PARAMETRIC, MODAL AND EXTERNAL EXCITATION IDENTIFICATION OF MACHINE-TOOL XY TABLE USING RECURSIVE ESTIMATORS

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Abstract. *The proposal of this work is to model the dynamics and to estimate the parameters of a XY table applied in conventional machine-tool to allow its control by using adaptive techniques. The proposed dynamic model will be simulated through Newmark method. The estimation of the parameters, modal parameters and excitation signal of the model will be established by using two methods - least-squares (LS) and recursive least-squares (RLS). The excitation signal will be Schroeder due its excellent characteristics of persistent excitation. The validation of the model will be observed through the curves and tables of the estimated parameters.*

Keywords. *Recursive Estimation, Machine-Tool, Identification, Numerical Method, Dynamic Modeling.*

1. Introduction

Recent paradigms of the industrialized world, such as quality, productivity and competitiveness, require flexible, fast and accurate process production. This fact demands high accuracy machine tools. XY tables, which are present in most machine-tools, are used to locate the workpiece during the machining process. Several works have been published concerning design and XY tables control of machine-tools objectifying to reach better requirements of accuracy, speed and reliability (Younkin, 1991; Tomizuka et all, 1992; Larsen et all, 1995; Smith et all, 1995; Tung et all, 1996; Chang et all, 1997; Jesus, 1999; Mei et all, 2001). In order to guarantee high accuracy positioning of this type of table, it is necessary to know its dynamics and main error source very well. Thus, it is possible to establish an appropriated control system. XY tables consist of several mechanical components as spindle, nut, bearing, and guides. In many cases, the dynamics characteristics these components are neither know nor supplied by machine-tool makers. It is possible to describe the dynamic behavior of a XY table through a differential equation and by means of numeric integration methods to obtain the solution of the system for an excitation signal and initial conditions. Once input and output signals, obtained from simulation process, are known it is possible to estimate unknown parameters of the model as well as the source of excitation action (Mariano, 1997) by using conventional and recursive estimation methods. In this work, dynamic modelling through differential equation and the Newmark method (Bathe, 1976) is used in the simulation process. The excitation signal is the Schroeder (Schroeder, 1970), due to its excellent characteristics of persistent excitation. Estimation techniques such as least square (LS) and recursive least square (RLS) are applied. Simulation techniques are fundamental during the design and advanced diagnosis process of structural behavior of a machine-tool when submitting to different operation conditions. In adaptive controllers the observations are obtained sequentially in real time. It is desirable to make computations recursively in order to save computation time. Recursive estimators techniques are quite applied in models of adaptive control due their capacity of estimating parameters in real time. Finally, dynamics, modal parameters and excitation signal of the model are identified.

2. XY table prototype

Most types of machine-tools use positioning systems of one (X), two (XY), or three (XYZ) axes, traveling from a few millimeters to several meters and requiring accuracy and repeatability as small as a nanometer. Examples of mechanical systems with positioners include lathes, milling machines, machining centers and coordinate measuring machines (CMM). The positioning table in study presents two spindles and it was part integrant of a conventional milling machine. Its structure shown in Fig. (1) is composed of sliding guides, mobile bases, sliding spindles, nuts and bearings.

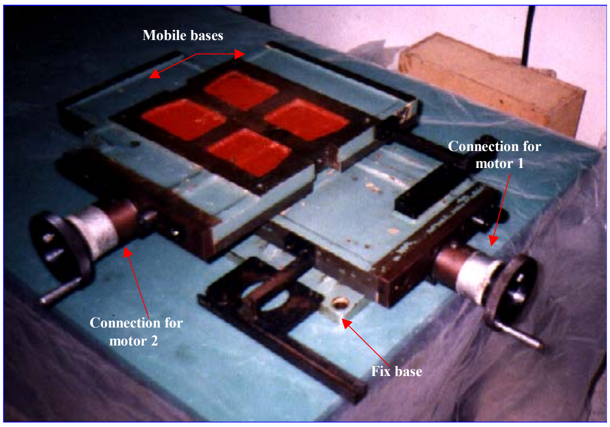

Figure 1. XY table

3. Modelling

A science fundamental problem is to explain physical observations starting from mathematical equations. Therefore, the process of obtaining a mathematical model that represents a physical phenomenon, whose behavior is the most reliable possible to a behavior of the real process, it is not a simple task. A mathematical model is a representation of the essential aspects of a system, that presents knowledge of that system in a usable form (Eykhoff, 1974). Effectively, the equation or equations group that compose the model are an approach of the real process. It means that the model cannot incorporate all macroscopic and microscopic characteristics of the real process. The relationship between input and output of a dynamic system can be expressed mathematically by transfer functions, differential equations, state space equations, among others.

