DEVELOPMENT OF ELECTRONIC CIRCUITS TO TEACH DYNAMIC SYSTEMS AND PROCESS CONTROL

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Abstract: The dynamic behavior of linear and non-linear systems can be reproduced from analog electronic circuits, where state variables are represented as electric signals. The use of these circuits on the development of didactic platforms presents several interesting aspects for the study of dynamic systems and process control. Since they are experimental assemblages, utilizing inexpensive and versatile electronic components easily found in market, many practical phenomena can be explored, which consideration is impossible on computational simulation. However, due to amplitude and frequency limitations of electrical signals, a direct implementation of a electronic circuit aiming to simulate a dynamic system can become very difficult. This paper presents a methodology to design and implementation of analog electronic circuits which reproduce the dynamic behavior of a dynamic system, using chaotic systems as example.

Keywords: Dynamic system, chaos, teaching systems

1. Introduction:

In undergraduate courses, the experience based on laboratory activities provides a better understanding for theoretical contents and encourages students to confront practical challenges. Thus, the education must be a mix of experimental and conceptual parts (Coelho et al., 2001). However, The implantation of a laboratory of control and automation implicates in several difficulties, such as space limitations, financial support, difficulties to build a real system, etc. In this context, the concept of analogy between systems can help experimental activities for students. Since the characteristics of a real system can be reproduced by an analogous system, which is easier to implement, this technique can be an interesting alternative for teaching of dynamic systems, process control and signal processing. The idea consists in implementing educational platforms which reproduce the dynamical behavior of a generic system, which can be mechanical, electrical, chemical, thermal, hydraulic, economical, biological, etc, using analogous electronic circuits. As an electronic circuit is robust, compact, versatile and inexpensive, it allows the implementation of a functional and didactic laboratory for study of dynamical systems, signal processing and process control.

Since voltage signals represent system variables and its derivatives, the system behavior can be directly observed and recorded directly on oscilloscopes and/or acquisition boards, avoiding the use of expensive sensors. Although an electronic prototype does not reproduce completely the real system, it incorporates several uncertainties of a practical implementation, such as unpredictable noises, measurement problems, failures, etc, which are very difficult to obtain in a realistic reproduction by computational simulation. Another advantage of the analogy philosophy for teaching is that an electronic circuit is easily assembled; also, its parameters can be easily controlled on-line (Dianese, 1984). However, a direct electronic implementation is relatively difficult because voltage signals are generally subjects to hard limitations aiming to assure a correct reproduction of the system and to maintain the integrity of the electronic devices.

This paper presents a methodology to design and to implement analog electronic circuits from a mathematical model of a dynamic system. If necessary, the original model must be modified aiming to restrict the amplitudes and frequencies of signals. It is desirable to obtain a simple electronic version of any system, using as few components as possible, since complex and intricate electronic circuits, with an excessive number of devices, increase the assembly difficulty, uncertainties and errors. A study of cases is presented, where this methodology is applied to the design of electronic circuits to reproduce the dynamic behavior of three mechanical systems that exhibit the phenomenon of chaos: forced Duffing System, Lorenz System and Rössler System.

2. Electronic Analogy

The evaluation of relationships between variables of a complex system is the first step towards understanding, analyzing, designing and controlling it (Dorf and Bishop, 2001, Ogata, 1982). A mathematical description of real system characteristics is called mathematical model, and the area of knowledge that studies ways to develop and implement mathematical models from real systems is the mathematical modeling (Ogata, 1982 , Aguirre, 2000). The physical laws, such as Newton's laws for mechanical systems, and Kirchoff's laws for electrical systems, can be used to elaborate a mathematical model taht describes the real system. This methodology, known as physical modeling of process, requires the previous knowledge of the system, aiming to obtain an adequate model and to describe its uncertainties and hypotheses related to practical operation of the real system (Dorf and Bishop, 2001, Aguirre, 2000). Independently of its nature, a mathematical model of a dynamic system is usually described in terms of ordinary differential equations. There are several applications for mathematical models (Aguirre, 2000): prediction, estimation, design, simulation, training, understanding and perception of phenomena observed in nature, social systems, biomedicine, equipments, etc. Once the mathematical model of a physical system is obtained, several analytical or computational tools can be used aiming the analysis and synthesis (Ogata, 1982).

