

ANALYSIS OF PERFORMANCE OF A HYDROPNEUMATIC SUSPENSION SYSTEM

Maurício Baldi

State University of Campinas
R. Medeleiv, s/n Cidade Universitária CEP:13083-970 Campinas-SP Brasil
baldi@fem.unicamp.br

Pablo Siqueira Meirelles

State University of Campinas
R. Medeleiv, s/n Cidade Universitária CEP:13083-970 Campinas-SP Brasil
pablo@fem.unicamp.br

Abstract: Vehicles used for transport of loads have efforts on their axles very close of the allowed limits, mainly driving on a rough road or during the corners. In this case, the use of conventional suspension system, where the stiffness is constant, can increase the axles overload phenomena. Hydropneumatic suspension system, when used in these vehicles takes an advantage to lead to a better load distribution per axle, decreasing the overload problem and increasing the ride comfort. The well known problem of the damper coefficient changes due to load variation in vehicles using conventional suspension system is even more observable when a hydropneumatic spring is applied due to its non-linearly, opposing the several advantages this spring type brings. And this problem is more accentuated in vehicles that load ranges significantly when they pass from an empty condition to a full load condition.

This study shows a mathematical model of the hydropneumatic spring as well as a numerical model of half vehicle. Through this model, a dynamic behavior comparison was performed between the model with conventional springs and using hydropneumatic springs under the same conditions, adopting different criteria to choose the passive (damping value constant) or adaptive damper value (semi-active suspension), the last based on simple control strategy.

Experimental validation was performed through the use of an agricultural trailer used to spray crop protection, equipped with a projected and assembled suspension system, in the mark of this study.

Hydropneumatic spring, Vehicle Suspension System, non-linear spring.

1. Introduction

Vehicles used for transport of loads have efforts on their axles very close of the allowed limits, mainly driving on a rough road or during the corners. In such cases, the use of conventional suspension system can increase the axles overload phenomena. Hydropneumatic suspension system leads to a better distribution of load per axle, decreasing the overload problem and increasing the ride comfort, as Felez and Vera (1987). The hydropneumatic suspension system consist of a hermetically sealed mass of gas in a chamber that is directly connected to the hydraulic cylinder, as showed in Fig.1. The difference, comparing with conventional suspension system, is the gas spring instead of mechanical spring, and the damping is achieved by the hydraulic fluid passing through the valve (hole), where the energy is dissipated without using additional dampers.

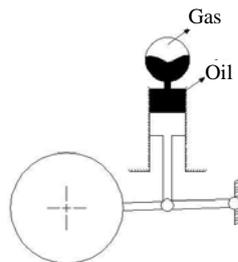


Fig.1 – Hydropneumatic suspension esqueme.

This system has been used in automotive industry aiming to increase the ride comfort and driving safety. The spring flexibility allows continuous and constant contact of the wheels with the ground, as Yohsuke (1999).

The most important item of this system is the gas chamber, therefore the stiffness will be defined, basically, by the volume and pressure of the gas. In some vehicles that use hydropneumatic suspensions, the chamber is built from two parts, that joined, take the shape of a sphere, and the gas is separated from the hydraulic fluid by a flexible diaphragm, as Yohsuke (1999). This diaphragm avoids gas leaks even having hydraulic fluid losses, if the system is not sealed perfectly. The hydraulic fluid drained of the system flows to a reservoir and return to the system through the use of a hydraulic pump, keeping the vehicle height constant. A semi-active control can be done through an adjustable valve that increases or decreases the damping, achieving an optimized system. An active suspension may be achieved by the control of the hydraulic fluid volume, adjusting the vehicle height. This configuration flexibility constitutes one of the most outstanding advantages of hydropneumatic suspension systems.

In this paper, a methodology for primary specification of critical parameters of a hydropneumatic suspension is presented, taking in account the characteristics of such application.

2. Theoretical determination of hydropneumatic spring characteristics

In the following development the gas is considered inert in order to avoid changes of the spring characteristics while it is working. The ideal gas model is considered as well as an isothermal process, which means, the process of compression and expansion of the gas occurs with constant temperature. This assumption does not represent exactly the reality, due to the gas heating. If the system is running in high frequencies, there is not enough time to allow the heat exchange with the environment, and a hysteresis phenomenon will occur, as showed by Els and Grobbelaar (1993). This effect will be not considered once it contributes to dissipate energy of the system.

According to the hypotheses mentioned above,

$$PV=nRT \quad (1)$$

$$PV = \text{constant} \quad (2)$$

where P is the pressure, V is the volume, T is temperature, n is number of moles and R is the universal gas constant.

To find the gas spring stiffness, a function that represents the force (F) done by the hydraulic cylinder as a function of the wheel displacement (x) must be developed. The initial state considered is where force is zero for null displacement. This condition occurs when the gas pressure inside of the chamber is atmospheric, which means, the force in both sides of the cylinder piston are the same. The Fig. 2(a) and (b) shows the forces and pressures for the initial condition and for generic condition, respectively.

