AIRFOIL SHAPE OPTIMIZATION USING GENETIC ALGORITHMS

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Abstract. Genetic Algorithms (GA) are search optimization methods that use principles of natural genetics and natural selection. In this method, the possible solutions for a certain problem in question is represented by a sort of biological population, which evolves over generations to adapt to an environment by selection, crossover and mutation. Instead of working with a single solution in each iteration, it works with a number of solutions, known as a population. Traditional optimization methods, such as gradient-based methods use derivative information to guide the search strategy. These methods quickly converge to an optimal state, but they tend to get stuck in suboptimal solutions and they are not efficient in non-differentiable or discontinuous problems. Aerodynamic design optimizations are often multimodal and nonlinear problems, because the flow field is governed by a system of nonlinear partial differential equations. In this way, a very robust and efficient tool is required for optimization. This work is focused on airfoil shape optimization using the GA concept. The MSES code, an Euler equation solver with boundary layer correction, is coupled to a genetic algorithm implementation. The objective function is based on the drag coefficient (Cd) and the procedure changes the shape of the input geometry using polynomials, also called shape functions, to minimize the Cd value.

Key Words: *Genetic Algorithms, Airfoil Shape Optimization, CFD.*

1. INTRODUCTION

In Darwinian terms, life is a struggle in which only the fittest survive to reproduce; not far away from the biological world we have the aeronautical industry. The development of new airliners has always been stimulated by the growth of the traffic volume and the improvement of technical and operational standards. With the passing years, the commercial airplane market has pushed the design of new airplanes to the current state-of-the-art. Faster and economical airplanes are desired and the penalties imposed for those designs that do not satisfy this requirement is simply not to 'survive'.

Simple questions such as: (a) how fast can it climb to a given altitude? (b) how long can it stay in the air?; (c) what is the maximum speed?; have the utmost importance in the airplane design characteristics. Few changes in some of these requirements might generate airplanes with complete different shape and size (Torenbeek,1982). Furthermore, independently of the shapes and the sizes, the airplane must achieve the best performance possible if it wants to be competitive. The field of aerodynamics is primarily responsible to assure a better airplane performance. A bad aerodynamic design hardly produces a successful airplane, but many times it is responsible for the death of the project. That is the reason why optimized aerodynamic shape is so important.

Although airfoils are two-dimensional representations of the wing profiles, their influence on the 3D design can be very effective. Therefore, the proposed design will be based solely on the profile modifications. It is not always that these optimized profiles produce an improvement in the wing design, due to the three-dimensional characteristics of the flow. Nevertheless, when a better wing is obtained, we have a considerable reduction in drag since the wing answers for 2/3 of the airplane drag at a typical cruise condition (Nixon, 1981).

This work is an effort to optimize airfoil shape using the Genetic Algorithm (Goldberg, 1989; Deb, 1995; Deb, 1999; Deb, 2001) concept. The optimization consists in changing the airfoil shape to reach the best aerodynamic performance. Here, a performance improvement is accepted as a decrease in the drag coefficient (Cd) and an increase in lift coefficient (Cl), in other words, a higher value of L/D. The modification in the airfoil shape is performed using polynomial functions with weight coefficients. The Genetic Algorithm (GA) is used to search the best set of coefficients to weight these functions. It is worth to say that this GA implementation is not inserted in a multiobjective problem, once we are optimizing a single objective function, which is the Cd. A NACA0012 and a PRO01 airfoils are used in this work for a range of Mach numbers and Cl. The flow conditions vary from low subsonic to transonic conditions. The main objective of this flow regime variation is to observe how the GA deals with compressible and incompressible flows in terms of robustness to search the best weight coefficients.

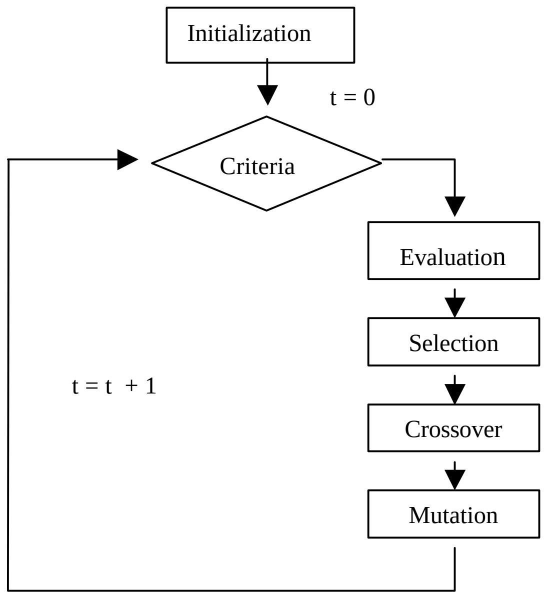
2. THEORETICAL FORMULATION

The GA works using the following sequence of concepts:

- An initial population of candidates is generated;
- The fitness of the population is evaluated;

- Those elements of the population which satisfy certain criteria are chosen to reproduce to the next generation;
- New individuals are created by the exchange of features from the individuals previously selected;
- Some of the individuals created can suffer mutation.

