopic form of the governing equations is obtained by taking the volumetric average of the entire equation set. In that development, the porous medium is considered to be rigid and satured by an incompressible fluid.

The macroscopic continuity equation is given by,

$$\nabla \cdot \bar{\mathbf{u}}_D = 0 \tag{1}$$

The Dupuit-Forchheimer relationship, $\bar{\mathbf{u}}_D = \phi \langle \bar{\mathbf{u}} \rangle^i$, has been used and $\langle \bar{\mathbf{u}} \rangle^i$ identifies the intrinsic (liquid) average of the local velocity vector $\bar{\mathbf{u}}$. The macroscopic time-mean Navier-Stokes (NS) equation for an incompressible fluid with constant properties is given as,

$$\rho \left[\frac{\partial \bar{\mathbf{u}}_D}{\partial t} + \nabla \cdot \left(\frac{\bar{\mathbf{u}}_D \bar{\mathbf{u}}_D}{\phi} \right) \right] = -\nabla \left(\phi \langle \bar{p} \rangle^i \right) + \mu \nabla^2 \bar{\mathbf{u}}_D + \nabla \cdot \left(-\rho \phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i \right) - \rho \beta_\phi \mathbf{g} \phi \left(\langle \bar{T} \rangle^i - T_{ref} \right) - \left[\frac{\mu \phi}{K} \bar{\mathbf{u}}_D + \frac{c_F \phi \rho |\bar{\mathbf{u}}_D| \bar{\mathbf{u}}_D}{\sqrt{K}} \right] \tag{2}$$

When treating turbulence with statistical tools, the correlation $-\rho \overline{\mathbf{u}'\mathbf{u}'}$ appears after application of the time-average operator to the local instantaneous NS equation. Applying further the volume-average procedure to this correlation results in the term $-\rho \phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i$. This term is here recalled the **Macroscopic Reynolds Stress Tensor** (MRST). Further, a model for the (MRST) in analogy with the Boussinesq concept for clear fluid can be written as:

$$-\rho \phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i = \mu_{t_\phi} 2 \langle \bar{\mathbf{D}} \rangle^v - \frac{2}{3} \phi \rho \langle k \rangle^i \mathbf{I} \tag{3}$$

where

$$\langle \bar{\mathbf{D}} \rangle^v = \frac{1}{2} \left[\nabla \left(\phi \langle \bar{\mathbf{u}} \rangle^i \right) + \left[\nabla \left(\phi \langle \bar{\mathbf{u}} \rangle^i \right) \right]^T \right] \tag{4}$$

is the macroscopic deformation rate tensor, $\langle k \rangle^i$ is the intrinsic average for k and μ_{t_ϕ} is the macroscopic turbulent viscosity. The macroscopic turbulent viscosity, μ_{t_ϕ}, is modeled similarly to the case of clear fluid flow and a proposal for it was presented in Pedras & de Lemos (2001a) as,

$$\mu_{t_\phi} = \rho C_\mu \frac{\langle k \rangle^{i\,2}}{\langle \varepsilon \rangle^i} \tag{5}$$

In a similar way, applying both time and volumetric average to the microscopic energy equation, for either the fluid or the porous matrix, two equations arise. Assuming further the **Local Thermal Equilibrium Hypothesis,** which considers $\langle \overline{T_f} \rangle^i = \langle \overline{T_s} \rangle^i = \langle \bar{T} \rangle^i$, and adding up these two equations, one has,

$$\begin{aligned} \left\{ (\rho c_p)_f \phi + (\rho c_p)_s (1-\phi) \right\} \frac{\partial \langle \bar{T} \rangle^i}{\partial t} + (\rho c_p)_f \nabla \cdot \left(\bar{\mathbf{u}}_D \langle \bar{T} \rangle^i \right) = \nabla \cdot \left\{ \left[k_f \phi + k_s (1-\phi) \right] \nabla \langle \bar{T} \rangle^i \right\} + \nabla \cdot \left[\frac{1}{\Delta V} \int_{A_i} \mathbf{n} \left(k_f \overline{T_f} - k_s \overline{T_s} \right) dS \right] \\ - (\rho c_p)_f \nabla \cdot \left[\phi \left(\langle {}^i \bar{\mathbf{u}} {}^i \overline{T_f} \rangle^i + \overline{\langle \mathbf{u}' \rangle^i \langle T_f' \rangle^i} + \langle \overline{{}^i \mathbf{u}' \, {}^i T_f'} \rangle^i \right) \right] \end{aligned} \tag{6}$$

