NONLINEAR TIME DOMAIN ANALYSIS OF RISERS

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Abstract. This work presents a time-domain model to analyze the mechanical behavior of cables. Firstly, the static problem of a riser submitted to a steady current is solved. Then, as the dynamic displacements are of a smaller magnitude order than the static span, the motion is treated as a small perturbation around the static configuration, resulting in a linear dynamic structural model.
Only two sources of nonlinearities are considered: the viscous damping, given by Morison's formula, and the unilateral contact between the riser and the seabed. The system of equations resulting from the finite element pipeline's discrete model is solved in time domain by using the Newmark Method.
This model was implemented in a software, the results of which are compared with a full nonlinear computer code. The results obtained were very good.

Keywords. Risers, Time-domain, Nonlinear, Newmark method, Dynamic.

1. Introduction

In the last years, the oil industry has given great attention to deepwater oil exploit, especially in Brazil, where ninety percent of the known oil reserves are in deepwater fields.

To check the technical (and economical) viability of the desired exploration, a great deal of research and analysis is needed. An important part of this research lies on the study of the mechanical behavior of risers, which connect the floating production system to the underwater oil field.

As one can imagine, the risers are submitted to high tension forces, even in the static state. And besides these, the risers should be able to uphold also the dynamic tension forces, which appear due to environmental conditions such as waves and the movement of the floating platform as well. Another point of interest is to analyze whether the resulting tension due to the dynamic influence becomes negative (compression) or not in any portion of the riser.

Physical experiments are very effective when done correctly. However, it demands a great deal of money, time and qualified staff. Also, the similitude analysis imposes a physical limit to the experiment. For example, in a 20m deep water tank it is very difficult to perform an experiment when we study deep-water conditions (typically 2000m). For a scale of 1:100, if we remember that risers are smaller than 0.4m (16 in.), we will need models of risers with diameter of 4 millimeters.

So, to perform these analyses, numerical approaches can be tried and that is why several specific computer programs have been developed. With these computational tools, the static span can be obtained and also the riser's dynamic behavior can be simulated. Of course, physical experiments play an important role here by validating the models implemented in these computational tools.

Yet, the system as a whole is essentially nonlinear: the structural model of the riser and also its iteration with the fluid (the seawater). This may imply in large computational costs. However, when analyzing the dynamic problem, only the two main sources of nonlinearity can be considered: the fluid drag along the suspended length of the riser and the unilateral contact between the riser and the seabed. The nonlinear dynamic problem can only be solved in time-domain.

This work presents a quick review of the most important characteristics in solving the two-dimensional static problem. It also shows how the linear dynamic model around the static configuration may be obtained and then how the two main nonlinearities can be included in the analysis. A homemade software performs both the static analysis and the dynamic nonlinear analysis in time domain, being the nonlinearities the ones described previously. The results for the dynamic tension are compared to a fully nonlinear software and the results are presented.

2. Static Problem Definition and Solution

The geometric configuration shown on Fig (1) represents a riser suspended at the sea level by the tension force $T_B = T(B)$. The forces acting upon the cable are of (i) gravitational nature (the cable is subjected to the gravitational field),(ii) hydrostatic nature (the cable is immersed in seawater) and (iii) hydrodynamic nature (the cable is subjected to a sea current[1]).