The main components of the XY table used in this work can be seen in Fig. (2), including the DC motor and coupling that are part of driving system. Each one of spindles that composed the table contains several elements, which possess mechanical characteristics such as axial elasticity, torsional elasticity, inertia, and friction. In additional, the table spindles are influenced by nonlinearities such as thermal deformation, stick-slip and backlash.

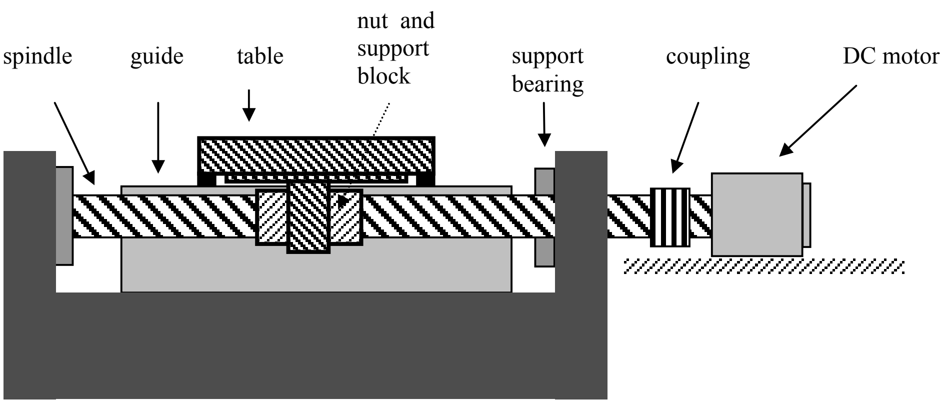

Figure 2. Principal components of the axis

For dynamics analysis of the spindle of Fig. (2), a mass-spring-damper model of the type shown in Fig (3) is proposed, where,

B	overall damping in the set
K	equivalent stiffness coefficient of the system
M1	sum of the inertia of the motor and of the spindle

M2	equivalent inertia of the table
F	difference between the torque of the motor and the load torque
x1, x2	angular displacements of the motor and of the spindle respectively

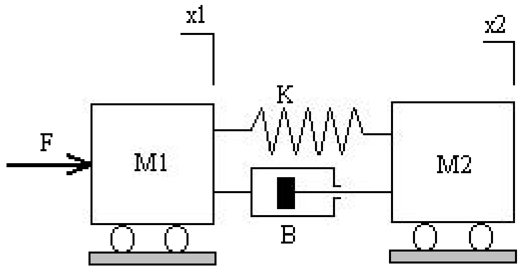

Figure 3. Mass-spring-damper model of the spindle

Applying the Newton's second law of motion in the system of Fig. (3), the following equations are originated,

$$\begin{aligned} M_1\ddot{x}_1 + B(\dot{x}_1 - \dot{x}_2) + K(x_1 - x_2) = F \\ M_2\ddot{x}_2 - B(\dot{x}_1 - \dot{x}_2) - K(x_1 - x_2) = 0 \end{aligned} \tag{1}$$

In matrix form, these equations can be introduced in the following way,

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} B & -B \\ -B & B \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \tag{2}$$

Making the due analogy with the rotary system, the Eq. (1) stay as follows,

$$\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} B & -B \\ -B & B \end{bmatrix}\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} T_m - T_c \\ 0 \end{Bmatrix} \tag{3}$$

The Eq. (3) is similar to that developed for (Lacerda, 1998). It can be attributed $T = T_m - T_c$ for simplification effect. The relationship between the linear displacement and the angular displacement of the system is given for $x = l\theta$, where l = 7,96x10^{-4} (m/rad) is the angular lead of the spindle.

4. System simulation

In order to obtain the temporal response of the dependent variables of a model by simulation, it is necessary that the input variables to be excited with adequate signals. In addition, the initial condition values of the dependent variables should be specified. Project of equipment, processes and plants and their respective control systems; control systems of processes and optimizing of the operational conditions of plants, are possible applications of the dynamics simulation (Garcia, 1997). For the execution of the simulation process with base in real values, it was used the data showed in Tab. (1). These data, which are related to a XY table, are provided by the table manufacturer, THOMSON Ind., according to (Lacerda, 1998).

Table 1. Parameter values

Parameter	**Value**	**Units**	**Description**
I_1	2,7e-05	kg.m^2	Sum inertial: motor and spindle
I_2	8,3e-06	kg.m^2	Equivalent inertia of the table – X axis
B	2,1e-03	N.m.s/rad	Damping of the system
K	1,1e+02	N.m/rad	Stiffness of the system

Important topics should be stood out in the simulation process; it can be mentioned the choice of the type of excitation signal, as well as, the selection of the numeric method to be used. Amongst the excitation signals that are used usually, it can be mentioned: harmonic excitation, impulse excitation and aleatory excitation that were discussed and used for (Treiguer, 1993). An indispensable characteristic in an excitation signal, in the process of systems identification, is the maintenance of the persistent excitation conditions. This, amongst other advantages, can be obtained by the application of the synthesized signal of the periodic type with the same statistical properties of a white noise (Schroeder, 1970). This signal was used in several works (Silva, 1999; Mariano, 1998) and it demonstrated enough efficiency in the identification process of the systems.

