NUMERICAL SOLUTION OF THE MOTION EQUATION OF RIGID ROTORS SUPPORTED BY SQUEEZE FILM DAMPER BEARINGS

Hudson A. Bode[1]
João Carlos Menezes[2]
Instituto Tecnológico de Aeronáutica - ITA
Div. de Eng. Mecânica-Aeronáutica, 12228-901, São José dos Campos - SP - Brazil
bode@mec.ita.cta.br[1]
menezes@mec.ita.br[2]

Abstract. *The performance of squeeze-film damper bearings as a flexible support of a bearing-rotor system is analyzed. The use of squeeze films as support in the system rotor-bearing, has been studied since the early 1960s and applications in turbines by the producers of gas-turbine. If appropriately designed, turbines using squeeze film bearings can go quickly by the critical speed of the axis rotor with substantial reduction of the vibrations of the axis, which could damage the equipment. The forces produced by dynamic pressure of the lubricant are obtained by the solution of the Reynolds equation for the fluid-film. The dynamical equations that govern the motion of the rotor supported by squeeze films are solved by Newmark method. An iterative procedure in the time domain is applied in order to have a solution of the nonlinear-coupled equations of motion. Parameters associated to the mass and rotational speed of the axis, physical and geometric characteristics of the bearing, such as viscosity of the fluid, radial clearance, length and diameter of the bearing, were varied to allow evaluations of the orbital behavior of the rotor. For chosen groups of parameters, the influence of the unbalanced force of the rotor was studied. Results reveal a sensitivity of motion stability and orbital size to all parameters values.*

Keywords: *Mechanical Vibrations, Squeeze Film Bearing and Numerical Methods.*

1. Introduction

The use of squeeze films as support in rotor-bearing systems, has been studied since the early 1960s and has found applications in turbines by the producers of gas-turbine (Cookson and Kossa, 1979). Basically, the squeeze film consists of a fluid confined between an axis-rotor (in this case, the axis of a turbine) and the outer race of a rolling contact bearing. That element type can, in much appraised conditions, attenuate the vibrations of the system rotor-bearings and promote the stability of the axis. If designed appropriately, turbines that use the squeeze film system can go quickly by the critical speed of the axis rotor with sensitive reduction of the vibrations of the axis, which could damage the equipment (Holmes, 1972).

Unlike the classic behavior in the hydrodynamic bearings analysis, where in the steady-state condition the axis assumes an eccentric constant position relative to the bearing, in the analysis of the squeeze film, the position of the axis varies with the time (Zhang, Litang and Li, 1992 and Nataraj and Ashrafioun, 1979). That characteristic is due to the presence of a centrifuge harmonic force caused by an unbalance of the rotor. Characteristically the position of the axis goes by a transient phase and progresses to a steady state orbit. The position and format of that orbit depends on the characteristics of the bearing, the lubricant oil, the weight and angular speed of the axis and the intensity of the unbalance force.

Notation

e	eccentricity between journal center and housing center
c	radial clearance (housing radius - journal radius)
h	oil film thickness
L	bearing land length
η	absolute viscosity
ε	eccentricity ratio (e/c)
Φ	attitude angle
t	time
g	gravitational acceleration
m	rotor mass (per bearing land)
r_a	bearing radius
r_b	journal radius
U	unbalance parameter ($F_u/mc\omega^2$)
B	bearing parameter ($12\eta L r_a^2/mc\omega^2$)
W	rotor weight (per bearing land)

$\overline{W}$	gravity (or weight) parameter ($W/mc\omega^2$)
w'_x	load component per unit width perpendicular to line of centers
w'_z	load component per unit width along line of centers
$\overline{F_r}$	non dimensional radial fluid-film force ($F_r/mc\omega^2$)
$\overline{F_t}$	non dimensional tangential fluid-film force ($F_t/mc\omega^2$)
F_r	fluid film force in radial direction
F_t	fluid film force in tangential direction
F_u	unbalance force
w_a, w_b	velocities of fluid in z direction acting at surface a and b, respectively, (m/s)
ω	rotational velocity of journal about sleeve center when eccentricity ratio is constant (rad/s)
ω_a	bearing angular speed of surface bearing (rad/s)
ω_b	rotor angular speed of surface journal (rad/s)
ϕ	angular distance from the positive x-axis in the fixed x-z coordinate set
ϕ_m	upper limit of the positive pressure
(')	$d/d(\omega t)$
(.)	d/dt