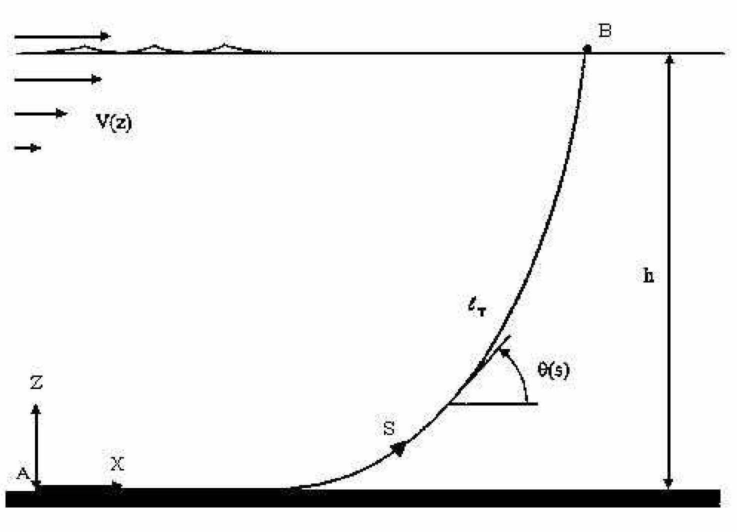

Figure 1. Two-dimensional static configuration and geometric definitions (adapted from Aranha et all, 1997)

The static problem can be stated as follows. Given the riser's physical properties (diameter D, weight per unit length q, friction coefficient between the riser and the seabed m), the geometry of the problem (x top coordinate X_T, z top coordinate Z_T, total length of the riser l_T), the environmental conditions (water depth h, water density r_A, gravity acceleration g, current velocity profile $V(z)$) and the drag coefficient c_D, one must obtain the static configuration of the riser.

Note that two hypotheses are assumed: the riser has (i) infinite axial stiffness ($EA = \infty$) and (ii) no bending stiffness ($EJ = 0$). The first hypothesis can be justified by saying that the riser always works in the elastic regime (small deformations). The second one can be justified by saying that, due to the high tension forces, the effect of the bending stiffness is important only at the top and touchdown points. Therefore, this effect can be incorporated later by a boundary-layer technique (Martins, 2000).

By modeling the static problem this way, it is obtained a set of first-order differential equations in the curvilinear coordinate s that can be solved numerically. The hypothesis of infinite axial stiffness is easily achieved by doing the static deformation $e_e = 0$.

$$\frac{dx_e}{ds} = (1 + e_e)\cos q_e \tag{1}$$

$$\frac{dz_e}{ds} = (1 + e_e) sin q_e \tag{2}$$

$$\frac{d}{ds}(T_e \cos q_e) = -F_{x,e} \tag{3}$$

$$\frac{d}{ds}(T_e sin q_e) = -F_{z,e} \tag{4}$$

$$\frac{dq}{ds} = \frac{F_{n,e}}{T_e} \tag{5}$$

$$e_e = \frac{T_e}{EA} \tag{6}$$

where the subscript "e" refers to static, F is the resulting force applied to the system and the subscript "n" refers to the transversal-to-cable direction. These static equations will be used to obtain the dynamical ones.

[1] Although the sea current is a dynamic load, its variation scale is of order of hours, much higher than the dynamic displacements time variation scale, of order of seconds. That is why we can consider the current load as a static load.

While the geometry of the riser is defined by the pair ($x(s)$; $z(s)$), the static equilibrium is defined by the tension force $T(s)$. In this way, the problem can be solved by using a fourth-order Runge-Kutta method with adaptive step, for example.

3. Nonlinear Dynamic Problem Definition

The dynamic effects appear mainly due to the top movement (motion of the floating platform) and due to the load caused by the waves[2]. These effects induce displacements along the cable's length and so dynamic tension forces.

In the dynamic model, one must take into account loads (i) of inertial nature[3], (ii) of viscous nature[4], (iii) due to the waves. The hypothesis of infinite axial stiffness applies no longer in the dynamic model. The physical explanation is that the cable cannot assimilate the top movement only by changing its catenary configuration and as a consequence, it has to accept some axial deformation. It is assumed here that the waves and the top movement are harmonic.

The dynamic problem then can be stated as follows. Given the static configuration of a riser, the axial stiffness EA, the wave parameters (period P_W , amplitude A_W), the top movement parameters (horizontal motion amplitude H_A , horizontal motion phase H_P , vertical motion amplitude V_A , vertical motion phase V_P) and the added mass coefficient c_M , one must evaluate the displacements and the dynamic tension along the riser's length.