A mathematical model can be considered as a mathematical analogous of a real system (Aguirre, 2000). The analogy concept has a great practical utility on system study, since the dynamic behavior of a specific kind of system can be more easily reproduced by an analogous system (Ogata, 1982). Particularly, the implementation of electrical and electronic systems is more easily realized than other physical systems, allowing an efficient experimental analysis. Thus, electronic circuits can be used to simulate the dynamical behavior of a real system, where the original variables are represented by continuous voltages. This concept is known as analog simulation and practically it died with development of fast digital computers and efficient software packages, such as Matlab/Simulink or VisSim, which show an excellent performance to simulate and to analyze mathematical models. Nevertheless, that technique represents an interesting solution to practical teaching of process control, signal processing and dynamic systems, since the implementation of a real system can be very difficult and expensive.

The electronic implementation for the reproduction of the dynamic behavior of any system consists of a set of electronic circuits which execute mathematical operations using analog voltage signals. When these electronic circuits are adequately connected, the final implementation is able to reproduce the mathematical model of the considered system.

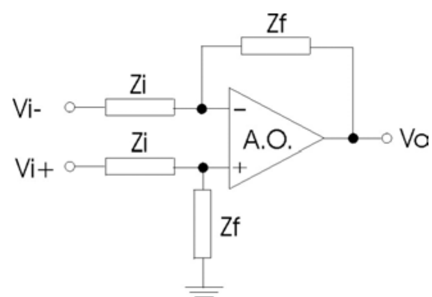


Figure 1. Generic configuration of a circuit with an operational amplifier.

The main electronic device used in these implementations is the operational amplifier, which characteristics allow the execution of several mathematical operations involving voltage signals when associated with specific electric/electronic structures. The operational amplifier consists of a multistage amplifier, with differential input, which characteristics resemble those of an ideal amplifier. Due to its high input impedance and its low output impedance, circuits with operational amplifier can be directly connected without interference between blocks. The generalized configuration of a circuit with operational amplifier (A.O.) is shown in Fig. 1. If impedances Z_i and Z_f are resistances R_i and R_f, respectively, the resulting configuration is known as subtractor, since:

$$V_O = \frac{R_f}{R_i}\left(V_{i+} - V_{i-}\right) \tag{1}$$

If a multiplication by constant is desired, then V_{i+} (non-inverting input) is connected to ground and the output voltage is given by:

$$V_O = -\frac{R_f}{R_i} V_{i-} \tag{2}$$

Note that the output of this configuration is inverted. Another configuration is the inverting weighted summer, which is obtained by connecting multiple input resistances to the operational amplifier. In this case, the output voltage is the weighted sum of all inputs. Considering three inputs, the transfer function of weighted summer is given by:

$$V_o = -R_f\left(\frac{V_A}{R_A}+\frac{V_B}{R_B}+\frac{V_C}{R_C}\right) \tag{3}$$

Since a dynamic mathematical model is a set of differential equations, the inverting weighted integrator is the more important cell for analog implementations of real systems. This configuration is obtained from weighted summer, changing the resistance R_f by a capacitance C. The transfer function of weighted integrator is:

$$V_o = -\frac{1}{C}\int\left(\frac{V_A}{R_A}+\frac{V_B}{R_B}+\frac{V_C}{R_C}\right)dt \tag{4}$$

Since the integrator output is inverted, it is advantageous to adopt the higher order term as negative for odd order equations. This procedure aims to simplify the final electronic implementation. Non-linear functions can be generated by splitting the function curve into line segments, which will be generated by circuits with polarized diodes (Dianese, 1984, Figini, 1982).

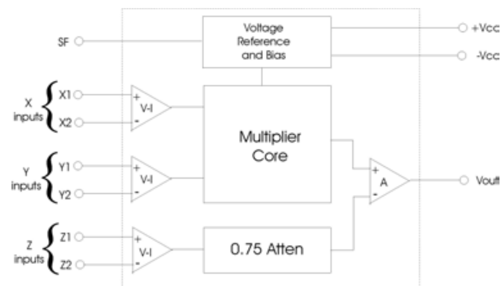


Figure 2. Schematic block diagram of an analog multiplier

Another important device for analog signal processing is the analog multiplier, which performs the product between two variables. However, it can generate several non-linear functions, operating as squarer, divider, square-rooter, trigonometric functions generator, etc (Burr-Brown, 1995). The schematic block of analog multiplier is shown in Figure 2, and its transfer function is given by:

$$V_{out} = A\left[\frac{(X_1 - X_2)(Y_1 - Y_2)}{SF} - (Z_1 - Z_2)\right] \tag{5}$$