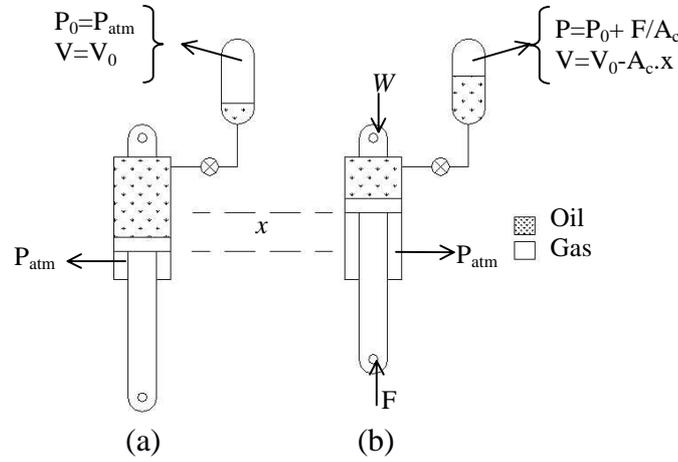


Figure 2. Forces and pressures acting on the hydropneumatic suspension.

To each piston displacement, due to force acting on the vehicle wheel, an oil volume will be displaced to inside or outside of the chamber, which will decrease or increase the gas volume since their initial value V_0 . As the oil volume displaced (V_d) is

$$V_d = A_c x \quad (3)$$

where A_c is the piston area, the final gas volume V inside of the chamber is

$$V = V_0 - x A_c \quad (4)$$

Considering Eq.(2) and Eq.(4) it is possible to see that

$$P_0 V_0 = P(V_0 - x A_c) \quad (5)$$

The force acting on the wheel changes the gas pressure inside the chamber to

$$P = \frac{F}{A_c} + P_0 \quad (6)$$

Replacing Eq.(6) in Eq.(5)

$$\frac{F}{A_c} + P_0 = \frac{P_0 V_0}{V_0 - x A_c} \quad (7)$$

so the force will be obtained multiplying Eq.(7) by the cylinder area A_c .

$$F = A_c \left(\frac{P_0 V_0}{V_0 - A_c x} - P_0 \right) = \frac{P_0 A_c^2 x}{V_0 - A_c x} \quad (8)$$

The Eq. (8) shows that the force as function of piston displacement has a non-linear behavior. Therefore, the spring stiffness coefficient is not constant, as in the conventional systems.

For each displacement x , with origin in the condition that $F(0)=0$, we will define the stiffness K_s as:

$$K_s = \frac{F}{x} \quad (9)$$

Replacing Eq.(8) in Eq.(9), the hydropneumatic spring is given by

$$K_s(x) = \frac{P_0 A_c^2}{V_0 - A_c x} \quad (10)$$

The Eq.(10) shows that the spring stiffness does not depend on the chamber shape where the gas is confined, but depends on the initial pressure, the initial gas volume and the variation of the gas volume. This is an important condition to achieve the best economic project optimization, once a chamber with a simple shape can be used, allowing project costs reduction. As the gas pressure depends on the piston area and the vehicle load, the easiest way to range the spring stiffness is changing the initial gas volume, therefore changes on the hydraulic cylinder can mean high costs. An important issue that must be considered at the spring project conception is when the vehicle load changes, the pressure and the volume of gas change together, resulting in variations on the gas spring stiffness.

The local stiffness K_l is usually intended, for non-linear systems, as the derivative of the force function with respect to the displacement x . Deriving Eq. (8) with respect to x is obtained K_l as:

$$K_l(x) = \frac{P_0 V_0 A_c^2}{(V_0 - A_c x)^2} \quad (11)$$

Therefore, it is interesting to see that this stiffness is suitable for local incremental analysis only. The product of $K_l(x).x$ does not give the value of $F(x)$, due to non-linearity. Then, this definition is not adequate for suspension analysis, because the hypothesis of small displacements is not acceptable, especially for high non-linearity.

For modeling and simulation aims, it is convenient to work with a new displacement variable z with origin at the static equilibrium position x_s and define a stiffness variable coefficient $K(z)$ that leads the difference between the force at a given displacement z and the force at $z=0$. Then,

$$z = x - x_s \quad (12)$$

$$K(z) = \frac{\Delta F}{z} = \frac{F(x) - F(x_s)}{x - x_s} \quad (13)$$

where $F(x)$ and $F(x_s)$ can be calculated using Eq. (8).

With this definition is satisfied that $\Delta F(z)=K(z).z$ for all z .

In numerical simulations the value of $K(0)$ is set by interpolation, avoiding divide by zero.

3. Definition of Initial Gas Volume and Chamber Dimensions

The geometric configuration of all suspension components, the material selection, the dampers valves adjustments and the dimensions of all components are the keys to the suspension system design. These parameters, including other like tire stiffness, suspension stroke, mass of wheel and tire set, define the vehicle response to external stimulation. This way, the optimization process of all components will define the quality of the suspension system. Frequently the behavior of the components are unknown analytically, but the main interest is on the vehicle response and not on the mathematical model of the system. To optimize the components, the manufactures usually confront the responses from systems empirically adjusted with the experience of trial pilots, and modify the characteristics of dampers, springs and other components to get a satisfactory system response, as Picado (1998).