There are two main sources of nonlinearity that arise in the dynamic problem. The first one is related to the viscous drag force, which is given by Morison's formula:

$$\vec{F}_D = -\frac{1}{2} r\, c_D D \left|\vec{v}_{r,n}\right| \vec{v}_{r,n} \tag{7}$$

where $\vec{v}_{r,n}$ is the normal component of the relative velocity between the cable and the fluid.

The second nonlinearity is related to the unilateral contact between the riser and the seabed. The unilateral contact is a boundary condition that imposes that (i) there is no restriction when the riser tends to lift from the seabed; (ii) it does not allow the riser to get any lower if the riser is already on the seabed (i. e., when $z = 0$). Aranha et all, 1997, showed that for practical cases, there is no impact load when a portion of the riser reaches the seabed and so, there are no additional external loads to be considered.

4. Nonlinear Dynamic Problem Solution

The equations of the complete dynamical problem are obtained from Eq. (1) to Eq. (6) by replacing (i) the static variables for the total ones and the (ii) total derivatives in respect to s for partial derivatives, once the total variables are also time-dependants. We assume here that the total variables are the static ones plus a small perturbation:

$$x(s,t) = x_e(s) + u(s,t) \tag{8}$$

$$z(s,t) = z_e(s) + w(s,t) \tag{9}$$

$$q(s,t) = q_e(s) + j(s,t) \tag{10}$$

$$T(s,t) = T_e(s) + t(s,t) \tag{11}$$

$$F_x(s,t) = F_{x,e}(s) + F_{x,d}(s,t) \tag{12}$$

$$F_z(s,t) = F_{z,e}(s) + F_{z,d}(s,t) \tag{13}$$

$$e(s,t) = e_e(s) + e(s,t) \tag{14}$$

In Eq. (8) to Eq. (14), the second terms on the right side are the dynamical components. The subscript "d" refers to dynamic.

By replacing the static variables in Eq. (1) to Eq. (6) for the total variables described in Eq. (8) to Eq. (14) and by replacing the total derivatives in respect to s for the partial ones, we obtain a set of partial differential equations of the

[2] This effect is important only at the surface, approximately from the sea level to 20m below.

[3] Also the added inertia must be considered.

[4] Drag forces due to the motion of the riser in the seawater.

disturbed system around the static configuration (the hypothesis of $e_e = 0$ is already included here; also, high-order terms are neglected):

$$\frac{\partial u}{\partial s} = e\cos q_e - j\ sin q_e \tag{15}$$

$$\frac{\partial w}{\partial s} = e\,sin q_e + j\ \cos q_e \tag{16}$$

$$\frac{\partial}{\partial s}\left(t\ \cos q_e - j\,T_e sin q_e\right) = -F_{x,d} \tag{17}$$

$$\frac{\partial}{\partial s}\left(t\ sin q_e + j\,T_e \cos q_e\right) = -F_{z,d} \tag{18}$$

$$e = \frac{t}{EA} \tag{19}$$

However, the dynamic displacements are typically of the order of some meters, being much smaller than the order of hundreds of meters of the static span. In this way, the dynamic motion can be treated as a small perturbation around the static configuration by using the Virtual Work Principle (Martins, 2000). Then, we can apply the Virtual Work Principle to the disturbed system described above:

$$\int_0^{l_T} f du ds = 0 \tag{20}$$

where f is the sum of all forces (inertia, viscous drag, wave and stiffness) and du is the virtual displacement. By doing this, the dynamic equation is obtained in its integral form.