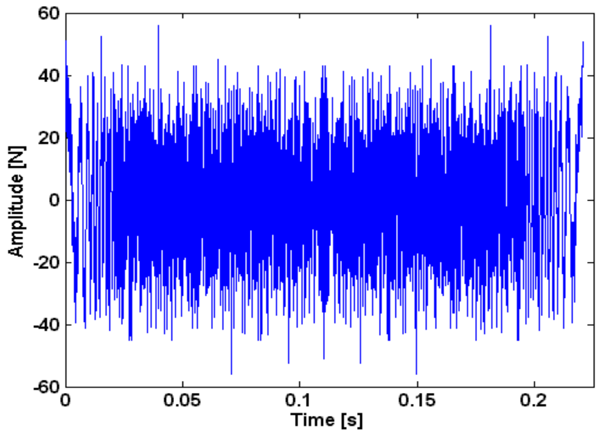

Figure 4. Schroeder excitation signal

Concerning the numeric integration method, the Newmark method, although being an unconditionally stable integrator and to present agreement among the maximum and minimum values of amplitudes corresponding of the numeric solution in relation to exact solution, it can be proven that an error exists in the vibration period, and that is a function of the integration time-step used. This and other topics referring to precision and stability of the direct integration methods, were analyzed thoroughly for (Nickell, 1973). These errors can constitute crucial problems in the process of the identification systems, however, a factor too much important that should be evaluated is the application of an appropriate integration time-step that can be reached by an efficient approach that contemplates the relevance of the dynamics of the system, allowing to obtain a great integration time-step. The more efficient the approach, the more exact it will be the dynamics response coming from the numeric integrator and consequentially more consistent it will be the identification results, as it is evidenced by the approach developed for (Oliveira, 1997).

The choice of the great numeric integration time-step is a primordial factor for a good performance of the numeric method and consequent consistency of the values of parameter identification and of forces in dynamic systems. This step is related to the dynamics of the system and it cannot be taken as an aleatory value.

In the direct integration algorithms, using a step-by-step numeric procedure solves Eq. (2) and (3); the direct term means that, to make the numeric integration, any transformation of the equations in another form is necessary. The Newmark integration method was applied by (Mariano, 1998) in the process of parameters identification and external disturbances in mechanical systems, having presented excellent results when compared to others numeric integration methods. The following equations are used,

$$\left\{\dot{X}^{t+\Delta t}\right\} = \left\{\dot{X}^{t}\right\} + \left[(1-\delta)\left\{\ddot{X}^{t}\right\} + \delta\left\{\ddot{X}^{t+\Delta t}\right\}\right]\Delta t \tag{4}$$

$$\left\{X^{t+\Delta t}\right\} = \left\{X^{t}\right\} + \left\{\dot{X}^{t}\right\}\Delta t + \left[\left(\tfrac{1}{2}-\alpha\right)\left\{\ddot{X}^{t}\right\} + \alpha\left\{\ddot{X}^{t+\Delta t}\right\}\right]\Delta t^{2} \tag{5}$$

Where α and δ are parameters that can determine precision and stability in the numeric integration process. The Newmark method originally proposed is an unconditionally stable algorithm in which case α = 1/4 and δ = 1/2. In addition to Eqs. (4) and (5), for solution of the displacements, velocities, and accelerations at time $t+\Delta t$, the equilibrium equations (3) at time $t+\Delta t$ are also considered,

$$[I]\left\{\ddot{X}^{t+\Delta t}\right\} + [B]\left\{\dot{X}^{t+\Delta t}\right\} + [K]\left\{X^{t+\Delta t}\right\} = \left\{T^{t+\Delta t}\right\} \tag{6}$$

Solving from Eq. (5) for $\left\{\ddot{X}^{t+\Delta t}\right\}$ in terms of $\left\{X^{t+\Delta t}\right\}$, and then substituting for $\left\{\ddot{X}^{t+\Delta t}\right\}$ into Eq. (4), we obtain equations for $\left\{\ddot{X}^{t+\Delta t}\right\}$ and $\left\{\dot{X}^{t+\Delta t}\right\}$, each in terms of the unknown displacements $\left\{X^{t+\Delta t}\right\}$ only. These two relations for $\left\{\dot{X}^{t+\Delta t}\right\}$ and $\left\{\ddot{X}^{t+\Delta t}\right\}$ are substituded into Eq. (6) to solve for $\left\{X^{t+\Delta t}\right\}$, after which, using Eqs. (4) and (5), $\left\{\ddot{X}^{t+\Delta t}\right\}$ and $\left\{\dot{X}^{t+\Delta t}\right\}$ can also be calculated.