This process can be represented by the flowchart below.



2.1 INITIALIZATION

The first step in the GA concept is to decide the strategy of coding and decoding the design variables in binary strings. The design variables in question are the weight coefficients of the shape functions and the following string is a form of binary representation of these coefficients for a sample case with 6 of such coefficients,

$$\underbrace{1010}_{a1}\ \underbrace{0011}_{a2}\ \underbrace{1100}_{a3}\ \underbrace{0010}_{a4}\ \underbrace{0001}_{a5}\ \underbrace{1110}_{a6} \qquad (1)$$

Each weight coefficient a_i is represented by a binary string within a certain length. The sum of all these binary parts generates the chromossome and the set of chromossomes forms the population. The name chromossome is an analogy with the natural biology and it is responsible for the core of the GA, since they contain the vital information from the design variables.

In the initialization stage, a predecessor population is created. This process consists in the generation of chromossomes, in which the binary strings are randomly sorted. The lower binary string is (0000) and the upper binary string is the (1111), and each one of these strings represents the $x_i^{\min}$ and the $x_i^{\max}$, respectively. The value of $x_i^{\min}$ and $x_i^{\max}$ are defined as the bounds of the solution domain. Any other sorted binary string is contained within these two exposed binary extremes and the x_i value for it can be obtained using the following formulae to decode:

$$x_i = x_i^{\min} + \frac{x_i^{\max} - x_i^{\min}}{2^{li} - 1} DV(a_i) \qquad (2)$$

where l_i is the length of the binary representation of the weight coefficient a_i and $DV(a_i)$ is the conversion of the binary string that represents the a_i coefficient to decimal. The length l_i can be computed as follows:

$$l_i = \log_2\left(\frac{x_i^{\max} - x_i^{\min}}{e_i}\right). \qquad (3)$$

Here e_i denotes the decimal precision of the weight coefficient a_i. Notice that the binary string length of each coefficient is a function of the desired decimal precision. So, the length of the binary string grows with the increase of the precision desired for the coefficient. The total length of the binary string is the sum of the lengths of each binary coefficient representation.

Notice that this procedure of random sort creates variability in the possible weight coefficient values. It creates possible solutions all over the solution domain and only those that are the fittest will pass to the next generation. The two chromossomes below are an example of how different a set of weight values can be produced with this binary random sort process:

$$\underbrace{01000}_{-0.0029}\ \underbrace{11110}_{0.0056}\ \textit{Chromossome}\ 1\,,$$
$$\underbrace{01001}_{-0.0025}\ \underbrace{11101}_{0.0052}\ \textit{Chromossome}\ 2\,. \tag{4}$$

In this specific example, $x_i^{\max} = 0.006$, $x_i^{\min} = -0.006$, and $e_i = 0.00375$.

2.2 EVALUATION

In this step an objective function is used to classify the solution represented by the chromossomes. The definition of the objective function depends on the problem in question. In this work, the objective function is the drag coefficient (Cd), which is evaluated by the MSES code for a given geometry. MSES (Drela, 1996) is an Euler solver with a coupled boundary layer routine that takes into account the most relevant viscous effects.

2.3 SELECTION

It is a stage where the chromossomes are selected for the reproduction phase. There is no rule about how many chromossomes must be selected, it can be the same number of the initial population or it can be more, in which case an increase in population is being allowed. The selection procedure is very important because, depending on the selection method adopted, there is the possibility to create super individuals for the next generation, provoking loss of diversity. On the other hand, a non-elitist method produces individuals that turn the optimization method ineffective.

In this work the roulette method (Goldberg,1989) is used. Basically, this method just takes into account the objective function value of the chromossome. In Table 1, we have five chromossomes with different fitnesses. The probability of a chromossome being chosen is given by the factor ***fi/Sum*** described in Table 1. The parameter ***fi/Average*** is an indication of how many samples of a certain chromossome are expected to be selected. We can see that the chromossome ***i*** has a probability of 35% to be picked and, if five individuals will be selected by the roulette method, there is the probability of having approximately two ***i*** chromossomes in this set.

Table 1. Selection process explanation for a population of five individuals.