$$\begin{aligned}
&\int_0^{l_T} r\ddot{u}_t du_t ds + \int_0^{l_T} (r + c_M r_A S)\ddot{u}_n du_n ds + \\
&\int_0^{l_T} \frac{1}{2} r_A D c_D \left|\dot{u}_n - v_{f,n}\right| \dot{u}_n du_n ds + \int_0^{l_T} \frac{1}{2} r_A D c_D \left|\dot{u}_t - v_{f,t}\right| \dot{u}_t du_t ds + \\
&\int_0^{l_T} EA \frac{\partial u_t}{\partial s}\frac{\partial du_t}{\partial s} ds - \int_0^{l_T} EA \frac{dq_e}{ds} u_n \frac{\partial du_t}{\partial s} ds - \int_0^{l_T} EA \frac{dq_e}{ds}\frac{\partial u_t}{\partial s} du_n ds + \int_0^{l_T} EA \left(\frac{dq_e}{ds}\right)^2 u_n du_n ds + \\
&\int_0^{l_T} T_e \frac{dq_e}{ds}\frac{\partial u_n}{\partial s} du_t ds + \int_0^{l_T} T_e \left(\frac{dq_e}{ds}\right)^2 u_t du_t ds + \int_0^{l_T} T_e \frac{\partial u_n}{\partial s}\frac{\partial du_n}{\partial s} ds + \int_0^{l_T} T_e \frac{dq_e}{ds} u_t \frac{\partial du_n}{\partial s} ds = \\
&\int_0^{l_T} \frac{1}{2} r_A D c_D \left|\dot{u}_n - v_{f,n}\right| v_{f,n} du_n ds + \int_0^{l_T} \frac{1}{2} r_A D c_D \left|\dot{u}_t - v_{f,t}\right| v_{f,t} du_t ds - \\
&\int_0^{l_T} \frac{1}{2} r_A D c_D \left|v_{c,n}\right| v_{c,n} du_n ds - \int_0^{l_T} \frac{1}{2} r_A D c_D \left|v_{c,t}\right| v_{c,t} du_t ds
\end{aligned} \tag{21}$$

where the subscript "e" refers to static values, the subscript "n" refers to the transversal-to-cable direction, the subscript "t" refers to the axial-to-cable direction; r is the cable's mass per unit length ($r = q/g$); v_f is the fluid total velocity and v_c is the current velocity; u is the dynamic displacement.

The Finite Elements Method can be used to obtain the discrete form of the dynamic equation

$$M\ddot{U} + C(\dot{U})\dot{U} + KU = R(t) \tag{22}$$

where M is the mass matrix, C is the damping matrix, K is the stiffness matrix, U is the dynamic displacement and is $R(t)$ the external load. The mass matrix refers to the first line of Eq. (21); the damping matrix refers to the second line of Eq. (21); the stiffness matrix refers to the third (axial stiffness) and fourth (geometrical stiffness) lines of Eq. (21); the external load refers to the right-side member of Eq. (21).

Note the dependency of the damping matrix C with the velocity. This is due to the viscous drag forces from Morison's formula.

It is important to note that the second nonlinearity is not included in Eq. (21) (and consequently, in Eq. (22)). The effect of the unilateral contact between the riser and the seabed must be included during the simulation, verifying at each time step if the displacements are physically consistent. For example, it must be avoided that the total z coordinate (considering dynamic displacements added to the static configuration) of any portion of the riser becomes negative.

5. Time Domain numerical methods. Selection of Newmark Method.

Numerical integration must be performed to solve Eq. (22) due to the nonlinearities. Some possibilities are found in the specialized literature: Central Difference Method (MET-1), Houlbolt Method (MET-2), Wilson-Theta Method (MET-3) and Newmark Method (MET-4) (Bathe, 1996).

All of these four possibilities were considered to choose the one that better fits to the problem. Features as (i) initial procedures, (ii) stability, (iii) accuracy, (iv) flexibility and (v) speed were taken into account. A simple computer program was created just to test and compare these methods and verify the possibility of their implementation. The qualitative comparison results are presented on Tab (1)[5].

Table 1. Numerical methods comparison.