5. System identification

It can be obtained a state-space model for the equation system (3) in the form $\dot{\theta} = A\theta + CT$. Choosing the states as being the angular displacements θ_1 e θ_2 and the angular speeds as being ω_1 e ω_2, then,

$$\begin{aligned} \omega_1 &= \dot{\theta}_1 \Rightarrow \dot{\omega}_1 = \ddot{\theta}_1 \\ \omega_2 &= \dot{\theta}_2 \Rightarrow \dot{\omega}_2 = \ddot{\theta}_2 \end{aligned} \tag{7}$$

Substituting the Eq. (7) in the equation system (3) is obtained the following model,

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\omega}_1 \\ \dot{\theta}_2 \\ \dot{\omega}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/I_1 & -B/I_1 & K/I_1 & B/I_1 \\ 0 & 0 & 0 & 1 \\ K/I_2 & B/I_2 & -K/I_2 & -B/I_2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \omega_1 \\ \theta_2 \\ \omega_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ -1/I_1 \\ 0 \\ 0 \end{bmatrix} \{T\} \tag{8}$$

Eq. (8) represented in the compact discrete form using $\theta_{k+1} = (I + A.Dt)\theta_k + Dt.C.T_k$ can be given for,

$$\begin{bmatrix} \theta_1(2) & \omega_1(2) & \theta_2(2) & \omega_2(2) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \theta_1(k+1) & \omega_1(k+1) & \theta_2(k+1) & \omega_2(k+1) \end{bmatrix} = \begin{bmatrix} \theta_1(1) & \omega_1(1) & \theta_2(1) & \omega_2(1) & T(1) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \theta_1(k) & \omega_1(k) & \theta_2(k) & \omega_2(k) & T(k) \end{bmatrix} \begin{bmatrix} 1 & -\frac{K}{I_1}Dt & 0 & \frac{K}{I_2}Dt \\ Dt & \left(1-\frac{B}{I_1}Dt\right) & 0 & \frac{B}{I_2}Dt \\ 0 & \frac{K}{I_1}Dt & 1 & -\frac{K}{I_2}Dt \\ 0 & \frac{B}{I_1}Dt & Dt & \left(1-\frac{B}{I_2}Dt\right) \\ 0 & Dt & 0 & 0 \end{bmatrix} \tag{9}$$

In its compact matrix form, it can be expressed as $[b]=[A][\phi]$, where,

$[b]$: rectangular matrix of Nx4 order that contains the displacement and speed vectors in the instant $t = k+1$;
$[A]$: rectangular matrix of Nx5 order that contains the excitation and response vectors in the instant $t = k$;
$[\phi]$: rectangular matrix of 5x4 order that contains the parameters that will be estimates.

The least-squares solution of $[A][\phi]=[b]$ is given for,

$$\hat{\phi} = \left[A^T A\right]^{-1} A^T b \tag{10}$$

So that $\hat{\phi}$ constitutes the least-squares estimator of the unknown parameters of ϕ.

5.1 Recursive least-squares estimation (RLS)

The Equation (10) presupposes that all input and output data for the system are known so that the estimation is done all at once (Aguirre, 2000). On the other hand, the state vectors of the system can be update sequentially in a discrete form allowing the recursive estimation of the parameters. The major advantages of the recursive techniques are the possibility to know and to monitor the parameters of the system at the time in that the process data are available. Also, they are less susceptible to problems of numeric order, such as the problem of the matrix inversion singularity present at the least-squares formulation, according to Eq. (10).

Computation of the least-squares estimation can be arranged in such a way that the results obtained at time $t-1$ can be used to get the estimation at time t. The solution in Eq. (10) to the least-squares problem will be rewritten in a recursive form (Aström, 1995). Let $\hat{\phi}(t-1)$ denote the least-squares estimation based on $t-1$ measurements. Admitting that the matrix $A^T A$ is nonsingular for all t. Defining $P(t)$ as being,

$$P(t) = \left[A^T(t)A(t)\right]^{-1} = \left[\sum_{i=1}^{t}\varphi(i)\varphi^T(i)\right]^{-1} \tag{11}$$

It follows that,

$$\begin{aligned} P^{-1}(t) &= A^T(t)A(t) = \sum_{i=1}^{t}\varphi(i)\varphi^T(i) \\ &= \sum_{i=1}^{t-1}\varphi(i)\varphi^T(i) + \varphi(t)\varphi^T(t) \\ &= P^{-1}(t-1) + \varphi(t)\varphi^T(t) \end{aligned} \tag{12}$$

The Equation (10) can be rewritten as,

$$\hat{\phi} = \left[\sum_{i=1}^{t}\varphi(i)\varphi^T(i)\right]^{-1}\left[\sum_{i=1}^{t}\varphi(i)b(i)\right] = P(t)\left[\sum_{i=1}^{t}\varphi(i)b(i)\right]$$