Individual	Fitness	Selection Probability	Expected count
	fi	**fi/Sum**	**fi/Average**
i	*7*	*0.35*	*1.75*
j	*6*	*0.30*	*1.50*
k	*4*	*0.20*	*1.00*
l	*2*	*0.10*	*0.50*
m	*1*	*0.05*	*0.25*
Sum	*20*	*1.00*	*5.00*
Max	*7*	*0.35*	*1.75*
Average	*4*	*0.20*	*1.00*

The selection is a kind of bingo where we put in a vector the chromossomes taking into account the objective value. In the example of Table 1, the i-th chromossome would have 7 samples, the j-th would get six samples and so on. The draw is done until we have the entire set of chromossomes that will generate the new population. Those with a bigger value of the objective function have more chance to be chosen. Therefore, it does not mean that they will always be selected. These selected chromossomes will perpetuate their characteristics for the next population. That is the way in which information is processed in a GA concept.

2.4 CROSSOVER

In this process new chromossomes are created by the change of features from the previous selected chromossomes. Frequently, this process leads to an improvement in the new population. There is a number of ways to execute the crossover (Deb and Agrawal,1995); but in all the methods two chromossomes are selected and in a certain position they exchange the binary strings. In this work the single-point crossover is implemented. The position where the chromossomes should exchange strings is randomly chosen. An illustration of a single-point operator can be seen below. Two chromossomes are chosen to create two new individuals.

1	0	0	0	1	0	1	0	1	1	0	1	1	1	0	1	Chromossome 1
1	1	1	0	0	0	1	0	1	0	1	0	0	1	1	1	Chromossome 2
1	0	0	0	1	0	1	0	1	0	1	0	0	1	1	1	New Chromossome 1
1	1	1	0	0	0	1	0	1	1	0	1	1	1	0	1	New Chromossome 2

Figure 2. Crossover procedure for the creation of new individuals.

2.5 MUTATION

It is a way to keep the diversity of the population. In this sequence, the binary value of a certain position in the string is changed. The mutation promotes a searching in the solution space that sometimes cannot be represented by the present chromossomes.

1	0	0	0	1	0	1	0	1	1	0	1	1	1	0	1	Chromossome n
1	0	0	0	1	0	1	1	1	1	0	1	1	1	0	1	New Chromossome n

Figure 3. Mutation at a certain position of the chromossome.

3 SHAPE FUNCTIONS

In the search process for the best aerodynamic configuration, some specific shape functions were used to modify the airfoil geometry. The modification was performed with the change in the weight coefficients of these functions, accomplished by the genetic algorithm. The NACA0012 airfoil was the first case of study in this work. For this profile, Legendre polynomials were used to modify its original shape. The Legendre polynomials are given by the expressions:

$$\begin{aligned}
&P_2 = 2X - 1\,,\\
&P_3 = 6X^2 - 6X + 1\,,\\
&P_4 = 20X^3 - 30X^2 + 12X - 1\,,\\
&P_5 = 70X^4 - 140X^3 + 90X^2 - 20X + 1\,,\\
&P_6 = 252X^5 - 630X^4 + 560X^3 - 210X^2 + 30X - 1\,,
\end{aligned} \tag{5}$$

and the modified airfoil coordinates are given by the following equation with the a_i's weight coefficients

$$Y_i^{NEW} = Y_i^{OLD} + [1 - X_i]^3 . \left[a_1\sqrt{X_i} + a_2(P_2+1) + a_3(P_3-1) + a_4(P_4+1) + a_5(P_5-1) + a_6(P_6+1)\right] \qquad i = 1...n, \tag{6}$$

where i represents the i-th airfoil point, and X_i is the airfoil ordinate of point i.

For the second test case, the PRO001 airfoil, satisfactory results were not obtained with the Legendre polynomials. In this way, a study of a variety of shape functions was performed. The best results were obtained with the Hicks-Hennes function for the lower surface, and another polynomial function for the upper surface. These functions are described below:

$$\begin{aligned}
&F_i^{upper} = 0.52\left[0.5X_i^3 - 1.5X_i^2 + X_i\right],\\
&F_i^{lower} = sin\left[p\ X_i^{1.357}\right]^3 .
\end{aligned} \tag{7}$$

The coordinate Y from the resultant geometry is obtained by the following relations,

$$
\begin{aligned}
Y_i^{new\_upper} &= Y_i^{old\_upper} + a_1\ F_i^{upper}\ , \\
Y_i^{new\_lower} &= Y_i^{old\_lower} + a_2\ F_i^{lower}\ ,
\end{aligned}
\tag{8}
$$

where a_1 and a_2 are the weight coefficients.

4 RESULTS

Initially, the drag rise curves for the NACA0012 profile for a set of Cl values are evaluated, as can be seen in Fig. 1. We can notice from Fig. 1 that the increase of the Cl value causes a decrease in the drag-divergence Mach number. The optimum airfoil shape is the one that can postpone the drag-divergence Mach number for a fixed lift value. The point where the airfoil should be optimized depends on operation design point of the airplane. If the airplane operates at a cruise Mach number (M) of 0.80, as an example, it is desirable to optimize the wing for this Mach number since, during the airplane mission; it is flying at this Mach number most of the time.