2. Mathematical development

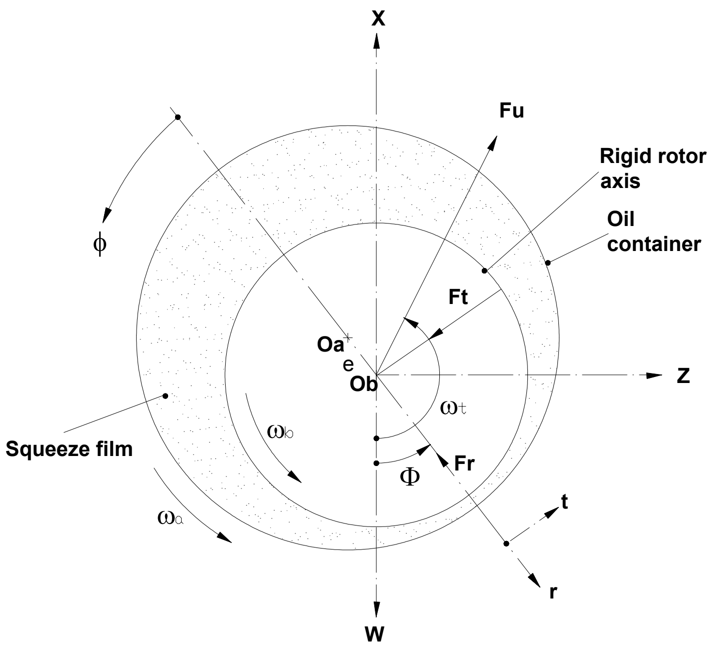

Figure 1. Squeeze film damper with dynamic forces and coordinates defined.

The general Reynolds equation governing the flow of the squeeze film oil is well known as (Cameron, 1981, Barret and Gunter, 1975, Kirk and Gunter, 1970):

$$\frac{\partial}{\partial x}\left(-\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(-\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)+\frac{\partial}{\partial x}\left[\frac{\rho h(u_a+u_b)}{2}\right]+\frac{\partial}{\partial y}\left[\frac{\rho h(v_a+v_b)}{2}\right]+$$

$$+\rho(w_a+w_b)-\rho u_a\frac{\partial h}{\partial x}-\rho v_a\frac{\partial h}{\partial y}+h\frac{\partial p}{\partial t}=0 \qquad (1)$$

where the following assumptions were made: (a) The fluid inertia terms in Navier-Stokes equations have been neglected due to their small magnitude; (b) The flow is laminar; (c) The fluid is Newtonian; (d) No slip exists at the fluid-solid interface; (e) The flow in the radial direction has been neglected; (f) The inclination of one surface relative to the other is so small that the sine of the angle of inclination can be set equal to the angle and the cosine can be set equal to unity.

The general Reynolds equation given in Eq. (1) can be applied to any section of the oil film and in this work only the dynamically loaded infinitely wide-journal-bearing solution will be presented. The film thickness can be described as (Hamrock, 1994, Bisson and Anderson, 1992, Dubois and Ocvick, 1953):

$$h = c(1 + \varepsilon \cos\phi) \tag{2}$$

if side-leakage is neglected, Eq. (1) can be rewritten and integrated while making use of Eq. (2) which gives:

$$\frac{\partial p}{\partial \phi} = \frac{12\eta\left(\frac{r_a}{c}\right)^2\left[\frac{\partial \varepsilon}{\partial t}\operatorname{sen}\phi - \varepsilon\cos\phi\left(\omega - \frac{\omega_a + \omega_b}{2}\right) - \widetilde{A}\right]}{(1+\varepsilon\cos\phi)^3} \tag{3}$$

where for $(p)_{\varnothing=0}=(p)_{\varnothing=2\pi}=p$,

$$\widetilde{A} = \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a + \omega_b}{2}\right) \tag{4}$$

replacing Eq. (4) in Eq. (3), one may write

$$\frac{\partial p}{\partial \phi} = \frac{12\eta\left(\frac{r_a}{c}\right)^2\left[\frac{\partial \varepsilon}{\partial t}\operatorname{sen}\phi - \varepsilon\cos\phi\left(\omega - \frac{\omega_a + \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a + \omega_b}{2}\right)\right]}{(1+\varepsilon\cos\phi)^3} \tag{5}$$