where to each underscored term on the right hand side of Eq. (6), the following significance can be attributed: I-**Tortuosity -** based on the stagnant heat path inside the porous medium, II-**Turbulent Heat Flux -** due to the macroscopic time fluctuations of the velocity and the temperature, III-**Thermal Dispersion** - associated to the spatial deviations of the time averaged microscopic velocity and temperature. Note that this term is also present in laminar flows in porous media. IV-**Turbulent Thermal Dispersion** - due to both time fluctuations and spatial deviations of the microscopic velocity and temperature.

A modeled form of equation (6) has been given in detail in the work of de Lemos & Rocamora (2002), and Rocamora & de-Lemos (2002), as,

$$\left\{ (\rho c_p)_f \phi + (\rho c_p)_s (1-\phi) \right\} \frac{\partial \langle \overline{T} \rangle^i}{\partial t} + (\rho c_p)_f \nabla \cdot \left(\mathbf{u}_D \langle \overline{T} \rangle^i \right) = \nabla \cdot \left\{ \mathbf{K}_{eff} \cdot \nabla \langle \overline{T} \rangle^i \right\} \tag{7}$$

where, $\mathbf{K}_{eff}$, given by:

$$\mathbf{K}_{eff} = \left[\phi k_f + (1-\phi) k_s \right] \mathbf{I} + \mathbf{K}_{tor} + \mathbf{K}_t + \mathbf{K}_{disp} + \mathbf{K}_{disp,t} \tag{8}$$

is the effective conductivity tensor. In order to be able to apply Eq. 7, it is necessary to determine the conductivity tensors in Eq. 8, *i.e.*, $\mathbf{K}_{tor}$, $\mathbf{K}_t$, $\mathbf{K}_{disp}$ and $\mathbf{K}_{disp,t}$. Following Kuwahara & Nakayama (1998), this can be accomplished for the tortuosity and thermal dispersion conductivity tensors, $\mathbf{K}_{tor}$ and $\mathbf{K}_{disp}$, by making use of a unit cell subjected to periodic boundary conditions for the flow and a linear temperature gradient imposed over the domain. The conductivity tensors are then obtained directly from the microscopic results for the unit cell (see Kuwahara & Nakayama (1998) for details on the expressions here used).

The turbulent heat flux and turbulent thermal dispersion terms, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, which cannot be determined from such a microscopic calculation, are modeled here through the Eddy diffusivity concept, similarly to Nakayama & Kuwahara (1999). It should be noticed that these terms arise only if the flow is turbulent, whereas the tortuosity and the thermal dispersion terms exist for both laminar and turbulent flow regimes.

Starting out from the time averaged energy equation coupled with the microscopic modeling for the 'turbulent thermal stress tensor' through the Eddy diffusivity concept, one can write, after volume averaging,

$$-(\rho c_p)_f \langle \overline{\mathbf{u}' T_f'} \rangle^i = (\rho c_p)_f \frac{\nu_{t_\phi}}{\sigma_T} \nabla \langle \overline{T}_f \rangle^i \tag{9}$$

where the symbol ν_{t_ϕ} expresses the macroscopic Eddy viscosity, $\mu_{t_\phi} = \rho_f \nu_{t_\phi}$, given by (5) and σ_T is a constant. According to equation 9, the macroscopic heat flux due to turbulence is taken as the sum of the turbulent heat flux and the turbulent thermal dispersion found by de Lemos & Rocamora (2002). In view of the arguments given above, the turbulent heat flux and turbulent thermal dispersion components of the conductivity tensor, $\mathbf{K}_t$ and $\mathbf{K}_{disp,t}$, respectively, are expressed as:

$$\mathbf{K}_t + \mathbf{K}_{disp,t} = \phi (\rho c_p)_f \frac{\nu_{t_\phi}}{\sigma_T} \mathbf{I} \tag{10}$$

In the equation set shown above, when the variable ϕ=1, the domain is considered as a clear medium. For any other value of ϕ, the domain is treated as a porous medium.