The basic implementation of analog multiplier is obtained by connecting the sum input Z_1 to V_{out},, when the transfer function becomes:

$$V_{out} = \frac{(X_1 - X_2)(Y_1 - Y_2)}{10} + Z_2 \tag{6}$$

Since the dependent variable or its derivatives are represented by voltage signals, the reproduction of a dynamic system using electronic circuits is subject to hard limitations. The first limitation is the maximum voltage admissible as input or output by electronic devices. These limits are usually determined by power supply. If voltage signals surpass these limits, the behavior of circuit does not present the necessary characteristics to accurately reproduce the system and the integrity of electronic devices is jeopardized. Another extreme is the minimum voltage value, where small signals can be confused with noise or errors signals generated by electronic devices. The frequencies of analog signals also represent another limitation, since they can not exceed the speed of electronic circuits or measurement devices. On the other hand, the real time to observe the dynamic behavior of slow systems can be unnecessarily too extended.

In most cases, the system variables do not respect the limits imposed by electronic implementation. The solution is the adoption of scaling factor in order to obtain a half scale condition, which is fundamental for a successful electronic implementation of any dynamical system, eliminating dangerous overloads on operational amplifiers, assuring a signal level above of noise level and allowing a comfortable time for observation. The amplitude scaling consists in the application of scale factors to force the system variable to stay within an adequate range of variation, while the time scaling is the adoption of a scale factor aiming to increase or decrease the operation speed in relation to real time.

5. Methodology for analogous electronic circuit design

1. To convert dynamic system model into state model,
2. To verify the maximum value reached by state variables,
3. If necessary, to perform the amplitude scaling,
4. To divide all system equation by greatest parameter to avoid any amplification,
5. The pu (per unity) values of resistors are the inverse of parameters,
6. The capacitor value is chosen according to desired speed for the analogous implementation,
7. To assembly each state equation using the analog weighted integrator and, if necessary, other sub circuits.

6. Study of cases

The main feature of a chaotic system is an unpredictable dynamics due to an extreme sensitivity to initial conditions and system parameters. The interest in chaotic system is always increasing, since chaotic behaviors were identified on dynamics of several important systems. Recent advances in the understanding of nonlinear circuits have shown that chaotic systems are controllable (Mahla and Torres, 2001, Jiang, 2002) and sometimes exhibit self-synchronization properties (Cuomo et al, 1993, Corron and Hahs, 1997). Both phenomena have many real applications on several areas, such as communication, epidemiology, chemistry, etc. In chaos control, external control inputs must be added in order to guide the chaotic dynamics aiming to stabilize periodic orbits embedded in a chaotic attractor. The self-synchronizing can be observed using a chaotic circuit (driving system) driving a similar system (receiving system) to obtain a correlated response. The objective of this section is to design analogous electronic circuits to reproduce the dynamic behavior of three period-doubling chaos mechanical systems. These implementations can be used to generate chaotic signals and to verify the control and synchronization of a chaotic system in didactic practical experiments. Texas Instruments IC's MPY634 (analog multiplier) and TL074 (quad op. amp.) are used in the electronic implementations. To assure more accuracy on system reproductions, precision resistors and styroflex capacitors were used. The phase plane and attractors were observed from a 20 MHz analog oscilloscope in X-Y mode.

6.1 The Forced Duffing Equation

The Duffing equation describes a specific non-linear pendulum moving in a viscous medium. The second-order differential equation to model the free velocity-damped vibrations of a mass *m* on a nonlinear spring is given by:

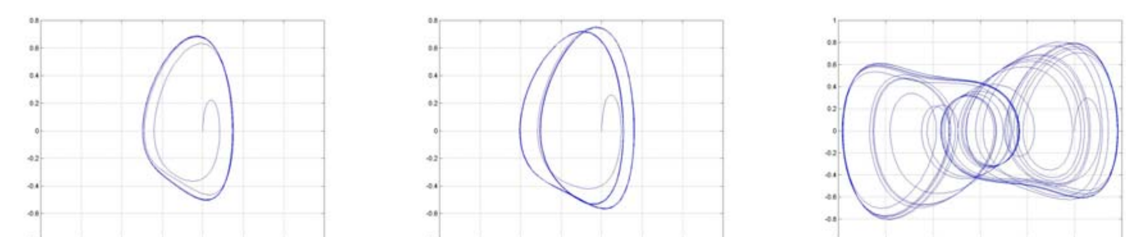
$$m\ddot{x} + c\dot{x} + kx + \beta x^3 = 0 \tag{7}$$