	MET-1	MET-2	MET-3	MET-4
Initial Procedures	*	*	***	***
Stability	*	***	***	***
Accuracy	**	**	***	***
Flexibility	*	*	**	***
Speed	***	***	*	**

At the end, Newmark Method was chosen. It does not need initial procedures, is unconditionally stable, has good accuracy (even when compared to analytical solutions), good flexibility (by changing its own integration parameters, see next section). Although it is not the fastest one, it is not too much slower when compared to simpler methods as Central Difference.

6. Newmark Method

Here the Newmark Method is presented and few considerations are made. This method assumes that the following relations are valid:

$$ {}^{t+\Delta t}\dot{u} = {}^{t}\dot{u} + \left[(1-d)\,{}^{t}\ddot{u} + d\,{}^{t+\Delta t}\ddot{u} \right] \Delta t \tag{23} $$

$$ {}^{t+\Delta t}u = {}^{t}u + {}^{t}\dot{u}\Delta t + \left[\left(\frac{1}{2} - a \right) {}^{t}\ddot{u} + a\,{}^{t+\Delta t}\ddot{u} \right] \Delta t^2 \tag{24} $$

where Δt is the time step; a and d are integration parameters.

Equation (23) can be solved for ${}^{t+\Delta t}\ddot{u}$; using this result, Eq. (24) can be solved for ${}^{t+\Delta t}\dot{u}$. By doing this, both ${}^{t+\Delta t}\ddot{u}$ and ${}^{t+\Delta t}\dot{u}$ may be written by means of ${}^{t+\Delta t}u$. So, Eq. (22) can be solved for the time $t+\Delta t$:

$$ M\,{}^{t+\Delta t}\ddot{U} + C\,{}^{t+\Delta t}\dot{U} + K\,{}^{t+\Delta t}U = {}^{t+\Delta t}R \tag{25} $$

This means that the values of ${}^{t+\Delta t}u$, ${}^{t+\Delta t}\dot{u}$ and ${}^{t+\Delta t}\ddot{u}$ may be obtained from the values of ${}^{t}u$, ${}^{t}\dot{u}$ and ${}^{t}\ddot{u}$.

The values of the integration parameters must obey the following relations:

$$ \begin{cases} d \geq 0{,}50 \\ a \geq 0{,}25(0{,}5+d)^2 \end{cases} \tag{26} $$

According to Newmark himself (Bathe, 1996), the values $a = 0.25$ and $d = 0.5$ are the better choice to avoid response distortion. Indeed, Rodriguez (1996), shows that for these values, the period elongation is minimized and there is no amplitude decay.

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[5] More stars means better evaluation.

By increasing the values of the integration parameters, the total time simulation is generally greatly decreased, but the response exhibits a larger period elongation and also an amplitude decay. No study was found in order to estimate these bad influences. The present work shall therefore adopt the values proposed by Newmark as better choice.

7. Implementation

Now some relative aspects to the implementation of the dynamic model by using the Newmark Method are discussed. It is considered here that the static problem is already implemented and operational. Thus, the static configuration is given; it is an input of the dynamic problem. As shown by Eq. (25), a set of nonlinear equations must be solved. To do this, the steps described below must be followed.

First Step. Build the mass (M), damping (C) [6] and stiffness (K) matrices.

Second Step. Initialize the displacement (U), velocity ($\dot{U}$) and acceleration ($\ddot{U}$) vectors [7].

Third Step. Given the time step Δt , calculate the auxiliary values [8] $a_k(\Delta t, a, d)$, $k = 0,1,\ldots,7$.