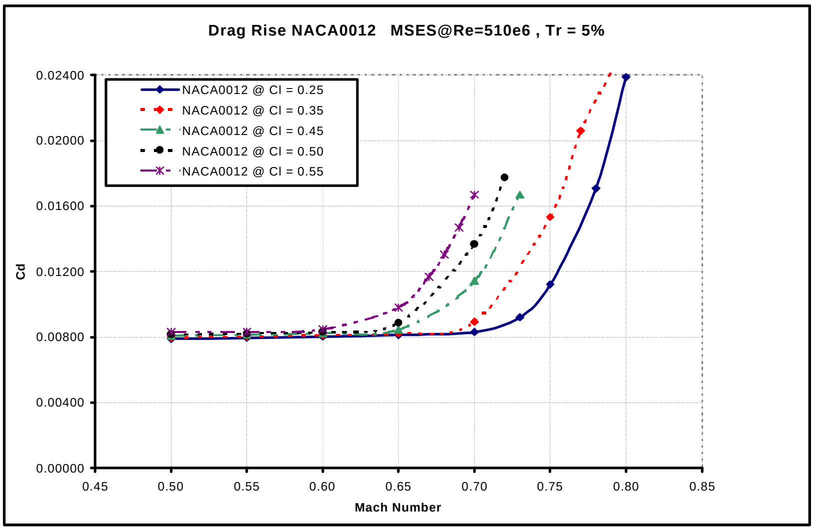


Figure 1. NACA0012 drag rise curves for a set of Cl values.

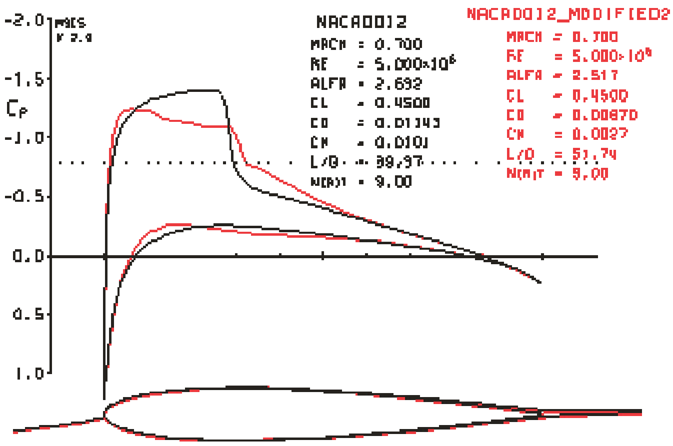


Figure 2. Comparison between the original NACA0012 airfoil and the optimized airfoil.

The first optimization test case was performed for a NACA0012 at Mach number of 0.7, lift coefficient (Cl) of 0.45 and Re of 5 million. The original profile at this point gave a drag coefficient (Cd) of 0.01143 and lift over drag ratio (L/D) of 39.37. For the optimized airfoil, it was obtained that Cd = 0.00870 and L/D = 51.74. The reader should observe that this result means a difference of 23.9% in the drag and 31.14% in the L/D value. Figure 2 shows the difference in shape and in the Cp distribution between the standard and the modified airfoils. In Fig. 3, one can observe the difference in the geometry due to the optimization procedure. The modified airfoil has a reasonable decrease in the Cd. However, we have to be concerned about how this new optimized shape will behave at other values of Mach number or Cl. A drag rise curve for this optimized profile was evaluated to observe if the same level of Cd decrease at

M = 0.7 would be present for other values of Mach number. Figure 4 is a comparison between the standard and the optimized airfoil at Cl = 0.45. It can be seen that the optimized airfoil has a drag creep for lower values of Mach number.

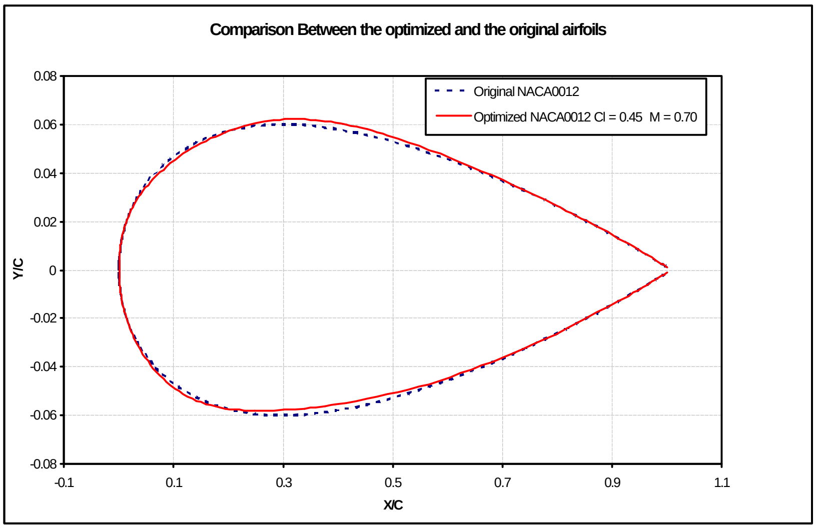


Figure 3. Comparison between the optimized and the original airfoil.