where the term *kx* represents the force exerted on the mass by a *linear* spring and x^3 is the nonlinearity of an actual spring. When an external and periodic force acts on the mass, *forced vibrations* arise on the system. With such a force adjoined to the system, for the displacement *x(t)* of the mass from its equilibrium position, the forced Duffing equation is given by:

$$m\ddot{x} + c\dot{x} + kx + \beta x^3 = F_O \cos(\omega t) \tag{8}$$

If $\beta=0$, then we have a linear equation with stable periodic solutions. To illustrate the quite different behavior of a nonlinear system, the system parameters were chosen to be $k=-1$ and $m=c=\beta=\omega=1$. The forced Duffing system can be described as state model by

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -y + x - x^3 + F_O \cos(t) \end{aligned} \tag{9}$$



(A) (B) (C)

Figure 3. Phase-plane for forced Duffing system: (A) F_O=0.6; (B) F_O=0.7; (C) F_O=0.8.

The objective now is to examine the dependence of the (presumably steady periodic) response *x(t)* upon the amplitude F_O of the periodic external force of period 2π. The figure 3 shows the phase planes where the amplitude of the external force is made to vary from $F_O=0.6$ to $F_O=0.8$. The figure indicates a simple oscillation about a critical point if $F_O=0.60$, and an oscillation with "doubled period" if $F_O = 0.70$. When $F_O=0.80$, a *period-doubling chaos* is finally obtained. In each case the equation was solved numerically with initial conditions $x(0)=1$, $x'(0)=0$. This *period-doubling toward chaos* is a common characteristic of the behavior of a *nonlinear* mechanical system according to appropriate physical parameter choice (such as m, c, k, β, F_O or ω) in Equation 8. No such phenomenon occurs in linear systems.

Although a direct electronic implementation of this Duffing system is possible, since maximum values of state variables are $|x_{max}| = 1.5$ and $|y_{max}| = 0.8$, the amplitude scaling is applied aiming to increase the signal amplitudes in a range of –6 to +6. This goal may be reached by redefining the state variables as $u = 4x$, $v = 8y$. The scaled Duffing system is given by:

$$\begin{aligned} \dot{u} &= \frac{1}{2}v \\ \dot{v} &= -v + 2u - \frac{25}{2}\left[\frac{w^3}{100}\right] + 8F_O\cos(t) \end{aligned} \tag{10}$$

Aiming to make the final implementation easier, the term $w^3/100$ is put in evidence. Both equations are divided by the value of the greatest parameter in order to avoid any signal amplification. This division implicates in a reduction of system speed. Thus, it is necessary to divide the frequency of external signal by 12.5. Considering F_o=0.8, the result Duffing system to be implemented is given by:

$$\begin{aligned} \frac{\dot{u}}{12.5} &= \frac{1}{25}v \\ \frac{\dot{v}}{12.5} &= -\frac{2}{25}v + \frac{4}{25}u - \left[\frac{u^3}{100}\right] + \frac{64}{125}\cos\left(\frac{2}{25}t\right) \end{aligned} \tag{11}$$