Once the pressure is known, load components can be evaluated. One may determine the components of the resultant load along and perpendicular to the line of centers, as:

$$w_x' = r_b \int_0^{\phi_m} \cos\phi \frac{dp}{d\phi} d\phi \tag{6}$$

$$w_z' = r_b \int_0^{\phi_m} \operatorname{sen}\phi \frac{dp}{d\phi} d\phi \tag{7}$$

taking into account Eqs. (8), (9), (10) and (11) as follows:

$$F_r = \overline{F_r} mc\omega^2 \tag{8}$$

$$F_t = \overline{F_t} mc\omega^2 \tag{9}$$

$$w_x' = F_t / L \tag{10}$$

$$w_z' = F_r / L \tag{11}$$

one may write:

$$\overline{F_r} = Lw_z' / mc\omega^2 \tag{12}$$

$$\overline{F_t} = Lw_x' / mc\omega^2 \tag{13}$$

replacing Eqs. (5), (6) and (7) into Eqs. (12) and (13) gives:

$$\overline{F_r} = B\int_0^{\phi_m} \frac{\left[\frac{\partial \varepsilon}{\partial t}\sin^2\phi - \varepsilon\sin\phi\cos\phi\left(\omega - \frac{\omega_a - \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a - \omega_b}{2}\right)\sin\phi\right]d\phi}{\left(1+\varepsilon\cos\phi\right)^3} \tag{14}$$

$$\overline{F_t} = B\int_0^{\phi_m} \frac{\left[\frac{\partial \varepsilon}{\partial t}\sin\phi\cos\phi - \varepsilon\cos^2\phi\left(\omega - \frac{\omega_a - \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a - \omega_b}{2}\right)\cos\phi\right]d\phi}{\left(1+\varepsilon\cos\phi\right)^3} \tag{15}$$

where the bearing parameter B is defined as:

$$B = 12\eta\left(\frac{r_a}{c}\right)^2 r_b L \Big/ mc\omega^2 \tag{16}$$

3. Governing equations for rigid rotors supported by squeeze film damper bearings

Figure (1) shows schematically a rigid rotor axis within the oil container, under the action of a steady load *W* due to the dead weight of the rotor it supports. Vibration arises from a centrifugal force Fu due to unbalance. The amplitude of orbital motion will depend on *W*, F_u, $\overline{F_r}$ and $\overline{F_t}$ .The latter two forces $\overline{F_r}$ and $\overline{F_t}$ *t*, are those arising hydro-dynamically from the squeeze film (Cookson and Kossa, 1979 and Gunter, 1966).

The following assumptions are made: (a) The rotor is rigid and symmetric; (b) The angular speed of rotation is constant; (c) No significant exciting forces are introduced by the rolling-contact bearings. Therefore, one may show that the governing equations of the motion for the rotor-bearing system are:

$$m\left(\ddot{e} - e\dot{\Phi}^2\right) = F_u\cos\left(\omega t - \Phi\right) + w\cos\Phi - F_r \tag{17}$$

$$m(e\ddot{\Phi} + 2\dot{e}\dot{\Phi}) = F_u\,\mathrm{sen}(\omega t - \Phi) - w\,\mathrm{sen}\,\Phi + F_t \tag{18}$$

Dividing these equations by $mc\omega^2$, one obtains

$$\varepsilon'' - \varepsilon\Phi'^2 = U\cos(\omega t - \Phi) + \overline{W}\cos\Phi - \overline{F_r} \tag{19}$$

$$\varepsilon\Phi'' + 2\varepsilon'\Phi' = U\,\mathrm{sen}\left(\omega_b t - \Phi\right) - \overline{W}\,\mathrm{sen}\,\Phi + \overline{F_t} \tag{20}$$

Replacing Eqs. (14), (15) and (16), into Eqs. (19) and (20) respectively, yields the following non-dimensional form of the equations of motion:

$$\varepsilon'' - \varepsilon\Phi'^2 = U\cos(\omega t - \Phi) + \overline{W}\cos\Phi - \\ - B\int_0^{\phi_m} \frac{\left[\frac{\partial \varepsilon}{\partial t}\mathrm{sen}^2\phi - \varepsilon\,\mathrm{sen}\,\phi\cos\phi\left(\omega - \frac{\omega_a + \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a + \omega_b}{2}\right)\mathrm{sen}\,\phi\right]d\phi}{\left(1+\varepsilon\cos\phi\right)^3} \tag{21}$$

$$\varepsilon\Phi'' + 2\varepsilon'\Phi' = U\,\mathrm{sen}\left(\omega t - \Phi\right) - \overline{W}\,\mathrm{sen}\,\Phi - \\ - B\int_0^{\phi_m} \frac{\left[\frac{\partial \varepsilon}{\partial t}\mathrm{sen}\,\phi\cos\phi - \varepsilon\cos^2\phi\left(\omega - \frac{\omega_a + \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a + \omega_b}{2}\right)\cos\phi\right]d\phi}{\left(1+\varepsilon\cos\phi\right)^3} \tag{22}$$