However, some parameters may be set to initial values that are near to the adequate value for the application. This is the case of the spring stiffness in conventional suspensions.

For hydropneumatic suspensions, the non-linear stiffness is function of initial pressure and volume of the gas, and the area of the cylinder. Assuming that the cylinder area is already specified, the gas volume at the atmospheric pressure will define the stiffness of the system. Obviously, the working volume and pressure depend of the cylinder area and vehicle weight, since

$$P_s = \frac{W}{A_c} \quad (14)$$

and

$$V_s = \frac{P_0 V_0}{P_s} \quad (15)$$

where W is the load in each wheel, P_s is the pressure in the static equilibrium, and V_s is the corresponding volume.

Then, additional criteria may be specified to determine initial volume V_0 , and consequently the non-linear stiffness function.

One important advantage in relation to conventional suspension system is that any required change of the stiffness from the initial project can be obtained through changing the initial volume of gas only.

The criteria used in this study to define the hydropneumatic spring stiffness and the initial gas volume assumes a maximum axle displacement (x_m), relatively to the vehicle frame, in the maximum attended load. This criteria reflects a limited suspension stroke capability and thus prevent excessive suspension bottoming, which can lead to deterioration of ride comfort and possible premature structural damage, as Hrovat (1997). The most severe load condition considered happens when the vehicle has its maximum static load (f_{em}) (this kind of vehicle can have a weight difference between empty and full loaded around five times) and the maximal dynamic load factor (f_d) is achieved. The Fig. 3 shows a scheme of the pressure, volume and load for each condition, used for the initial gas volume setup.

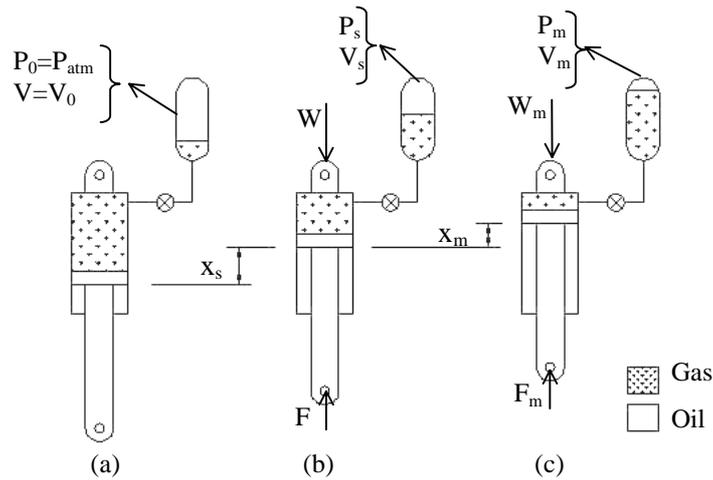


Figure 3. Criteria to define the initial volume of gas.

The condition (a) means force equal zero acting at the suspension. In other words, the pressure inside of the chamber is atmospheric and the displacement of the cylinder is zero. Starting from this position, any load change will result in an alteration of the static position. At condition (b), the maximum static load that the vehicle will transport is acting, resulting in a pressure (P_s) and a volume of gas (V_s) inside of the chamber. The static displacement of the piston in this condition is x_s . At the condition (c), the dynamic load is acting, which means, the bigger load static condition times the dynamic load factor, that results in a pressure (P_m) and a volume of gas (V_m) inside of the chamber. As described before, the criteria used to define the initial volume of gas allows a maximum displacement for the axle, therefore the displacement x_m is assumed as the maximum allowed displacement, happened in the maximum load condition acting on the wheel.

Considering conditions (a) and (b), in Fig. 3, and Eq.(2), is obtained:

$$P_0 V_0 = P_s V_s = P_s (V_0 - A_c x_s) \quad (16)$$

and

$$V_0 = \frac{A_c x_s P_s}{P_s - P_0} \quad (17)$$

To determine the x_s , the condition (a) and (c) must be considered, since the x_m is known, doing:

$$P_0 V_0 = P_m V_m = P_m (V_0 - A_c (x_s + x_m)) \quad (18)$$

Isolating x_s in Eq.(18) and replacing it in Eq.(17), the expression to calculate the initial volume of gas inside of the chamber at atmospheric pressure, following the criteria above, is given by

$$V_0 = \frac{A_c P_m P_s x_m}{P_0 (P_m - P_s)} \quad (19)$$

Note that P_0 is the atmospheric pressure, as mentioned, and P_s and P_m can be easily calculated by

$$P_s = \frac{W}{A_c} \quad (20)$$

and

$$P_m = \frac{W_m}{A_c} \quad (21)$$

Figure 4 shows the theoretical curves of hydropneumatic spring stiffness considering four different load conditions as function of the piston displacement in the range of -50mm to 50mm . The parameters used to define the initial volume are: Internal diameter of the cylinder (d) $0,05\text{m}$, Maximum vehicle static load (W) 8829N , Maximum allowed displacement (x_m) $0,05\text{m}$ and Dynamic load factor (f_d) 3 . Equation (19) gives an initial volume $V_0=0,0067\text{ m}^3$.