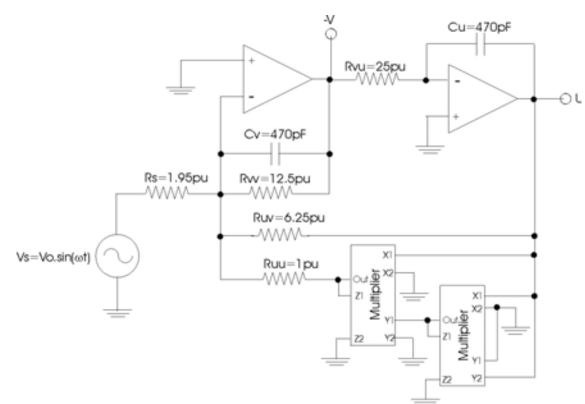


Figure 4. Forced Duffing based chaotic circuit

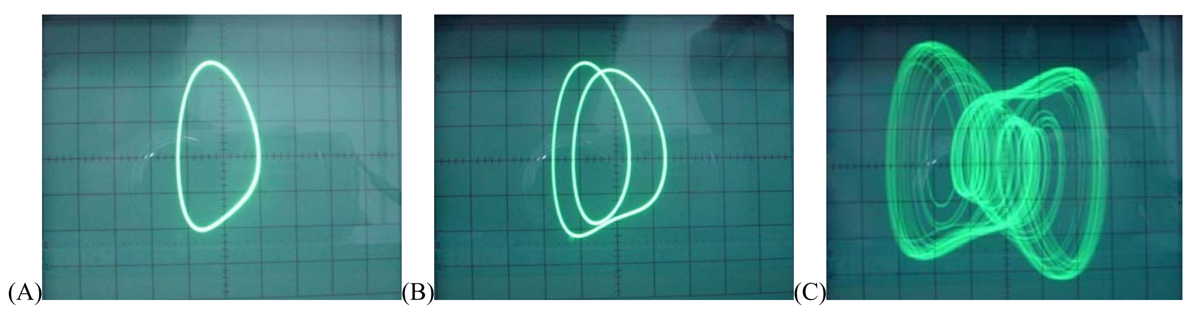


Figure 5. Phase-plane for forced Duffing system observed on oscilloscope (CH X = 1V/div and CH Y = 1V/div): (A) V_O=0.7; (B) F_O=0.8; (C) F_O=1.1.

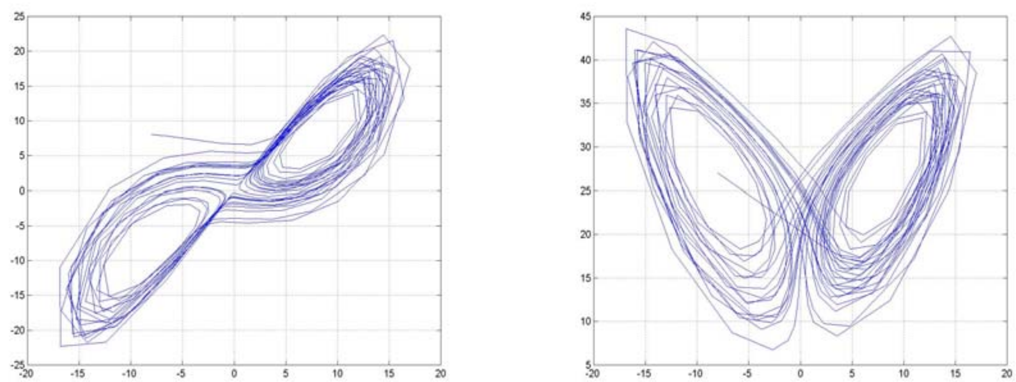
The exact electronic implementation of the equation (11) is shown in Figure 4. It needs only three IC's (two analog multiplier and one quad operational amplifier), two capacitors and five resistors. The resistor values are presented in p.u. (per unit) and the capacitors determine the system dynamics. In the experimental implementation, the base resistance is 10 kΩ and all capacitors are of 470pF, which means an increase in the dynamics of the system. Thus, the frequency of the excitation signal has to be around 2.7kHz to reproduce the original system. This signal is obtained from a signal generator. Figure 5 shows the phase-plane, observed on an oscilloscope operating in X-Y mode, of the forced Duffing system for V_o=0.7V, 0.8V and 1.1V, which demonstrates that that analogous electronic implementation successfully reproduces the dynamic behavior of original forced Duffing system.

6.2 The Lorenz Strange Attractor

The *Lorenz system* is a classic example of autonomous system with chaotic behavior. This system was first studied by Edward N. Lorenz, a meteorologist, around 1963. It was derived from an extremely simplified model of convection in the earths atmosphere. It also arises naturally in models of lasers and dynamos. The system is most commonly expressed by 3 coupled non-linear differential equations:

$$\begin{aligned} \dot{x} &= -s(x-y) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \end{aligned} \tag{12}$$

where x, y and z are the state variables and s, r and b represent the system parameters.



(A) (B)

Figure 6. Chaotic Lorenz attractor: (A) projection on *xy*-plane; (B) projection on *xz*-plane

A solution curve in *xyz*-space is best visualized by looking at its projection into some *plan*e, typically one of the three coordinate planes. The Figure 6 shows the projections into the *xy* and *xz*-plane of the solution obtained by numerical integration of Lorenz system from $t=0$ to $t=30$ with the values $s=10$, $r = 28$ and $b = 8/3$ and the initial values $x(0)=-8$, $y(0)=8$ and $z(0)=27$. As the projection in this figure is traced in "real time", the moving solution point $P(x(t), y(t), z(t))$ appears to undergo a random number of oscillations on the right followed by a random number of oscillations on the left, then another random number of oscillations on the right followed by a random number on the left, and so on. A close examination of such projections of the Lorenz trajectory shows that it is *not* simply oscillating back and forth around a pair of critical points (as the figure may initially suggest). Instead, as $t \to \infty$, the solution point $P(t)$ on the trajectory wanders back in forth in space coming closer and closer to a certain complicated set of points whose detailed structure is not yet fully understood. This elusive set that appears somehow to "attract" the solution point is the famous *Lorenz strange attracto*r.