The angle $\varnothing_m$ is the upper limit of the positive pressure, which is obtained numerically. Equations (21) and (22) of motion of the center journal are numerically solved by Newmark's method to give the journal position, velocity and acceleration.

3.1. Integration of the pressure profile

The forces arising in the fluid film have been expressed as an integral over the circumference of the journal. The forces are given by Eqs. (14) and (15). The expressions under the integral are now representative of the pressure in the film and hence will be equated to zero when its value is less than zero. This is equivalent to keeping only those pressures greater than ambient. This will avoid the sub ambient pressure contributions that appear in closed-form solutions. According to this approach, one needs to calculate the extent of the positive pressure region. Figure (2) illustrates the pressure variation along the circumferential positions at several time positions for a case where F_u=0. The exact region of film cavitations and the resulting pressure therein are by no means well understood or well defined in the literature. Dubois and Ocvirk (1953) argued that in the absence of high datum pressures, the effect of any negative pressure (not exceeding atmospheric) could be neglected as being negligible in comparison to the positive pressure region.

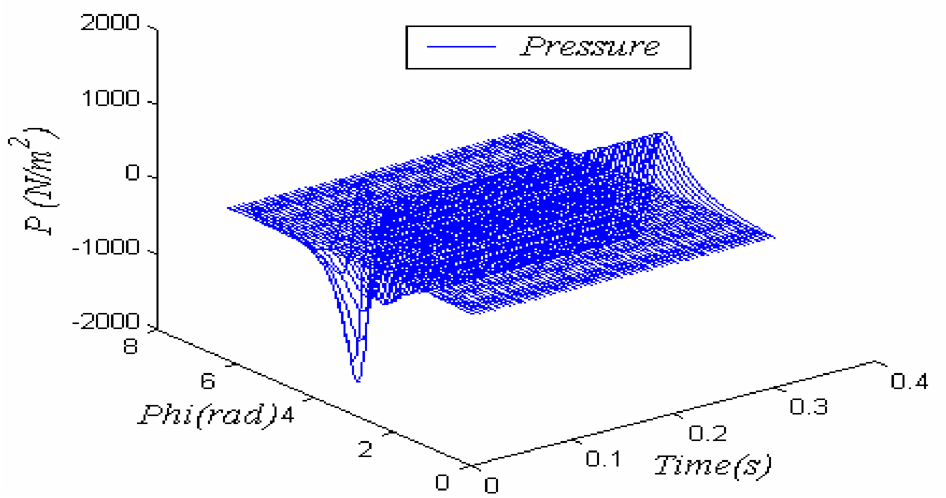

Figure 2. Illustration of pressure variation

A numerical method is used to obtain the fluid forces from the integral above. An appropriate method for this purpose is the well-known trapezoidal method, which can be expressed as follow:

$$\int_{x_0}^{x_i} f(x)\,dx = \Delta x\left[\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \cdots + \frac{f(x_i)}{2}\right] \tag{23}$$

The error of the above formula is directly related to the increment Δx and therefore a proper the number of points has to be chosen in the evaluation, according to the order of curve that is being integrated.

3.2. Integration of the Equations of Motion

The most basic self-starting method is simply a Taylor Series Expansion truncated after some arbitrary number of terms. By truncating the series, which is known as Newmark's Method (Rao et al., 1995) one may obtain:

$$\dot{x}_{t+\Delta t} = \dot{x}_t + \Delta t\left[(1-\delta)\ddot{x}_t + \delta\ddot{x}_{t+\Delta t}\right] \tag{24}$$

$$x_{t+\Delta t} = x_t + \Delta t\dot{x}_t + (\Delta t)^2\left[\left(\frac{1}{2}-\beta\right)\ddot{x}_t + \beta\ddot{x}_{t+\Delta t}\right] \tag{25}$$

where δ and β are parameters that can be determined to obtain integration accuracy and stability. When δ=1/2 and β=1/4, Eqs. (24) and (25) correspond to the linear acceleration method.