The least-square estimation is given for,

$$\hat{\phi} = P(t)\left[\sum_{i=1}^{t}\varphi(i)b(i)\right] = P(t)\left[\sum_{i=1}^{t-1}\varphi(i)b(i) + \varphi(t)b(t)\right] \tag{13}$$

It follows from Eqs. (12) and (13) that,

$$\sum_{i=1}^{t-1}\varphi(i)b(i) = P^{-1}(t-1)\hat{\phi}(t-1) = P^{-1}(t)\hat{\phi}(t-1) - \varphi(t)\varphi^T(t)\hat{\phi}(t-1)$$

The estimation at time t can now be written as,

$$\begin{aligned} \hat{\phi}(t) &= \hat{\phi}(t-1) - P(t)\varphi(t)\varphi^T(t)\hat{\phi}(t-1) + P(t)\varphi(t)b(t) \\ &= \hat{\phi}(t-1) + P(t)\varphi(t)\left[b(t) - \varphi^T(t)\hat{\phi}(t-1)\right] \\ &= \hat{\phi}(t-1) + K(t)\varepsilon(t) \end{aligned}$$

where,

$$\begin{aligned} K(t) &= P(t)\varphi(t) \\ \varepsilon(t) &= b(t) - \varphi^T(t)\hat{\phi}(t-1) \end{aligned}$$

The residual $\varepsilon(t)$ can be interpreted as the error in predicting the signal $b(t)$ one step ahead based on the estimation $\hat{\phi}(t-1)$. To proceed, it is necessary to derive a recursive equation for $P(t)$, the following lemma is useful,

Lemma 1. Let X, Y, and $Y^{-1} + ZX^{-1}W$ be nonsingular square matrices. Then $X + WYZ$ is invertible, and

$$(X + WYZ)^{-1} = X^{-1} - X^{-1}W(Y^{-1} + ZX^{-1}W)^{-1}ZX^{-1}$$

Applying Lemma 1 to $P(t)$ and using Eq. (12), we get,

$$\begin{aligned} P(t) &= \left[A^T(t)A(t)\right]^{-1} = \left[A^T(t-1)A(t-1) + \varphi(t)\varphi^T(t)\right]^{-1} \\ &= \left(P(t-1)^{-1} + \varphi(t)\varphi^T(t)\right)^{-1} \\ &= P(t-1) - P(t-1)\varphi(t)\left[I + \varphi^T(t)P(t-1)\varphi(t)\right]^{-1}\varphi^T(t)P(t-1) \end{aligned}$$

This implies that,

$$K(t) = P(t)\varphi(t) = P(t-1)\varphi(t)\left[I + \varphi^T(t)P(t-1)\varphi(t)\right]^{-1}$$

Theorem 1. Admitting that the matrix $A(t)$ has full rank, that is, $A^T(t)A(t)$ is nonsingular, for all $t \geq t_0$. Given $\hat{\phi}(t_0)$ and $P(t_0) = \left[A^T(t_0)A(t_0)\right]^{-1}$, the least-squares estimate $\hat{\phi}(t)$, then, satisfies the recursive equations,

$$\hat{\phi}(t) = \hat{\phi}(t-1) + K(t)\left[b(t) - \varphi^T(t)\hat{\phi}(t-1)\right] \tag{14}$$

$$K(t) = P(t)\varphi(t) = P(t-1)\varphi(t)\left[I + \varphi^T(t)P(t-1)\varphi(t)\right]^{-1} \tag{15}$$

$$P(t) = P(t-1) - P(t-1)\varphi(t)\left[I + \varphi^T(t)P(t-1)\varphi(t)\right]^{-1}\varphi^T(t)P(t-1) = \left[I - K(t)\varphi^T(t)\right]P(t-1) \tag{16}$$

The Equations (14), (15) and (16) define, therefore, the recursive least-squares estimator (RLS), appropriate to be used, for example, in the parameters estimation in real time at some models of adaptive controllers.

The Equation (9) can be represented in a recursive form as follows,

$$y(t) = y(t-1)\delta + hu(t) + e(t) \tag{17}$$

Where $y(t)$: matrix $[b]$; $y(t-1)$: matrix $[A]$ being eliminated the last column; δ : matrix $[\phi]$ being eliminated the last line; $e(t)$: residue vector; $u(t)$: $T(t)$.