The appearance of this drag creep brought up the question of wheter the same sort of behavior would be present in other values of Cl. Therefore, this optimized airfoil was evaluated for lower values of Cl, as can be seen in Figs. 5 and 6. One can see in Fig. 5 the optimized airfoil evaluated at Cl = 0.35. At this value of Cl, one can observe that the drag creep present at Cl = 0.45 does not appear anymore and a similar behavior occurs for the Cl = 0.25, as shown in Fig. 6. For these lower values of Cl, the drag creep disappears but the difference in Cd, at the Mach number for which the airfoil was optimized, decreases. Nevertheless, a postponement in the drag-divergence Mach number can still be observed in those cases. This result might lead to the following conclusion: one must optimize the airfoil for a Cl value above the design Cl if one wishes to have no drag creep. Now, supposing one would need an airfoil to operate at Cl = 0.45, the previous optimization is not satisfactory because there is drag creep for lower Mach numbers. Therefore, one should optimize the airfoil for a Cl value above 0.45 and take this optimized airfoil and evaluate it for Cl = 0.45.

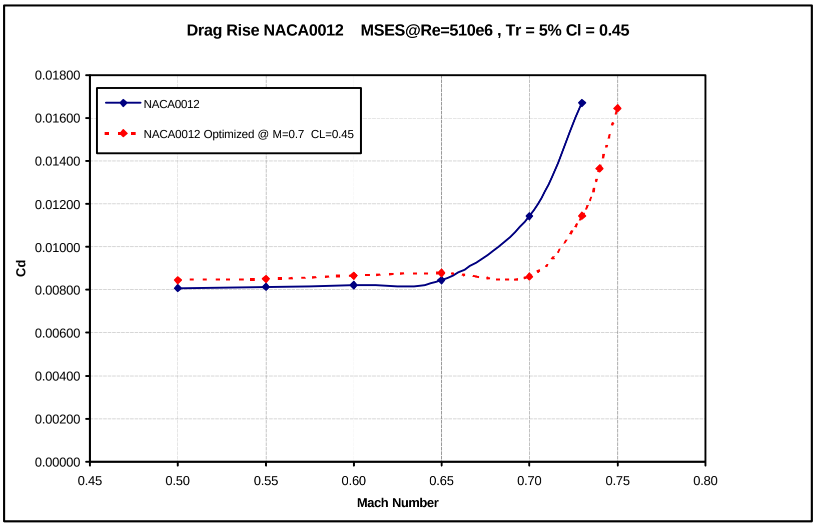


Figure 4. Drag rise evaluation for the optimized NACA0012 airfoil.

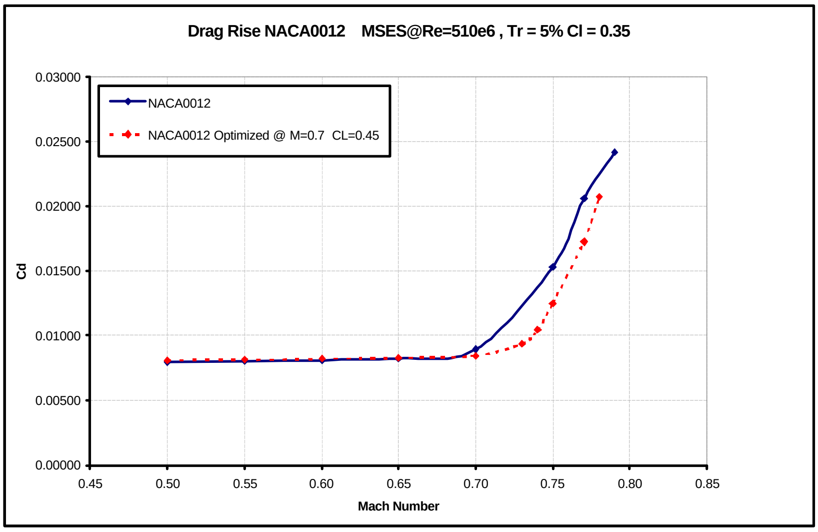


Figure 5. Optimized NACA0012 airfoil evaluated for Cl=0.35.