All variables in Eqs. (21) and (22) have to be considered at the same time step. For this purpose they are written as

$$\varepsilon''_{t+\Delta t} - \varepsilon_{t+\Delta t}\Phi'^{2}_{t+\Delta t} = \gamma_{t+\Delta t} \tag{26}$$

$$\varepsilon_{t+\Delta t}\Phi''_{t+\Delta t} + 2\varepsilon'_{t+\Delta t}\Phi'_{t+\Delta t} = \alpha_{t+\Delta t} \tag{27}$$

Solving Eq. (25) for $\ddot{x}_{t+\Delta t}$ in terms of $x_{t+\Delta t}$ and then replacing $\ddot{x}_{t+\Delta t}$ into Eqs. (26), one can obtain equations for $\ddot{x}_{t+\Delta t}$ and $\dot{x}_{t+\Delta t}$, each one in terms of the unknown displacements $x_{t+\Delta t}$ and the previous velocity and acceleration. Replacing the second derivatives in Eqs. (26) and (27) properly, according to Eqs. (28) and (29) below, one may obtain Eqs. (30), (31), (32) and (33).

$$\varepsilon''_{n+1} = \frac{4}{(\omega\Delta t)^2}\left(\varepsilon_{n+1} - \varepsilon_n\right) - \frac{4}{(\omega\Delta t)}\varepsilon'_n - \varepsilon''_n \tag{28}$$

$$\Phi''_{n+1} = \frac{4}{(\omega\Delta t)^2}\left(\Phi_{n+1} - \Phi_n\right) - \frac{4}{(\omega\Delta t)}\Phi'_n - \Phi''_n \tag{29}$$

$$\left[\frac{4}{(\omega\Delta t)^2} - \left(\Phi'_{n+1}\right)^2\right]\varepsilon_{n+1} = \gamma_{n+1} + \frac{4}{(\omega\Delta t)^2}\varepsilon_n + \frac{4}{(\omega\Delta t)}\varepsilon'_n + \varepsilon''_n \tag{30}$$

where

$$\Phi'_{n+1} = \frac{2}{(\omega\Delta t)}\left(\Phi_{n+1} - \Phi_n\right) - \Phi'_n \tag{31}$$

$$\left[\frac{4}{(\omega\Delta t)} - \left(\frac{\varepsilon_{n+1}}{(\omega\Delta t)} + \varepsilon'_{n+1}\right)\right]\Phi_{n+1} = \alpha_{n+1} + 2\varepsilon'_{n+1}\left(2\frac{\Phi_n}{(\omega\Delta t)} + \Phi'_n\right) + \varepsilon_{n+1}\left[\frac{4}{(\omega\Delta t)}\left(\frac{\Phi_n}{(\omega\Delta t)} + \Phi'_n\right) + \Phi''_n\right] \tag{32}$$

where

$$\varepsilon'_{n+1} = \frac{2}{(\omega\Delta t)}\left(\varepsilon_{n+1} - \varepsilon_n\right) - \varepsilon'_n \tag{33}$$

4. Results

A computer code, based on Newmark approach, was written. Eqs. (30) and (32) are solved simultaneously, and therefore, an interactive routine had to be created to get convergence at each time step.

Some cases were devised and for each case one or more system parameters were varied. These cases and the correspondent parameters values are listed in Tab. (1):

Table (1). System parameters for a rigid rotor supported in squeeze-film damper bearings.

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Initial eccentricity (e/c)	0.8	0.8	0.8	0.8	0.8	0.8	0.8
Journal weight (kg)	33.6	33.6	33.6	33.6	33.6	33.6	33.6
Clearance (m)	2.540e-4	2.540e-4	2.540e-4	2.540e-4	2.540e-4	2.540e-4	2.540e-4
Bearing radius (m)	2.540e-2	2.540e-2	2.540e-2	2.540e-2	2.540e-2	2.540e-2	2.540e-2
Bearing length (m)	5.080e-2	5.080e-2	5.080e-2	5.080e-2	5.080e-2	5.080e-2	5.080e-2
Unbalance force (N)	100.0	100.0	100,0	100,0	300.0	100.0	100.0
Journal speed (rpm)	2000	6000	2000	5000	5000	2000	4000
Viscosity (Ns/m^2)	2.622e-3	2.622e-3	8.276e-3	8.276e-3	8.276e-3	2.530e-2	2.530e-2