Since amplitudes of state variables for this Lorenz system reach high values ($|x_{max}| = 18$, $|y_{max}| = 24$ e $|z_{max}| = 45$), a direct electronic implementation of this Lorenz system is technically inconceivable. To solve this problem, an amplitude scaling is applied, aiming to restrict the signal amplitudes to a range of –5 to +5. Redefining the state variables as $u = x/3.6$, $v = y/4.8$ and $w = z/9$, the scaled Lorenz system is given by:

$$
\begin{aligned}
\dot{u} &= -u + 1.3333v \\
\dot{v} &= 2.1u - 0.1v - 6.75\left(\frac{uw}{10}\right) \\
\dot{w} &= 1.92\left(\frac{uv}{10}\right) - 0.2667w
\end{aligned} \tag{13}
$$

The product of state variables is put in evidence to aid on final implementation. Although the state variables of scaled Lorenz system remain within an acceptable range, it is advisable to avoid any signal amplification since intermediate signals can surpass the voltage limits. This condition is obtained dividing all system equations by the value of greatest parameter, which affects only system dynamics. The Lorenz system to be implemented is then given by:

$$
\begin{aligned}
\frac{\dot{u}}{6.75} &= -0.1481u + 0.1975v \\
\frac{\dot{v}}{6.75} &= 0.3111u - 0.0148v - \left(\frac{uw}{10}\right) \\
\frac{\dot{w}}{6.75} &= 0.2844\left(\frac{uv}{10}\right) - 0.0395w
\end{aligned} \tag{14}
$$

The exact electronic implementation of the equation (14) is shown in Figure 7. It needs only three IC's (two analog multiplier and one quad operational amplifier), three capacitors and seven resistors, which demonstrates the simplicity of this Lorenz based circuit if compared to other similar electronic implementations, such as presented in reference (Cuomo et al., 1993). The resistor values are presented in p.u. (per unit). The system dynamics is determined by values of base resistance and capacitance. The parameters *s*, *r* and *b* of Lorenz system can be independently adjusted through of C_u, R_{uv} and R_{ww}, respectively. In the experimental implementation, the base resistance value is 10 kΩ and all capacitors, 470pF. The projections of the Lorenz attractor, observed in an oscilloscope on X-Y mode, are shown in Figure 8. The analogous electronic implementation successfully reproduces the dynamic behavior of original Lorenz system. Note that attractor amplitudes are restricted within ±5 V range, indicating that the signals do not surpass the designed limits. Other strange attractors may be observed when R_{uv}, R_{ww} and C_u values are changed.

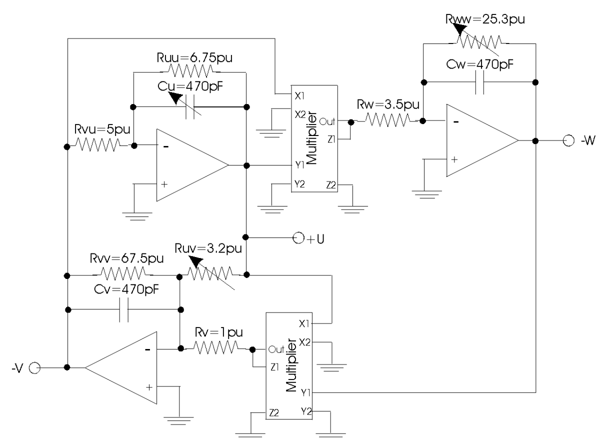
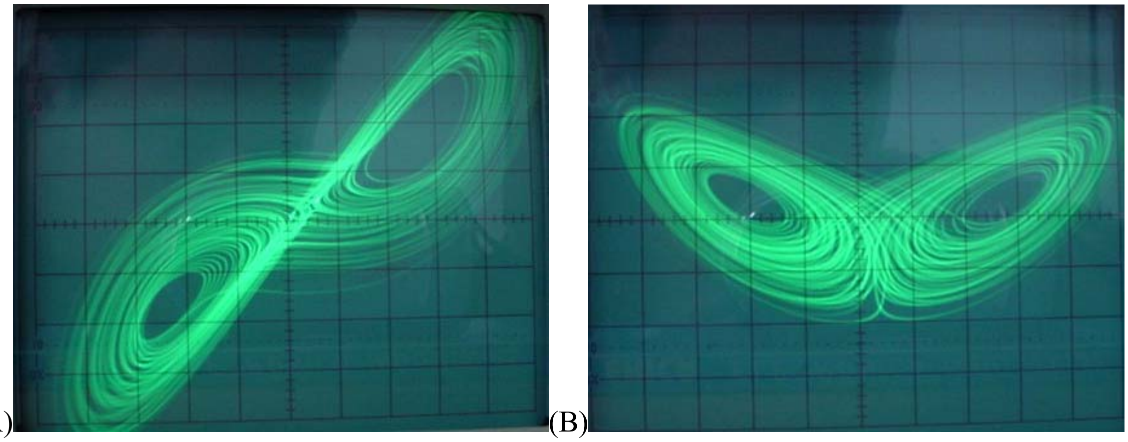