Figure 4(a) shows the stiffness obtained when the vehicle is empty. Figure 4(d) when the vehicle is fully loaded, and Fig.4(b) and (c) in two intermediate conditions of load. The respective loads are on the graphics.

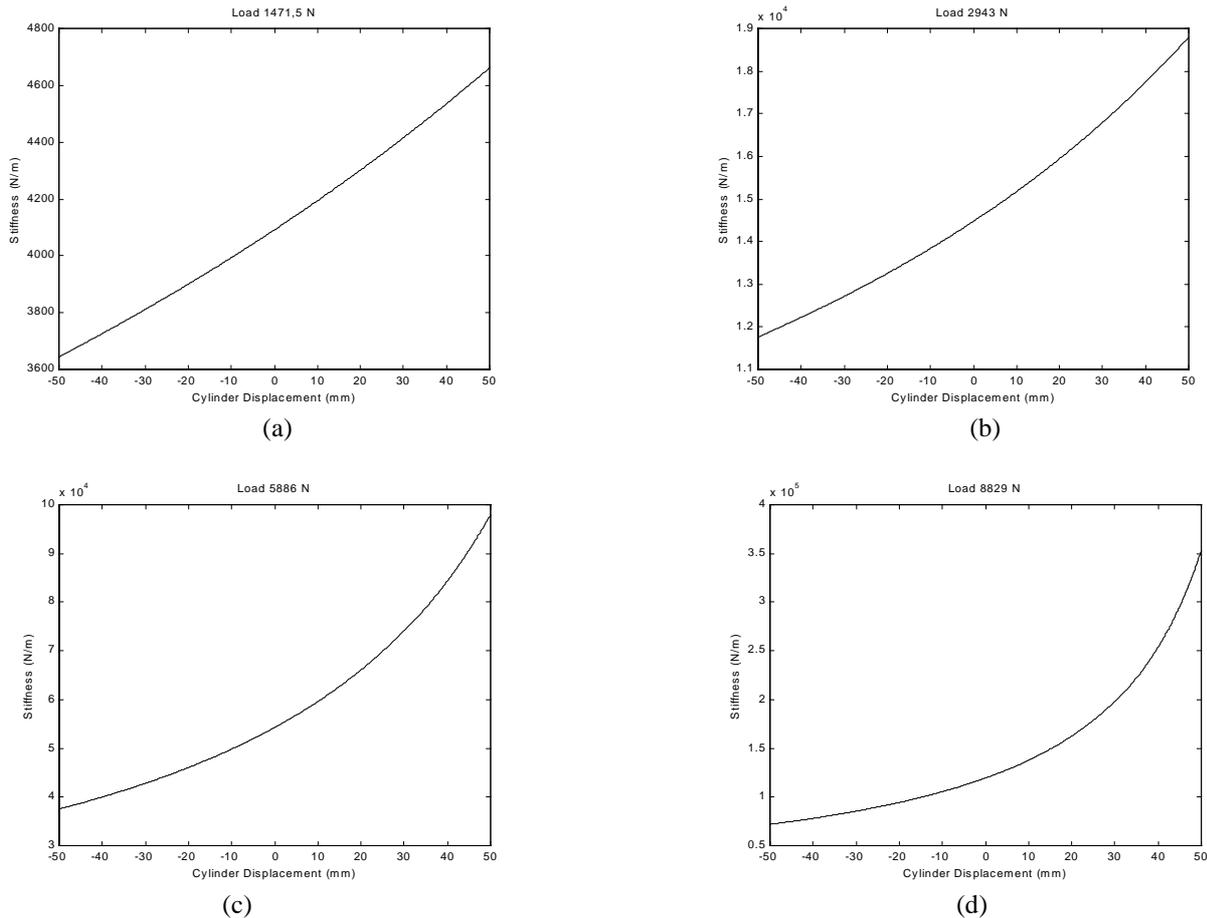


Figure 4. Curves of hydropneumatic spring stiffness.

The cylinder displacement equal zero means that it is at equilibrium position (the vehicle is at correct height). Positives values mean that the gas is being compressed (vehicle is lowering) and expanded for negatives values (vehicle is arising). An important observation that must be taken in account is for each different load condition, the static displacement of the cylinder (x_s) is calculated and added to displacement vector, which means, the oil volume displaced

to the chamber due to increase of vehicle load is subtracted to the initial volume of gas. In practice, it means for each increase of the vehicle static load and consequent displacement of the piston to new equilibrium position, the oil volume must be increased to bring the piston to the zero, which means, the adjustment of vehicle height is obtained modifying the oil volume. Another important point is the addition of oil in the system does not change its stiffness, since this will not change the pressure and volume of gas inside of the chamber.

When the vehicle load decreases, the stiffness decreases too, because the pressure becomes lower, the gas inside of the chamber is expanded, and the wheel displacement x becomes lower, increasing the denominator in Eq.(10) and (11). In a similar way, stiffness increases when the vehicle load rises.

The stiffness rises drastically when the piston is achieving its end, because in this situation, the volume of gas becomes very small and over high compression. It is another advantage of the hydropneumatic suspension system, because if there is a displacement of the wheel bigger than the previewed during the project, the suspension becomes tight very fast, avoiding the piston achieving its end, what can result in damages to the vehicle.