4. Numerical Method and Solution Procedure

The numerical method employed for discretizing the governing equations is the control-volume approach with a collocated grid. A hybrid scheme, Upwind Differencing Scheme (UDS) and Central Differencing Scheme (CDS), is used for interpolating the convection fluxes. The well-established SIMPLE algorithm (Patankar & Spalding, (1972)) is followed for handling the pressure-velocity coupling. Individual algebraic equation sets were solved by the SIP procedure of Stone, (1968).

5. Turbulence Model

Transport equations for $\langle k \rangle^i = \langle \overline{\mathbf{u}' \cdot \mathbf{u}'} \rangle^i / 2$ and $\langle \varepsilon \rangle^i = \mu \langle \overline{\nabla \mathbf{u}' : (\nabla \mathbf{u}')^T} \rangle^i / \rho$ in their so-called High Reynolds number form are proposed in Pedras & de Lemos (2001a) and extended in de Lemos & Braga (2003) to incorporate the buoyant effects as:

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle k\rangle^i\right)+\nabla\cdot\left(\bar{\mathbf{u}}_D\langle k\rangle^i\right)\right]=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla\left(\phi\langle k\rangle^i\right)\right]+P^i+G^i+G^i_\beta-\rho\phi\langle\varepsilon\rangle^i \quad (11)$$

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\varepsilon\rangle^i\right)+\nabla\cdot\left(\bar{\mathbf{u}}_D\langle\varepsilon\rangle^i\right)\right]=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right]+c_1P^i\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}+c_2\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}G^i+c_1c_3G^i_\beta\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}-c_2\rho\phi\frac{\langle\varepsilon\rangle^{i\,2}}{\langle k\rangle^i} \quad (12)$$

where c_1, c_2 , c_3 and c_k are constants, $P^i=\left(-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\bar{\mathbf{u}}_D\right)$ is the production rate of $\langle k\rangle^i$ due to gradients of $\bar{\mathbf{u}}_D$, $G^i=C_k\rho\frac{\phi\langle k\rangle^i|\bar{\mathbf{u}}_D|}{\sqrt{K}}$ is the generation rate of the intrinsic average of k due to the action of the porous matrix and $G^i_\beta=\phi\frac{\mu_{t_\phi}}{\sigma_t}\mathbf{g}\beta_\phi\nabla\langle\bar{T}\rangle^i$ is the generation rate of $\langle k\rangle^i$ due to the buoyant effects.

6. Results and Discussion

When a substance that is a mixture of two or more constituents undergoes a phase change from liquid to solid, or vice-versa, a partially solidified region often forms. In many circumstances, the solid forms a rigid framework with respect to which the liquid may move and the region is called mushy-zone. An elegant manner to study that region is to consider it as a porous medium.

The composite cavity is isothermally heated from the left side and cooled from the opposing side. The other two walls are insulated. The porous medium is considered to be rigid and saturated with an incompressible fluid. The Rayleigh number is calculated as in the case of a clear cavity.

Calculations for turbulent flow were performed for all cases using a 120x80 grid with several points inside the boundary layer. Figures (2) and (3) show the isotherms and streamlines of a composite square cavity for *Ra* number ranging from 10^4 to 10^{10}.

For *Ra*=10^4, the isotherms are almost parallel to the heated wall indicating that the main mechanism of heat transfer is conduction, Fig. (2a). The streamlines, not shown here, are a single vortex confined only in the clear region and the flow circulation in the porous medium is almost none.

Increasing the *Ra* number to 10^6, the isotherms in the clear region starts to be distorted due to the increasing of the natural convection. However, in the porous region, the main mechanism of heat transfer is still conduction. Fig. (2b). The streamlines are now stronger than those for *Ra*=10^4, but the flow structure still remains mainly in the clear region. Fig. (3a).

Further increasing *Ra* to 10^8, the isotherms in the clear region are stratified and the convection mechanism is fully developed in such region. In the porous region, the isotherms become distorted due to the part of fluid that crosses significantly the porous layer inducing the natural convection, Fig. (2c). The fluid motion in the clear region is now very intense and the center of the single vortex is moved toward to the heated wall. In the porous region the fluid starts to permeate the porous matrix inducing the natural convection in that region, Fig. (3b).