Fourth Step. Build the effective stiffness matrix

$$\hat{K} = K + a_0 M + a_1 C \tag{27}$$

Fifth Step. For each time step, until the final integration time is not reached,

- Rebuild the damping matrix (C) accordingly to the instantaneous riser's and fluid's velocities;
- Rebuild the effective stiffness matrix ($\hat{K}$) accordingly to Eq. (27);
- Build the effective load

$${}^{t+\Delta t}\hat{R} = {}^{t+\Delta t}R + M(a_0\ {}^{t}U + a_2\ {}^{t}\dot{U} + a_3\ {}^{t}\ddot{U}) + C(a_1\ {}^{t}U + a_4\ {}^{t}\dot{U} + a_5\ {}^{t}\ddot{U})\ ; \tag{28}$$

- Solve the matricial equation

$$\hat{K}\ {}^{t+\Delta t}U = {}^{t+\Delta t}\hat{R}\ ; \tag{29}$$

- Obtain the velocities and accelerations vectors by using Eq. (23) and Eq. (24);
- Verify the unilateral contact condition.

The dynamic tension forces can be obtained by using the relation below:

$$t = EA\left(\frac{\partial u_t}{\partial s} - u_t \frac{dq_e}{ds}\right) \tag{30}$$

where t is the dynamic tension (Martins, 2000).

8. Tests and Numerical Results

At last, a case using our homemade software (SOFT-1) can be simulated and the same case can be simulated using a commercial software (SOFT-2), which is Visual Orcaflex 7.0, by Orcina Software. This way, we may compare the dynamic tension forces resulting from both softwares. The input parameters are given in Tab. (2).

Table 2. Input test parameters

Riser's physical properties	*Problem Geometry*	*Environmental Conditions*	*Wave Parameters*	*Top Movement*	*Coefficients*
$D = 0.1037\,m$	$X_T = 470.0m$	$h = 500.0m$	$P_W = 10.0s$	$H_A = 0.0m$	$c_D = 1.1$
$q = 0.2138\,kN/m$	$Z_T = 508.3m$	$r_A = 1024\,kg/m^3$	$A_W = 1.0m$	$H_P = 0^{\circ}$	$c_M = 1.0$
$EA = 158{,}000kN$	$l_T = 850.0m$	$g = 9.807\,m/s^2$		$Z_A = 1.0m$	
$m = 0.0$		$V(z) = 0.5m/s$		$Z_P = 0^{\circ}$	

In SOFT-1, a time step $\Delta t = 0.001s$ was used; the size of the element $\Delta s = 1.5m$ was used (400 elements in the suspended length) and 30 periods were simulated.

[6] At time $t = 0$ seconds.

[7] These vectors may be initialized with all zeros, for example; Newmark method should converge to the right solution.

[8] These auxiliaries are all described by Bathe, 1996.

In SOFT-2, a time step $10^{-6} \le \Delta t \le 10^{-5}$ was used; the size of the element was $\Delta s = 2m$ in the first 150m of the riser (from anchor to top); $\Delta s = 1m$ in the next 400m; $\Delta s = 2$m in the next 150m and finally $\Delta s = 1m$ in the last 200m of the riser. Four periods were simulated.

It is important to emphasize the difference between SOFT-1 and SOFT-2. While SOFT-1 works with a linear dynamic model with the nonlinearities included during the simulation, SOFT-2 works with a full nonlinear code. This reflects mainly in the total simulation time. While SOFT-1 performs the analysis in 1 hour, SOFT-2 needs at least 9 hours to complete the whole simulation.

The results for the dynamic tension are shown in Figs. (2) and (3). Here, the dynamic tension t is plotted against the curvilinear coordinate s. Figure (2) shows the maximum values of the dynamic tension and Fig. (3) shows the minimum values. Table (3) shows the values for the dynamic tension (maximum and minimum) for the top and anchor positions. Note that the difference is practically constant. The percentual difference is also shown.