4. Dynamic Behavior Analysis

The purpose of this part of this work is to assess the hydropneumatic suspension system performance, compare with a conventional suspension system and identify what parameters have significantly influences in its performance. The adopted mathematical model to perform the dynamic analysis was a one-quarter of vehicle, showed in Fig.(5). This model is the same for both kinds of suspension, being the suspension spring stiffness K_s the only difference, which means, K_s constant for conventional suspension and given by Eq.(11) for hydropneumatic suspension.

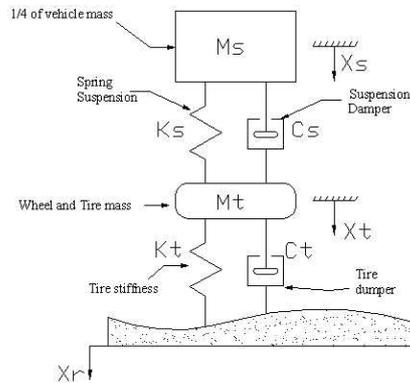


Figure 5. Model of on-quarter of vehicle.

The Tab.(1) describes all variables of the model

Table 1. Variables used in the 1/4 vehicle model.

Variable	Unite	Description
k_s	N/m	Spring stiffness of the suspension
k_t	N/m	Tire stiffness.
c_s	Ns/m	Suspension damper.
c_t	Ns/m	Tire damper.
x_s	m	Vertical displacement of the vehicle mass.
x_t	m	Vertical displacements of the tire and wheel mass.
x_r	m	Profile of the ground.
m_s	kg	Mass of 1/4 of vehicle.
m_t	kg	Mass of wheel and tire.

The equations that describe the model above are in matrix form in Eq.(22).

$$\begin{bmatrix} m_s & 0 \\ 0 & m_t \end{bmatrix} \begin{Bmatrix} \ddot{x}_s \\ \ddot{x}_t \end{Bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s + c_t \end{bmatrix} \begin{Bmatrix} \dot{x}_s \\ \dot{x}_t \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \begin{Bmatrix} x_s \\ x_t \end{Bmatrix} = \begin{Bmatrix} 0 \\ c_t \dot{x}_r + k_t x_r \end{Bmatrix} \quad (22)$$

The variable x_r means the displacement of the wheel due to soil imperfections and the Eq.(22) takes in account the transferred efforts to the system as result of these imperfections. It is assumed as the only external efforts acting on the system and for this reason appear zero in the first coordinate of the independents terms vector. Equation (22) does not take in account the initial displacement due to the weight of the vehicle, once this does not have influence on the dynamic behavior. It means the displacements are measured from the equilibrium state.

The integration method adopted in this work was the Newmark integration method, because it is a direct implicit integrator in the time domain and adequate to analyse non-linear systems. The algorithm used to implement this integrator is showed by Bathe and Wilson (1976), who suggest the parameters α and δ 0,25 and 0,50, respectively, to assure the stability of the system. The integration increment (df) must be 1/10 of the lesser period of the excitation signal. The lesser period corresponds the largest frequency. As the Nyquist frequency is equal $1/(2d_f)$ the condition is satisfied if it is assured that the largest frequency in the signal is less than 1/5 of the Nyquist frequency, as Pablo (1989).

5. Influencies of the damper

The damping choose of a system is extremely important to assure the vehicle designed characteristics of confort and safe. It is not a easy task to be solved mathematically due to several existent variables must be take in account in the math model. The ideal damping is choosed by pratical trials with experienced pilots resopnsable to optimize the suspension, as Picado (1998).

To determine the damper in this work, the Eq.(23), defined in Thomson (1978) for a one-degree of freedom system, is used as a good approach. With this calculated value, the curve time x vehicle mass displacement for an initial pulse is obtained and, if necessary, the damper is changed to adjust this response as desired. The damping coefficient ξ adopted as ideal in this work is 0,7.

$$\xi = \frac{c_s}{2\sqrt{k_s m_s}} \quad (18)$$

In the Eq.(18), can be seen that the damping coefficient can be changed by three variables: the vehicle mass, the damper and the suspension stiffness. Considering the hydropneumatic suspension system, a load change has more influence on the damping coefficient, when compared with conventional systems, because for the first, a load change means a stiffness change as well, as Eq.(11).

To perform the dynamic analysis a trailer used to spray crop protection is considered due to its load can range from 2746,8N up to 17461,8N and brings large changes on the damping coefficient for the hydropneumatic suspension. Three cases are analyzed to determine how much the load change affects the damping coefficient and what possible suggestions to minimize this problem. The ideal damping coefficient considered is 0,7 and the stiffness is achieved by Eq.(11) considering the static condition, once during cyclic excitation, the stiffness is around this value.

1st Case: Damping is defined by the medium vehicle load.

The damper is calculated by Eq.(23), considering a medium load condition, and kept constant for all other load condition. The Tab.(2) shows the values used for this analysis. Can be seen that the damper coefficient has largest changes in the hydropneumatic suspension system.