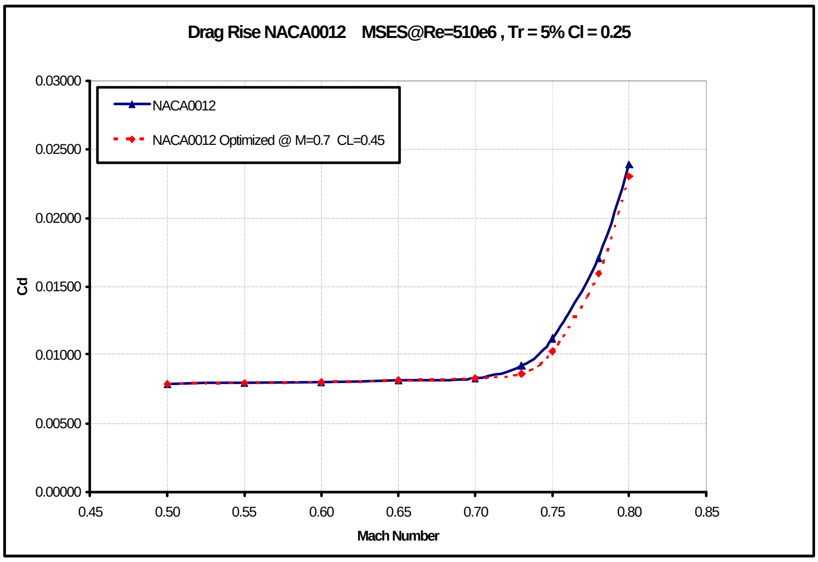


Figure 6. Optimized NACA0012 airfoil evaluated for Cl=0.25.

Figure 7 presents the comparison of the drag rise curves for two optimized NACA0012 airfoils, the first one optimized for Cl = 0.45 and M = 0.7 and the other one optimized for Cl = 0.50 and M = 0.7, when both airfoils are evaluated at Cl = 0.45. One can observe that the NACA0012 airfoil optimized for Cl = 0.50 produced a worse behavior than the one optimized for Cl = 0.45. The assumption that the optimization should be performed in a Cl value higher than the design Cl does not seen to be very applicable. These optimized NACA0012 airfoils were also evaluated at other two different Cl values, Cl = 0.35 and Cl = 0.25. Figure 8 shows that, as the design Cl decreases, the difference between the airfoil optimized for Cl = 0.5 and the airfoil optimized for Cl = 0.45 increases.

In Fig. 9, the drag polar curves at M = 0.7 were traced for the two optimized NACA0012 airfoils and the original NACA0012. One can observe that the original NACA0012 airfoil reaches the drag-divergence Mach number at Cl = 0.35, while the optimized airfoils have not reached the drag-divergence Mach number even at Cl = 0.45. For that reason, both optimized airfoils can operate at higher values of Cl without divergence of the drag coefficient. However, it can be seen that, when the airfoil optimized for Cl = 0.5 and M = 0.7 is operating at a Cl values lower than 0.35, it is producing more drag than the original NACA0012. Figure 10 shows the drag polar curve at M = 0.65 for the three airfoils. One can notice that the airfoil optimized for Cl = 0.5 and M = 0.7 produces the higher Cd value for all the Cl range.

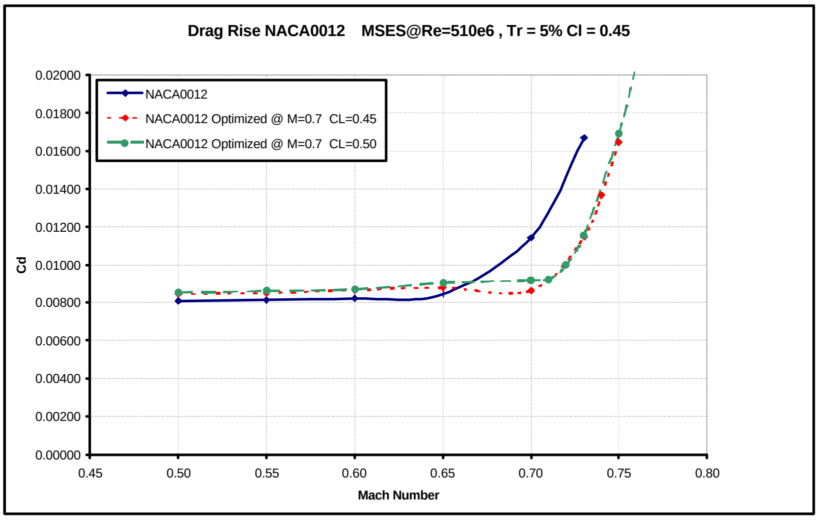


Figure 7. Comparison of the drag rise curves between different optimized NACA0012 airfoils evaluated at Cl=0.45.