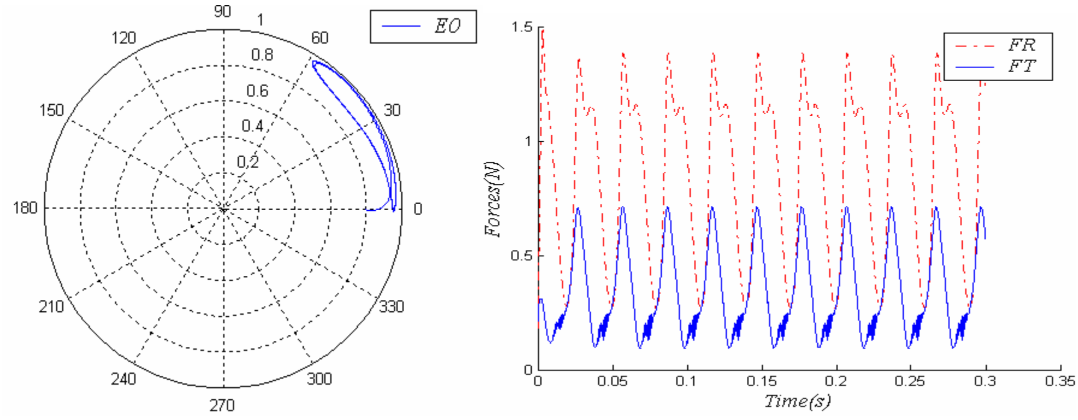

Figure 3. Case I: ε=0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=100 N, V_b=2000 rpm, η=2.622e-3 Ns/m^2.

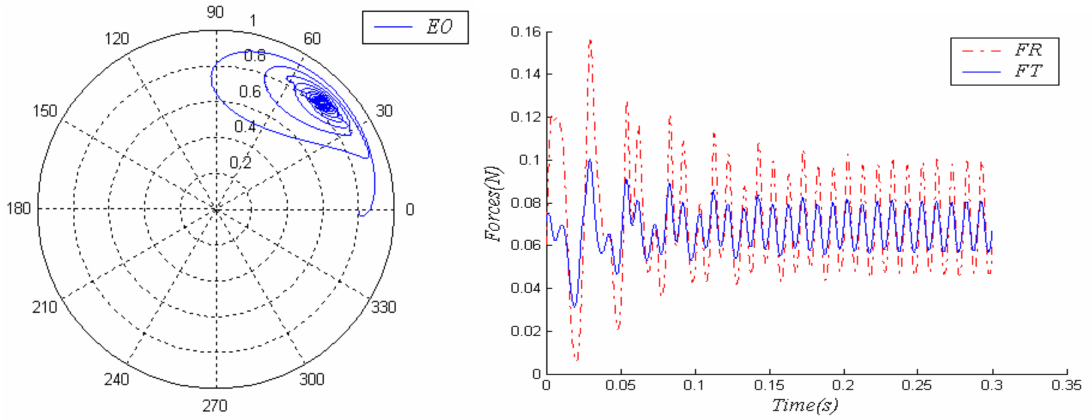

Figure 4. Case II: ε= 0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=100 N, V_b=6000 rpm, η=2.622e-3 Ns/m^2.

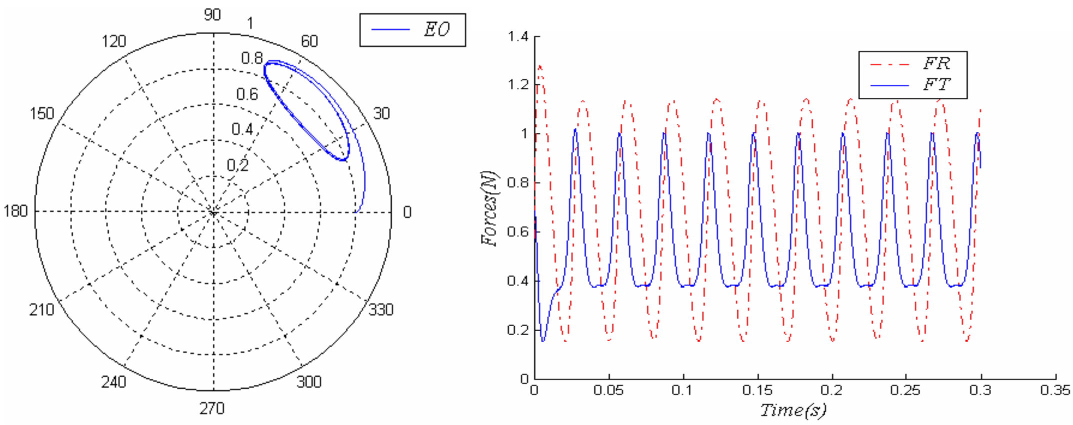

Figure 5. Case III: ε=0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=100 N, V_b=2000 rpm, η=8.276e-3 Ns/m^2.