Considering the inertia $\left[\hat{I}\right]$, damping $\left[\hat{B}\right]$ and stiffness $\left[\hat{K}\right]$ estimated matrices, after to the parameters identification process of the system, it is developed the excitation signal identification process which is given by the following equation,

$$\left\{\hat{T}(t)\right\} = \left[\hat{I}\right]\left\{\ddot{\theta}(t)\right\} + \left[\hat{B}\right]\left\{\dot{\theta}(t)\right\} + \left[\hat{K}\right]\left\{\theta(t)\right\} \tag{18}$$

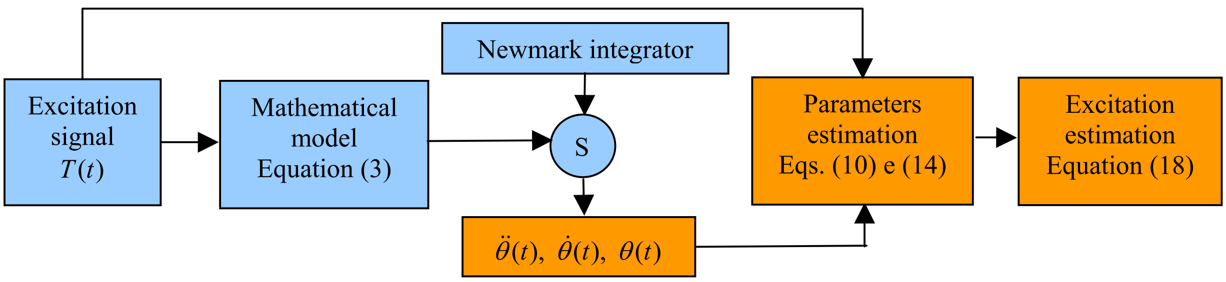

Figure 5. Block diagram for the estimation process

6. Simulation results

The Table (2) presents the data for the calculation of the integration time-step using the criterion $Dt = \left(2\pi / N\omega_n\right)k$, where N is the number of discretization points, ω_n is the natural frequency of the system. The excitation frequency is $\omega_0 = \left(\omega_n / k\right) = \left(2\pi / NDt\right)$. This criterion, as can be seen, considers the dynamics of the system in the calculation of Dt, what makes possible the selection of a great value.

Table 2. Data for the calculation of Dt

N	k	ω_n (rad/s)	ω_0 (rad/s)	Dt (s)
512	10	4,1626e+03	416,2582	2,9481e-05
2048	146	4,1626e+03	28,5108	1,0761e-04

The values of k were selected after a search process that consisted in the keep fixed the value of N varying k to determine the best value for Dt that favored the smallest estimation error. This can be checked through Tab. (2).

For the data of the Tab. (2) was obtained the response as function of displacement, speed and acceleration from Eqs. (4), (5) and (6) that constitute the Newmark integrator.

With the data of the system response and of the excitation signal it was possible to determine from Eq. (10) the parameters of the system and compare them to the exact values, which are presented in Tab. (3). It should be note that the real values in Tab. (3) are those showed in Tab. (1). Substituting the real parameters and the estimated parameters values in the states matrix of Eq. (8) and solving the determinant $|A-\lambda I|$ and $|\hat{A}-\hat{\lambda} I|$, where A e $\hat{A}$ are the real states matrix and estimate states matrix, respectively, then the real eigenvalues and estimate eigenvalues are determined. Substituting the eigenvalues in the equation $(A-\lambda I)x=0$, the eigenvectors can be determined easily (Lalanne, 1984). The calculation of the eigenvalues allows through the formulations $-\zeta\omega_n \pm j\omega_d$ and $-\hat{\zeta}\hat{\omega}_n \pm j\hat{\omega}_d$ the obtaining of $\zeta, \omega_n, \omega_d$ and $\hat{\zeta}, \hat{\omega}_n, \hat{\omega}_d$, which are damping factor, natural frequency and damped natural frequency, real and estimated, respectively, of the system. In this case, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and $\hat{\omega}_d = \hat{\omega}_n\sqrt{1-\hat{\zeta}^2}$. Table (4) presents the results of the real and estimated parameters, which were obtained by using these formulations.

Table 3. Estimated parameters values – LS Estimator

Parameters	$N=512 \quad k=10$			$N=2048 \quad k=146$		
	Real	Estimated	Error (%)	Real	Estimated	Error (%)
I_1 (kgm^2)	2,7e-5	2,7133e-5	4,9e-1	2,7e-5	2,702e-5	7,6e-2
I_2 (kgm^2)	8,3e-6	8,2999e-6	7,05e-6	8,3e-6	8,29999e-6	1,06e-7
B (Nms/rad)	2,1e-3	1,4e-03	3,509e+1	2,1e-3	7,4e-3	2,52e+2
K (Nm/rad)	1,1e+2	1,0905e+2	8,5e-1	1,1e+2	1,03e+2	6,36e0

Table 4. Modal parameters estimated values – LS Estimator

Modal parameters	$N=512 \quad k=10$			$N=2048 \quad k=146$		
	Real	Estimated	Error (%)	Real	Estimated	Error (%)
ω_{n1} (rad/s)	4,1626e+3	4,1423e+3	4,8e-1	4,1626e+3	4,0276e+3	3,24e0
ω_{n2} (rad/s)	4,1626e+3	4,1423e+3	4,8e-1	4,1626e+3	4,0276e+3	3,24e0
ω_{d1} (rad/s)	4,1593e+3	4,1410e+3	4,4e-1	4,1593e+3	3,9852e+3	4,18e0
ω_{d2} (rad/s)	4,1593e+3	4,1410e+3	4,4e-1	4,1593e+3	3,9852e+3	4,18e0
ζ_1	3,97e-2	2,59e-2	3,48e+1	3,97e-2	1,447e-1	2,64e+2
ζ_2	3,97e-2	2,59e-2	3,48e+1	3,97e-2	1,447e-1	2,64e+2