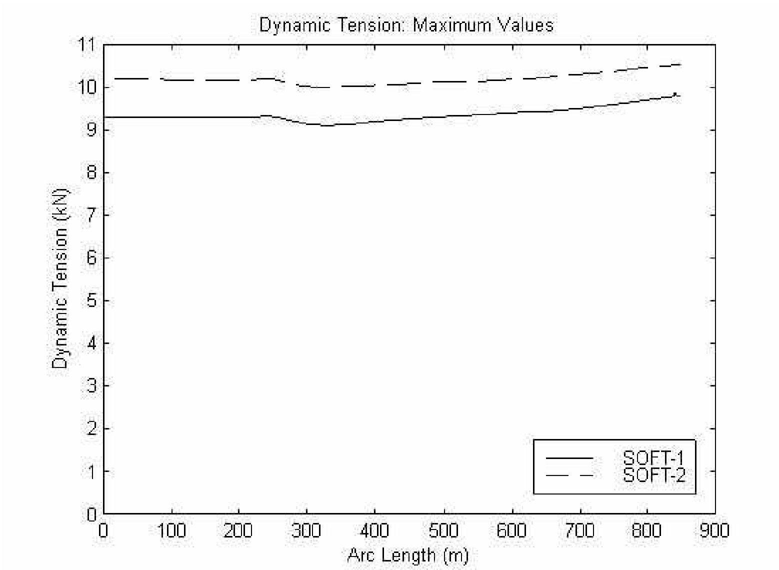

Figure 2. Maximum dynamic tension forces comparison

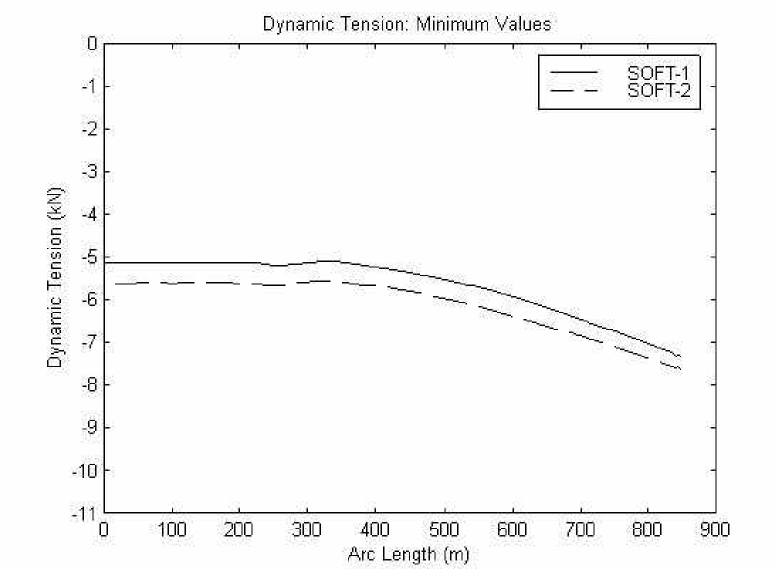

Figure 3. Minimum dynamic tension forces comparison

Table 3. Comparison of dynamic tension forces at end points

	SOFT-1	SOFT-2	Difference
Anchor Minimum Dynamic Tension (kN)	-5.12	-5.61	8.7%
Anchor Maximum Dynamic Tension (kN)	9.29	10.20	8.9%
Top Minimum Dynamic Tension (kN)	-7.31	-7.60	3.8%
Top Maximum Dynamic Tension (kN)	9.82	10.55	6.9%

9. Conclusions

This paper presented the dynamic equation of a riser by using the Virtual Work Principle around the static configuration. From this equation, the nonlinear dynamic model of the riser submitted to waves and top movements is obtained quite easily. This model can be solved in time domain by using the Newmark Method, which was also shown in this work, along with its basic implementation.

As shown in the last section, the dynamic tension of our homemade software (SOFT-1) is very close than the one from the commercial software (SOFT-2): the difference is always smaller than 10%. In fact, this was expected once that SOFT-2 is completely nonlinear. Even so, the results can be considered as very good ones. Besides, SOFT-1's time simulation is much smaller than SOFT-2's.

10. Acknowledgments

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