Figure 7. Lorenz based chaotic circuit



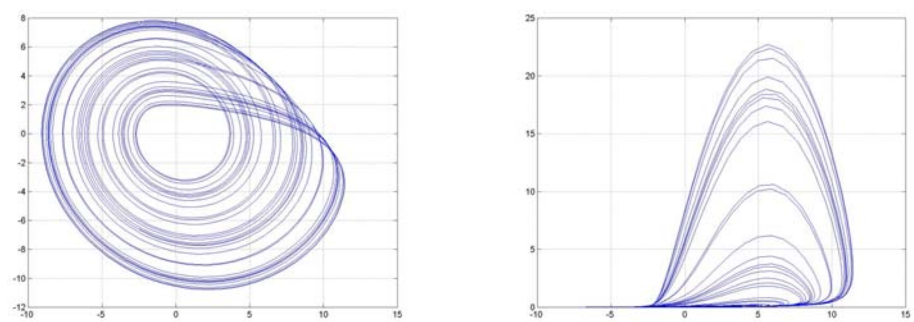
(A) (B)

Figure 8. Chaotic Lorenz attractor observed on oscilloscope (CH X = 1V/div and CH Y = 1V/div): (A) projection on *uv*-plane; (B) projection on *uw*-plane

6.3 The Rössler System

Another example of autonomous system with chaotic behavior is the Rössler system, which was derived from chemical kinetics. The attractor is formed with a set of Navier-Stokes equations. It is credited to Otto Rössler, a non-practicing medical doctor who approached chaos with a bemusedly philosophical attitude. The Rössler system is described with 3 coupled non-linear differential equations:

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \qquad (15)$$



(A) (B)

Figure 9. Rössler Band: (A) projection on *xy*-plane; (B) projection on *uw*-plane

The Rossler's attractor is a rather nice attractor which draws a neat picture. The unique part of this attractor is an arrangement of circles known as "banding". Figure 9 shows an *xy and xz*-projection of the *Rössler band*, a chaotic attractor obtained with the values $a = b = 0.2$ and $c = 5.7$ of the parameters in equation (15). Another interesting fact about Rossler's attractor is that it has a half-twist in it, which makes it look somewhat like a Möbius strip (what you get when you take a strip of paper, half-twist it once, and tape the ends together. A trick that almost everybody knows is to cut along the middle of the band, you will end up with a double-loop; and if you cut in the middle of that double-loop, you will end up with two separate, linked rings). In the *xy*-plane the *Rössler band* looks "folded," but in space it appears twisted like a Möbius strip.

As can be seen in Figure 9, state variables reach the values $|x_{max}| = 11.43$, $|y_{max}| = 10.76$ e $|z_{max}| = 22.84$, which surpass the acceptable limits of an electronic implementation. Applying the amplitude scaling to Rössler's system, the state variables are redefined as $u = x/2.5$, $v = y/2.5$ e $w = z/5$, in order to restrict its variation within a range between +5 and –5. The system is then converted into:

$$
\begin{aligned}
\dot{u} &= -v - 2w \\
\dot{v} &= u + 0.2v \\
\dot{w} &= 0.04 + 25\left(\frac{uw}{10}\right) - 5.7w
\end{aligned}
\tag{16}
$$

To avoid the amplification of any intermediate signal, the Rössler's system is normalized dividing its equations by the greatest parameter, resulting in:

$$
\begin{aligned}
\frac{\dot{u}}{25} &= -\frac{1}{25}v + \frac{2}{25}w \\
\frac{\dot{v}}{25} &= \frac{1}{25}u + \frac{1}{125}v \\
\frac{\dot{w}}{25} &= \frac{1}{625} + \left(\frac{uw}{10}\right) - \frac{57}{250}w
\end{aligned}
\tag{17}
$$