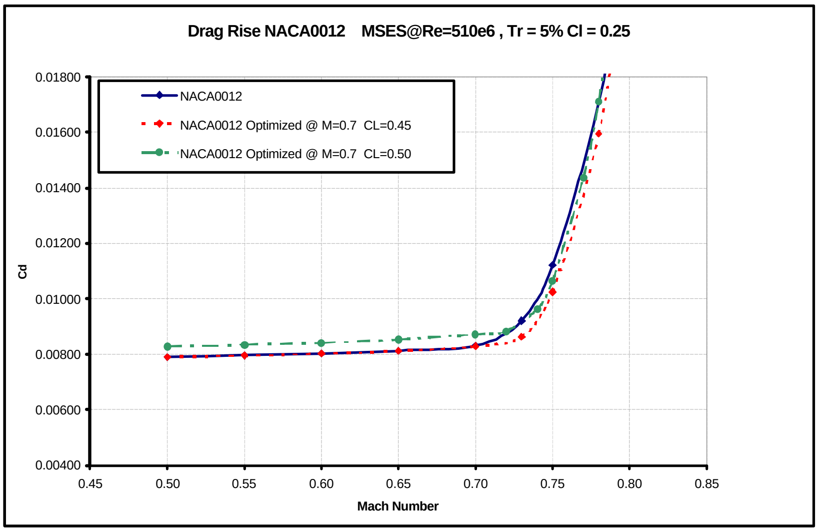


Figure 8. Drag rise curves for the optimized NACA0012 airfoils evaluated at Cl=0.25.

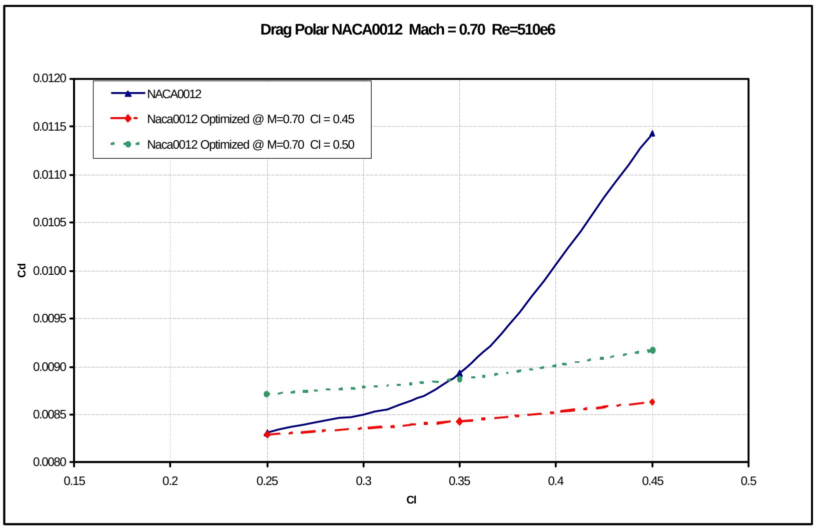


Figure 9. Drag polar curves for original and optimized NACA0012 airfoils at Mach number of 0.70.

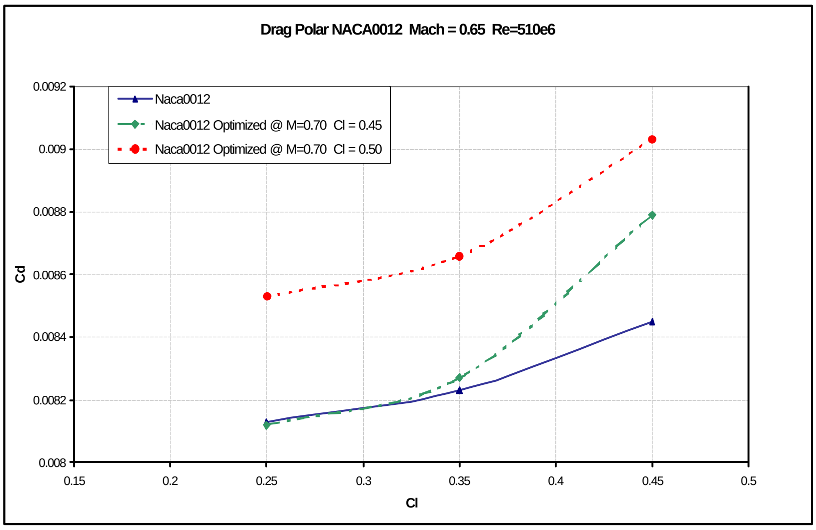


Figure 10. Drag polar curves for original and optimized NACA0012 airfoils at Mach number of 0.65.

Figure 11 shows the drag rise curve evaluated at Cl = 0.70 for the original PRO01airfoil and the optimized PRO01 airfoil at M=0.75 and Cl=0.60. For this optimization, one can observe that the resultant geometry does not produce a drag creep for lower values of Mach number, as observed in the optimized NACA0012 airfoils. However, the optimized airfoil has little difference in terms of drag coefficient with relation to the original airfoil, about 0.0002. When this optimized airfoil was evaluated for CL=0.40, as can be seen in Fig 12, the difference in the drag coefficient maintained constant at the same 0.0002 level.