Table 2. Parameters used in the first case.

m_s [N]	Hydropneumatic Suspension			Conventional Suspension		
	k_s [N/m]	c_s [Ns/m]	ξ	k_s [N/m]	c_s [Ns/m]	ξ
8829	119475,5	5245,78	0,25	529740	21615,52	0,49
5886	54276,8	5245,78	0,46	529740	21615,52	0,61
4414,5	31199,8	5245,78	0,7	529740	21615,52	0,7
2943	14471,0	5245,78	1,26	529740	21615,52	0,86
1471,5	4090,4	5245,78	3,35	529740	21615,52	1,21

The responses for a initial displacement are showed in Fig.(6a) and (b) for the hydropneumatic and conventional system, respectively.

2nd Case: Damping is modified for each static load condition to keep the same ξ for all load condition.

In this case, for each static load, the damper is changed to achieve the desired damping coefficient kept constant during the dynamic analysis. The damping coefficient is kept constant 0,7 for all static load condition. The Tab.(3) shows the values used for this case.

The responses for initial displacement for both type of suspension system are showed in fig.(7) below.

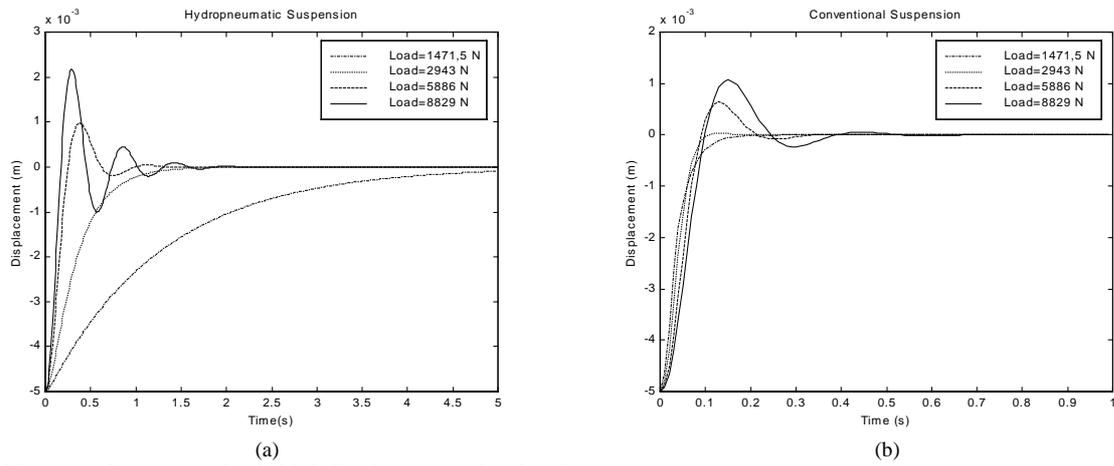


Figure 6. Response for initial displacement for the first case.

Table 3. Parameters used in the second case.

m_s [N]	Hydropneumatic Suspension			Conventional Suspension		
	k_s [N/m]	c_s [Ns/m]	ξ	k_s [N/m]	c_s [Ns/m]	ξ
8829	119475,5	14517,40	0,70	529740	30568,96	0,70
5886	54276,8	7989,34	0,70	529740	24959,45	0,70
2943	14471,0	2917,01	0,70	529740	17649,00	0,70
1471,5	4090,4	1096,62	0,70	529740	12479,73	0,70

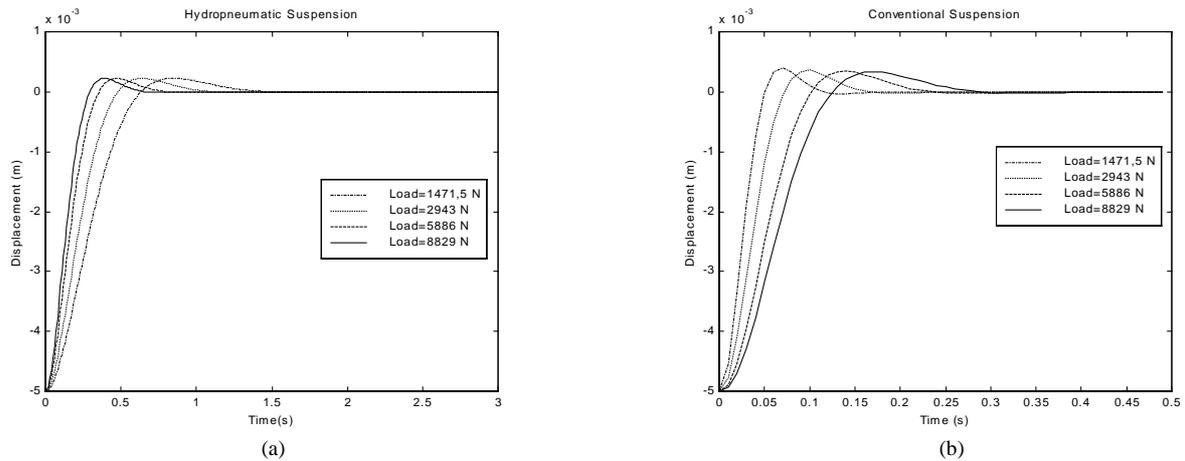


Figure 7. Response for initial displacement for the second case.