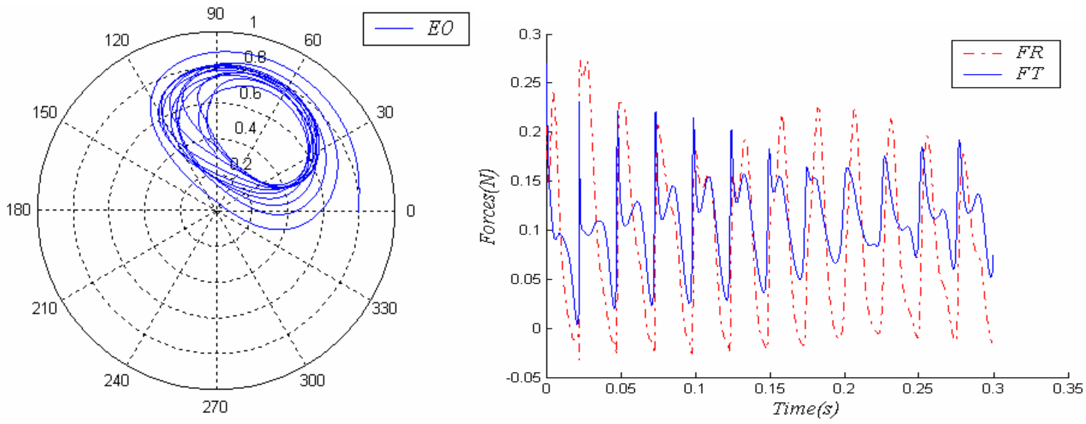

Figure 6. Case IV: ε=0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=100 N, V_b=5000 rpm, η=8.276e-3 Ns/m^2.

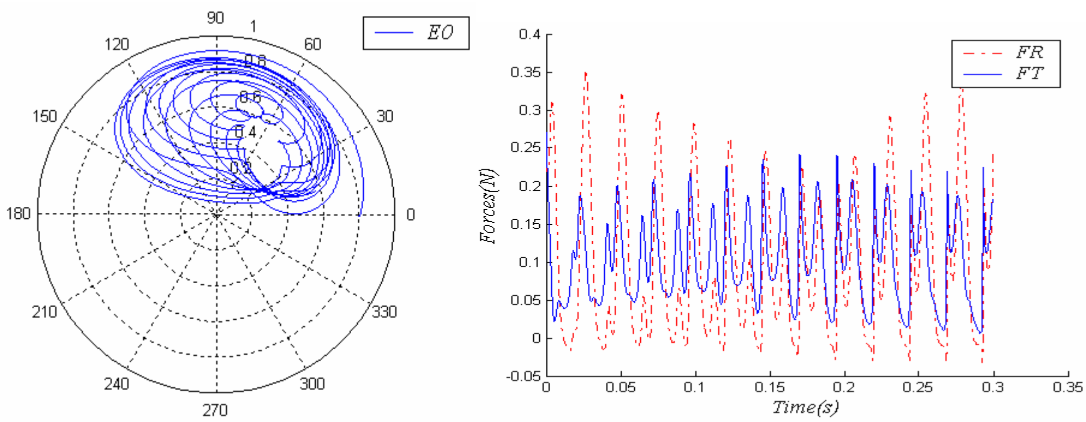

Figure 7. Case V: ε=0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=300 N, V_b=5000 rpm, η=8.276e-3 Ns/m^2.