The estimated parameters of Tab. (3) were applied in Eq. (18) for estimation of the excitation signal and comparison with the real signal, what can be seen in Figs. (7) and (8). For the case of the recursive least-squares estimator the procedure was similar to above accomplished. Thus, with the response data of the system and of the excitation signal was possible to determine through Eq. (14) the parameters of the system and compare them to the exact values; this is presented in Tab. (5). The determination of the eigenvalues, eigenvectors, $\zeta, \omega_n, \omega_d$ and $\hat{\zeta}, \hat{\omega}_n, \hat{\omega}_d$ follows the same previous procedure. The Table (6) presents the results of estimation of these parameters. It is opportune to visualize the behavior of the estimated parameters along the recursive process graphically, what is presented in Figs. (9), (10), (11) and (12) respectively. The estimated parameters of Tab. (5) were also applied in Eq. (18) for the estimation of the excitation signal and comparison to the real signal.

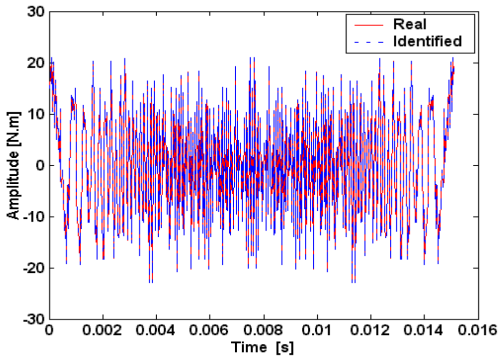

Figure (7). Identified excitation - $N=512 \quad k=10$

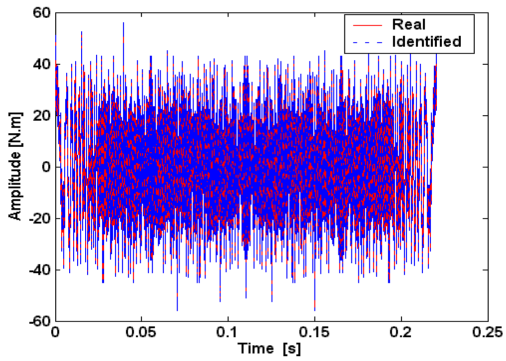

Figure (8). Identified excitation - $N=2048 \quad k=146$

Table 5. Estimated parameters values – RLS Estimator

Parameters	$N = 512 \quad k = 10$			$N = 2048 \quad k = 146$		
	Real	Estimated	Error (%)	Real	Estimated	Error (%)
I_1 (kgm^2)	2,7e-5	2,699e-5	3,67e-2	2,7e-5	2,69998e-5	6,81e-4
I_2 (kgm^2)	8,3e-6	8,299e-6	2.75e-4	8,3e-6	8,29999e-6	3,6e-7
B (Nms/rad)	2,1e-3	2,08e-3	6,64e-1	2,1e-3	2,0998e-3	5,0e-3
K (Nm/rad)	1,1e+2	1,0994e+2	4,89e-2	1,1e+2	1,0999e+2	5,8e-4

Table 6. Modal parameters estimated values - RLS Estimator

Modal parameters	$N = 512 \quad k = 10$			$N = 2048 \quad k = 146$		
	Real	Estimated	Error (%)	Real	Estimated	Error (%)
ω_{n1} (rad/s)	4,1626e+3	4,1617e+3	2e-2	4,1626e+3	4,16257e+3	2,11e-4
ω_{n2} (rad/s)	4,1626e+3	4,1617e+3	2e-2	4,1626e+3	4,16257e+3	2,11e-4
ω_{d1} (rad/s)	4,1593e+3	4,1585e+3	1,9e-2	4,1593e+3	4,15928e+3	2,04e-4
ω_{d2} (rad/s)	4,1593e+3	4,1585e+3	1,9e-2	4,1593e+3	4,15928e+3	2,04e-4
ζ_1	3,97e-2	3,95e-2	6,36e-1	3,97e-2	3,973e-2	4,6e-3
ζ_2	3,97e-2	3,95e-2	6,36e-1	3,97e-2	3,973e-2	4,6e-3

It is observed with base in the results of the Tabs. (3), (4) and of the Figs. (7) and (8) that a number smaller of discretization points N associated with a value smaller for Dt presented better estimation results for the method LS, although the damping estimation error having been reduced, still continue high. However, it is necessary to point out that different values are being used for k . For the RLS method, analyzed the results of the Tabs. (5) and (6) and of the Figs. (9), (10), (11) and (12), can be concluded by the superiority of the estimation results, when compared to the estimation results by the LS method. It should be pointed out that in this latter case a larger value of N had an influence more positive in the reduction of the estimation errors.