This PRO01 is a typical rear-loading airfoil and it basically has some geometric characteristics of transonic airfoils. For this peculiar sort of geometry, the optimization procedure showed to be not a simple task. The Legendre polynomials did not produce a good result and, therefore, other types of shape functions had to be used. The results presented here for the PRO01 airfoil can probably be further improved if better shape functions could be found for this case.

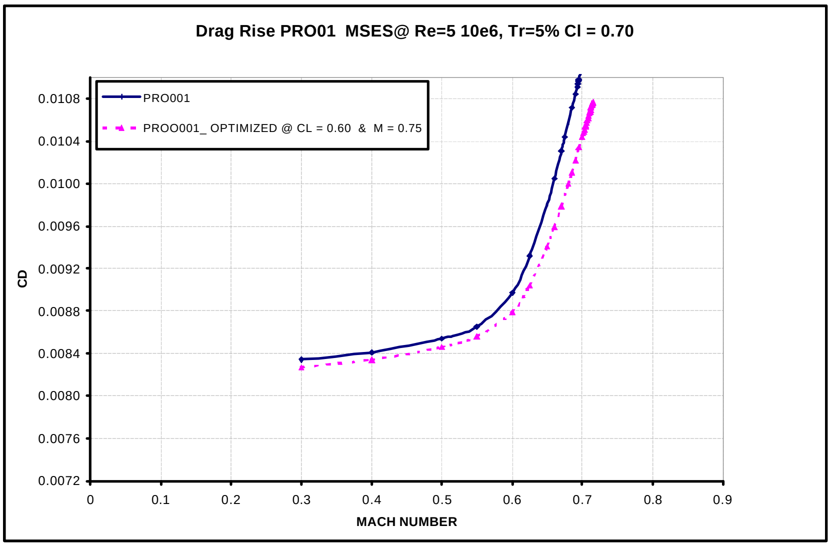


Figure 11. Drag rise curves for the original and optimized PRO01 airfoils at Cl=0.70.

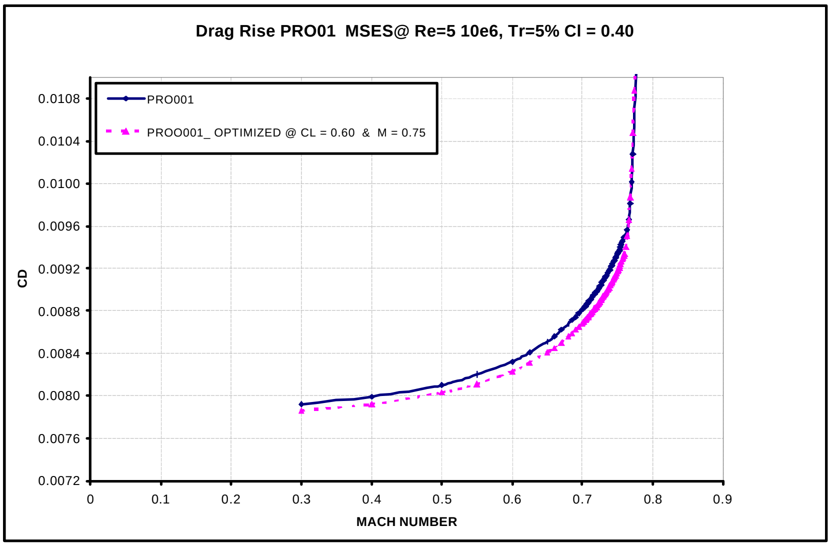


Figure 12. Drag rise curves for original and optimized PRO01 airfoils at Cl=0.40.

5 CONCLUSIONS

The optimization process with the GA is very satisfactory but sometimes it has a high computational cost. In this specific work, the computational costs were not so high since we were using a two-dimensional formulation (MSES) to obtain the objective function. Real improvements in the resulting geometries were obtained, at least in the optimization point. Therefore, to obtain this improvement, an exploration of the fittest shape function had to be performed. There is no a universal function that can always lead to optimized geometries. For each type of airfoil there exists a shape function that can produce the best results. The PRO001 airfoil is a clear proof of this statement as it could not be optimized with the Legendre polynomials. On the other hand, the use of the Hick-Henne shape function produced some improvement in the airfoil performance.

Another important concern is about the optimization point. Which is the best design point to optimize? The results presented gave the indication that a single point optimization is not a very adequate way to make optimizations. For the NACA0012, it could be notice that the optimization for a specific point produced 'collaterals effects' in other values of Cl and Mach number. Based on this experience, the ideal optimization should take into account more than one design point and the overall mission of the airplane. On the other hand, these 'collateral effects' were not present in the optimized airfoil PRO001, besides the fact that it was optimized just for one design point. Perhaps, the modification in the airfoil was not big enough to allow the appearance of these 'collateral effects'. This is a question that needs to be further investigated.

6. ACKNOWLEDGEMENTS

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