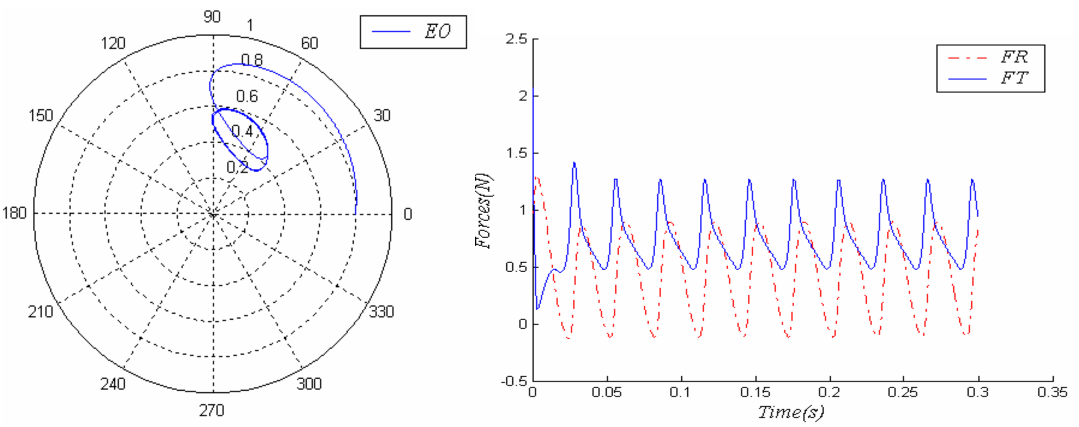

Figure 8. Case VI: ε=0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=100 N, V_b=2000 rpm, η=2.53e-2 Ns/m^2.

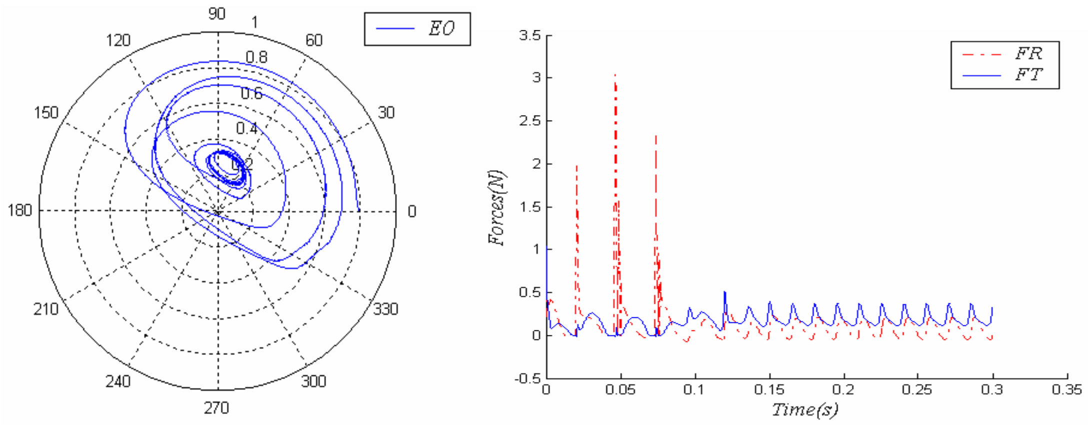

Figure 9. Case VII: ε=0.8, W=33.6 kg, c=2.54e-4 m, r_b=2.54e-2 m, L=5.08e-2 m, F_u=100 N, V_b=4000 rpm, η=2.53e-2 Ns/m^2.

5. Conclusion

Performing a preliminary analysis of the results for the motion, one may verify, that in a situation where the unbalanced force is not taken into account, the rotor center converges to a steady-state position at a certain eccentricity and attitude angle. Under certain circumstances, journal develops a characteristic eccentric orbit. Figure (2) illustrates the variation of the pressure profile, and most important, the variation of the limits of the positive pressure. The increase on journal speed, as one can compare Case I and Case II, results in a more centralized orbit. Case: III, IV, V, VI and VII consider the same clearance, bearing radius, and bearing length. For such cases, the increase in journal speed, comparing Case IV to Case III, results also in a more centralized orbit, although in these cases, the orbit increased in size. The increase of the unbalanced force in Case V produced less consistent orbits. In Case VI, the unbalanced force and the journal speed were reduced, which produced a smaller and more centralized orbit. In Case VII, the journal velocity was set to 4000 rpm, producing a final less eccentricity orbit. One may conclude that all the listed parameters interfere on the journal behavior. The size, position and shape of the orbit are a result of the combined values of these parameters. It can be shown that in the absence of unbalance forces, the journal center converges to a steady position. Under special circumstances one can obtain instability of the journal.

6.Acknowledgments

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