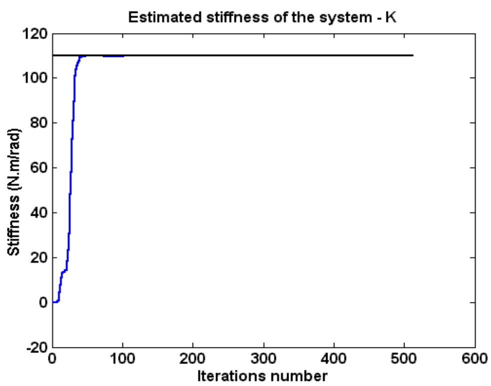

Figure (9) – Recursive estimation of stiffness

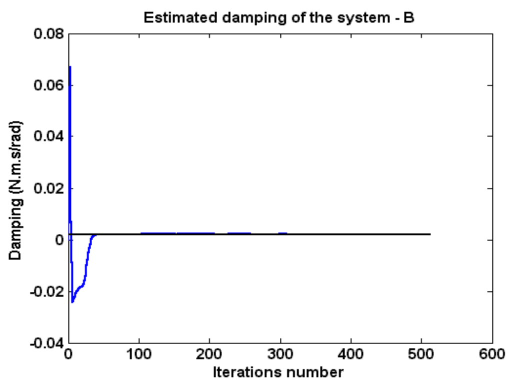

Figure (10) – Recursive estimation of damping

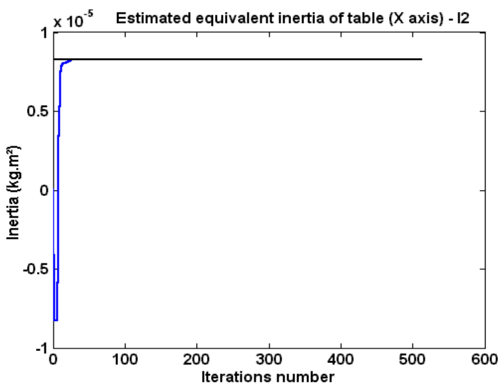

Figure (11) – Recursive estimation of inertia

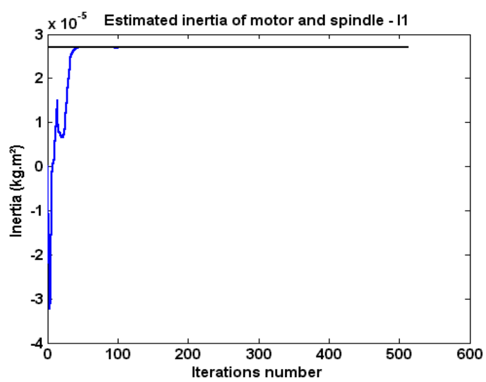

Figure (12) – Recursive estimation of motor inertia

7. Conclusions

In this work, a dynamic model for a machine-tool spindle was developed and the response to a signal with appropriate characteristic of persistent excitation was obtained. Simulation results in terms of state vectors (displacement and speed) along with the input vector were applied for the identification process of the system and input signal by using two techniques of parameters estimation, which were RLS and LS techniques. The dynamic modelling of the table spindle and the proposed identification method demonstrated to be effective.

In this study, several situations were simulated and the obtained results showed the superiority of the RLS technique compared to of the LS technique.

The influence of the selection of an appropriate time-step of discretization was observed in the performance of the numeric integration method in the solution of the differential equation that governs the dynamic behavior of the system.

This study showed that Newmark method was adequate for systems that present this type of dynamic behavior.

Although, the values of the parameters applied in the simulation process were relate to a high precision table the dynamic model and identification method which were proposed in this study can be applied to others models of table.

Since that the dynamics characteristics of the table, in other words, the plant, are identified, it is possible to project the controller to meet the requirements of performance of the system with respect to positioning accuracy and response speed. As the dynamics of the table is variable with the time due to influence of several factors as friction, for instance, the adaptive controllers are a quite attractive option in these applications. Techniques of adaptive control are already been studied with this objective, also, the dynamic model of the spindle will contemplate important effects of nonlinearities of the process that has not be considered in this work, yet.

This study could be extended through the evaluation of other estimators, for instance, extended least squares (ELS) and recursive least squares with exponential forgetting. Besides, others numeric integration methods, for instance, Houbolt, Wilson θ and Runge-Kutta could be appraised as well as others excitation signals.

Further work is been carried out in order to observe the influence of the white noise in the results of recursive estimate. In additional, the RLS estimation technique will be investigated together with other recursive techniques in the experimental identification of the table in study objectifying the application of adaptive techniques in the control of position of the system.

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