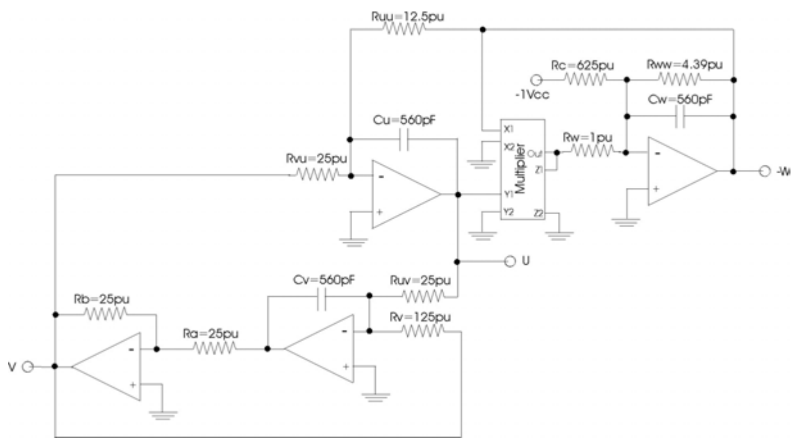
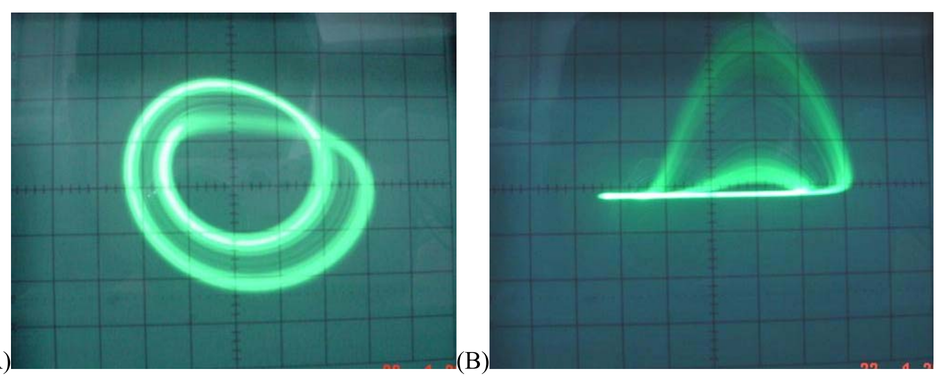


Figure 10. Rössler based chaotic circuit



(A) (B)

Figure 11. Rössler Band observed on oscilloscope (CH X = 2V/div and CH Y = 2V/div: (A) projection on *uv*-plane; (B) projection on *uw*-plane

Due to the transfer function of analog multiplier, the division by 10 of the variable products is put in evidence. The final schematic diagram for Rössler electronic circuit, using two IC's (one analog multiplier and one quad operational amplifier), three capacitors and nine resistors, is shown in Figure 10. The system dynamic is determined by base resistance (0.4 kΩ) and by capacitance values (560pF). The Rössler band obtained experimentally is observed in an analog oscilloscope as show in Figure 11. Note that attractor's projection are confined inside the region of ±5 V.

7. Conclusion

The purpose of this paper is not to revive the analog computation, which did not survive to digital computation, but to show a real possibility of easy practical implementations of dynamic systems using analogous electronic prototypes. Analog electronic implementations can easily reproduce the dynamical behavior of several mechanical, electrical, chemical, thermal, hydraulics, economics or biological systems using robust, compact, versatile and inexpensive prototypes, if compared with other options for practical implementations. Which represents an interesting alternative to experimental education on dynamic systems, process control and signal processing. A methodology to design and to implement these electronic prototypes is presented and applied on the design of electronic circuits that reproduce the dynamic behavior of three mechanical systems that exhibit the phenomenon of chaos, and the results show an efficient reproduction of the chaotic behavior of forced Duffing System, Lorenz System and Rössler System. Although an analogous electronic prototype may not reproduce completely the real system, it is a practical implementation that demonstrates several problems that, otherwise, would be very difficult to have a realistic reproduction by computational simulation. Thus, the proposed analog electronic implementations constitute an interesting alternative that can be used as experimental platform to observe many practical aspects of a particular dynamic system, as control techniques and synchronization phenomenon.

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