3rd Case: Damping is modified the whole time to always maintain the damping factor constant.

The behavior of the suspension is analyzed being the damper value changed constantly to keep the damping factor constantly equal 0,7, which means, whenever it happens an increase of the force of the spring, the force of damping will be reduced and vice-versa. This analysis will not be performed for a conventional suspension, because once the damping was adjusted in the static condition, it will not be changed due to the spring stiffness is constant. The Tab.(4) shows the values for this case, considering the range from the lower wheel position up to the higher wheel position (from -0,05m up to 0,05m).

Table 4. Parameters used in the third case.

m_s [N]	k_s [N/m]		c_s [Ns/m]		ξ
	$k_s(-0,05)$	$k_s(0,05)$	$c_s(-0,05)$	$c_s(0,05)$	
8829	71900,0	351160,0	11261,95	24888,68	0,70
5886	37536,0	97971,0	6643,97	10733,77	0,70
2943	11762,0	18801,0	2629,84	3324,9	0,70
1471,5	3644,2	4661,1	1035,08	1170,63	0,70

The Fig.(8) shows the response for a initial displacement.

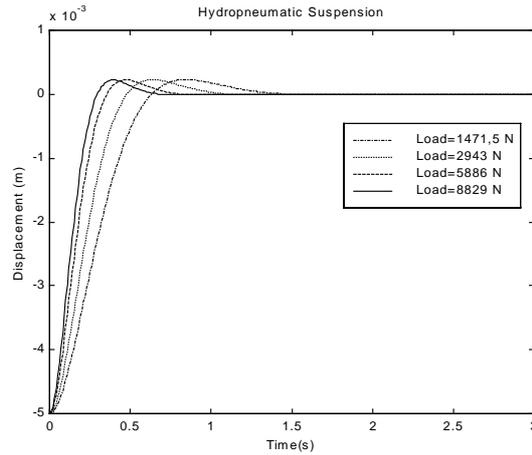


Figure 8. Response for initial displacement for the third case.

In the three cases above, the results suggest that the gain to be obtained with the use of a control of damping semi-active should be more significant for the hydropneumatic than for the conventional suspension system. Only the semi-active control can avoid high damping, when the vehicle is empty, or damping exaggeratedly small when the vehicle is completely charged, in cases where the rate between the vehicle load capacity and the vehicle net weight is very high.

6. Experimental Analysis

Laboratory tests were conducted to verify the damping coefficient variation of a hydropneumatic suspension system through the use of trailer used to spray crop protection, with a projected and assembled hydropneumatic suspension. The experimental validation was carried out through the use of a SCHENK Hydropuls® equipment to excite the trailer. The Fig. 9 shows a schematic diagram of the experimental setup used in this work. Note that the actuator displacement is transferred to wheels by the folded lever assembled on bearings.

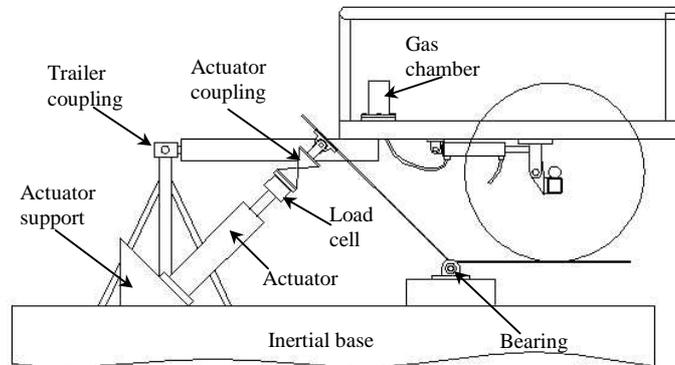


Figure 9. Experimental setup diagram.

The objective of the tests was verify the damping coefficient changes due to load changes in trailer. For convenience, the energy dissipation factor, defined as the ratio between the energy dissipated in each cycle and the elastic energy of deformation is evaluated. Four situation was tested: full loaded, half loaded, empty, and full loaded without suspension. The last setup allows to evaluate the energy dissipated in mechanisms others than the suspension.

The system was excited with constant 5 mm amplitude sinusoidal displacement. The diagram Load/Displacement was registered and are shown in figure 10. We can see the different weight on booth sides. The average, for booth wheels excitation, energy dissipation factors are shown in table 5.

Table 5: Energy dissipation factor in each condition.

Condition of load	Empty	Half loaded	Full	Full without suspension
Energy dissipation (%)	18.99	5.135	4.175	2.644

The three values of the energy dissipation ratio obtained for the system with the hidropneumatic suspension clearly shown that the damping decreases when load increases. The fourth result shown that, ever if other mechanism of energy dissipation are presents in the system, the suspension is responsible for a significant part of the energy absorption.