Finally for *Ra*=10^{10}, the isotherms in both clear and porous region are stratified and the heat transfer in the solid region as a high temperature gradient, Fig. (2d). The fluid movement is stronger in both regions and the natural convection is also developed in the porous medium, Fig (3c).

As proposed by Pedras (2000), the porous matrix contributes with the generation of turbulent kinetic energy such that a new term in the $\langle k\rangle^i$ transport equation was introduced. For a fixed value of the Darcy velocity through a porous bed, the amount of mechanical energy converted into turbulence should depend on the medium properties. For the limiting case of high porosity and permeability media ($\phi\to1\Rightarrow K\to\infty$) no fraction of this available mechanical energy is expected to generate turbulence. The flow, in this situation, behaves like clear fluid flow. As the flow resistance increases, by increasing $\phi/\sqrt{K}$, gradients of local **u** within the pore will contribute to increasing $\langle k\rangle^i$. However, in this case, no extra turbulent kinetic energy is generated due to the presence of the porous medium. In other types of configurations the porous matrix contributes to increasing $\langle k\rangle^i$ and promotes an earlier transition between the laminar and turbulent regimes, like shown in Braga & de Lemos (2003).

Table (1) shows the average Nusselt number in the heated wall for *Ra* ranging from 10^4 to 10^{10}. Table (1) shows that for the range of *Ra* analyzed there are no significant variation between the laminar and turbulent model solution. Probably for higher values of *Ra*, a bifurcation can be found.

Table 1 - Average Nusselt numbers at the hot wall for a composite square cavity for *Ra* ranging from 10^4 to 10^{10} with ϕ=0.95 and D_p=1 mm.

Model Applied \ *Ra*	10^4	10^6	10^8	10^{10}
Laminar solution	1.447	2.132	7.423	19.027
Turbulent solution	1.450	2.160	7.423	19.027

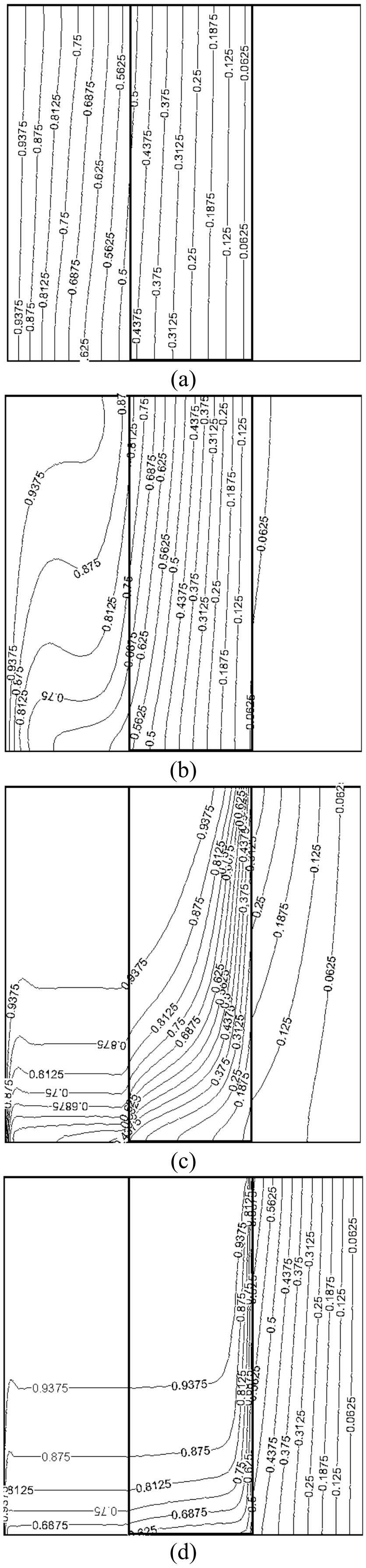

Figure 2 - Turbulent isotherms of a composite square cavity for Ra =10^4, 10^6, 10^8 and 10^{10} with ϕ=0.95 and D_p=1 mm.