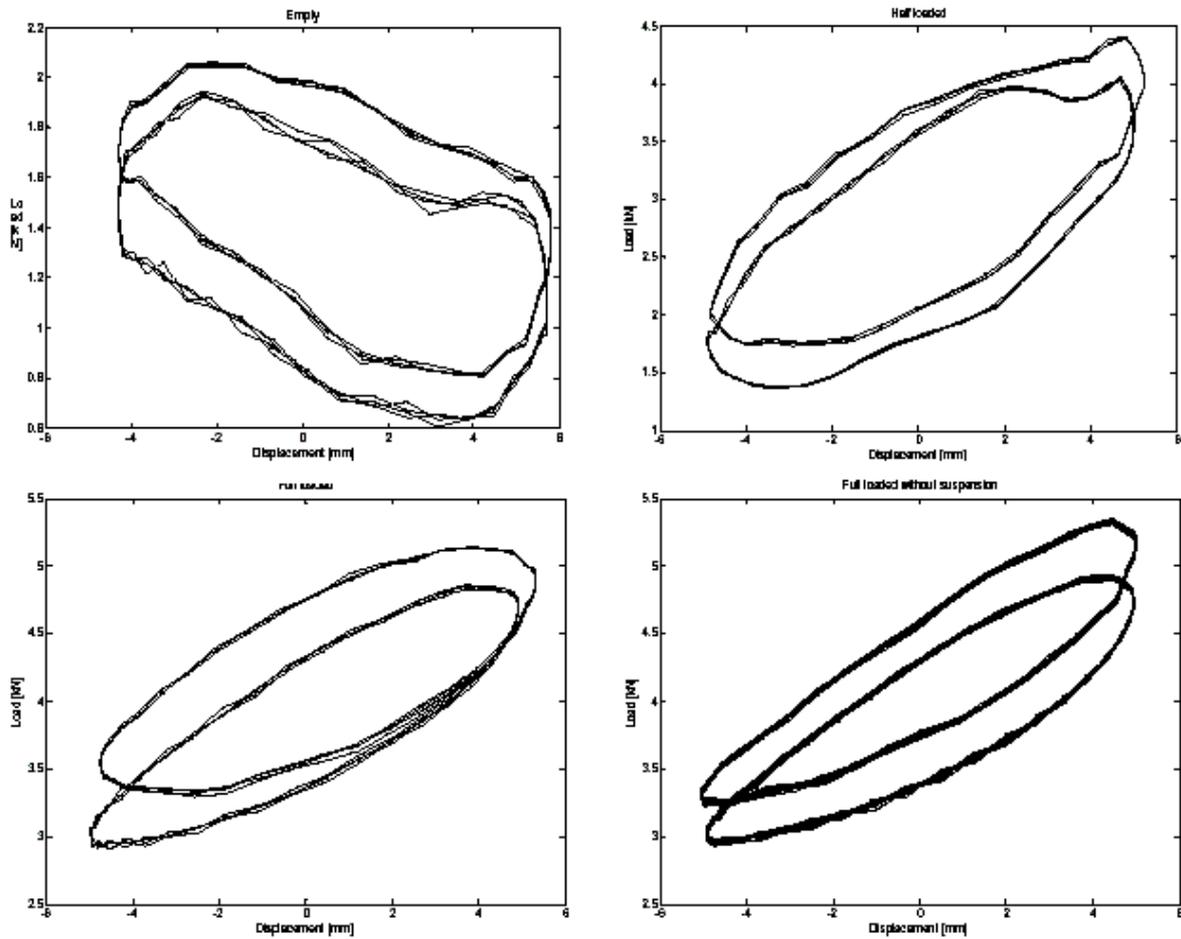


Figure 10: Force x displacement diagram in each condition.

7. Conclusions

This work discusses the characteristics of hidropneumatic suspensions. A model for the nonlinear stiffness is presented, and also a procedure for the setup and design of the chambers and critical components. A study of the damping behavior is developed, and a experimental verification of the predicted behavior is presented. Some experiments and complementary studies are being done currently.

8. References

- Bathe, K. J. and Wilson, E. L., 1976, "Numerical Methods in Finite Element Analysis", Prentice-Hall Inc.
- Els, P.S., Grobberlaar, B., 1993, "Investigation of the Time and Temperature Dependency of Hydro-Pneumatic Suspension System," SAE Technical Paper Series n. 930265, Vehicle Suspension and Steering Systems, SAE Special Publication SP-256, pp. 55-65.
- Felez, J., Vera, C., 1987, "Bond Graph Assisted Models for Hydro-pneumatic Suspension in Crane Vehicles," Vehicle System Dynamic, 16, pp.313-332.
- Hrovat, D., 1997, "Survey of Advanced Suspension Developments and Related Optimal Control Applications," Automatica, 33, pp. 1781-1817.
- Meirelles, P.S., 1989, "Simulação Experimental de Vibrações para Teste Dinâmico de Estruturas com Não Linearidade," Ms. thesis, Campinas State University, Campinas
- Picado, R.M., 1998, "Controle Semi-ativo em Suspensões Automotivas," Ms. thesis, Campinas State University, Campinas
- Thomson, W. T., 1978, "Teoria da Vibração com Aplicações", Ed. Interciência Ltda., Rio de Janeiro, Brazil.
- Yohsuke, A., 1999, "Suspension Control," Automotive Electronic Handbook, Mac GRAW HILL, 2nd Edition, pp. 18.1-18.19.