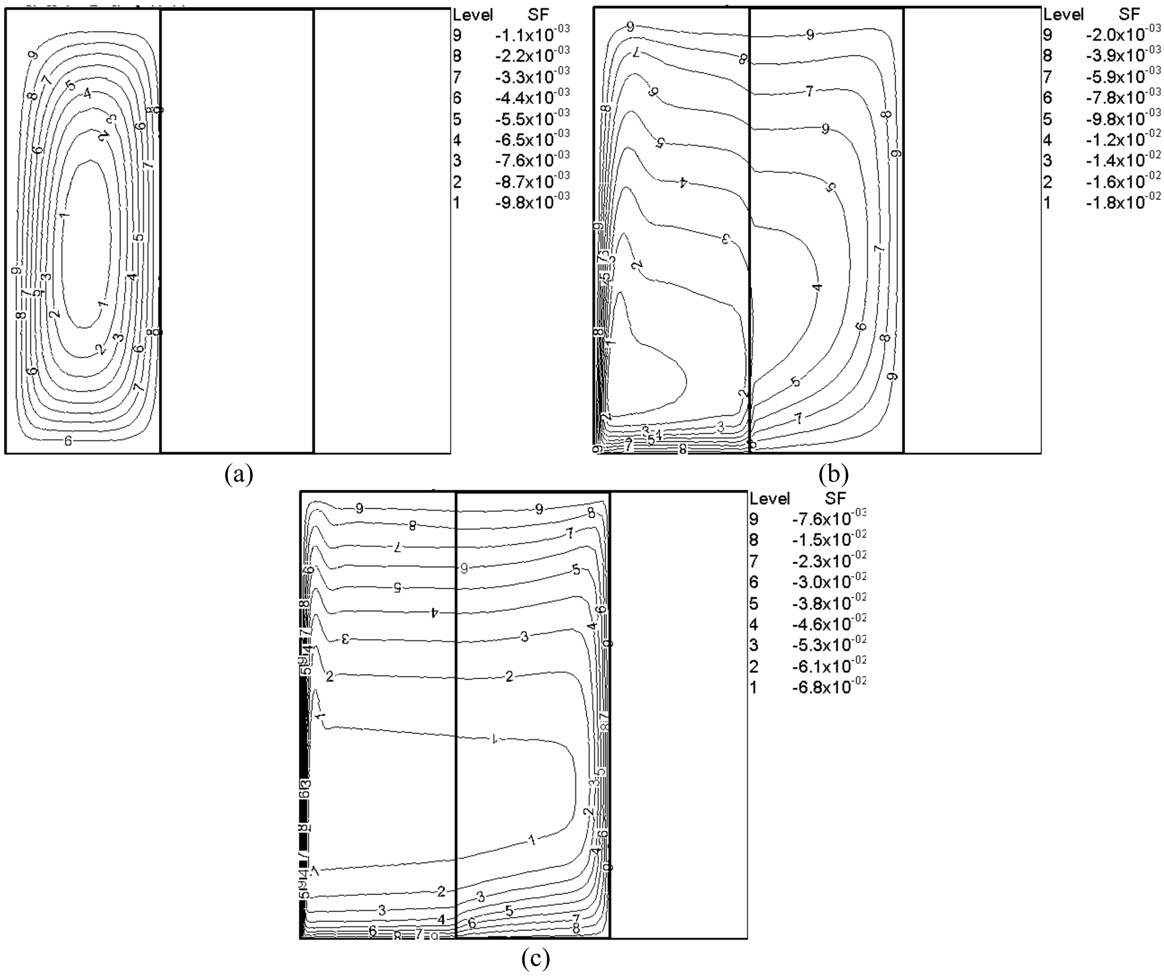

Figure 3 - Turbulent streamlines of a composite square cavity for $Ra=10^6$, 10^8 and 10^{10} with $\phi=0.95$ and $D_p=1$ mm.

7. Concluding Remarks

This paper presented computations for laminar and turbulent flows with the standard k-ε model with a wall function for natural convection in a square composite cavity. It is clearly seen from the figures that the fluid begins to permeate the porous medium for values of Ra greater than 10^6. Nusselt numbers for a square composite cavity show that for the range of Ra analyzed there are no significant variation between the laminar and turbulent model solution. Probably for higher values of Ra, a bifurcation from the laminar branch could be found.

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