

HOPF BIFURCATION OF FPSO MOORED SYSTEMS WITH TERMS OF TYPE V|V|: AN ANALYSIS USING THE CENTRAL MANIFOLD THEOREM AND INTEGRAL AVERAGING TECHNIQUES

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Abstract: This work treats the problem of dynamic equilibrium bifurcation of systems that exhibit terms of type $\dot{x}|\dot{x}|$. Examples of this kind are common in mechanics, particularly in those involving viscous fluid forces that appear in mechanical, aeronautical and ocean engineering. Nowadays the current use of computer simulators enables the analyst to address the problem under an exhaustive non-linear time domain approach. Alternatively, some other techniques of Applied Mathematics as the Central Manifold Theorem (CMT) and the Integral Averaging Method (IAM) may be proper to qualify and quantify Hopf bifurcation scenarios. For systems with terms of type $\dot{x}|\dot{x}|$ a direct application of such techniques are not straightforward, though. The analyst must treat the low differentiability, of order one, at $\dot{x}=0$. In this paper a simple regularization technique, based on polynomial approximations is proposed, enabling a local treatment of the post-critical behaviour for this kind of problems. Examples extracted from engineering, as the fishtailing instabilities of a moored offshore vessel can be viewed as practical applications.

Keywords: *Hopf Bifurcation, Central Manifold Theorem, Integral Averaging, Dynamical Systems, FPSO moored systems.*

1. Introduction

Differential equations modeling systems, which involve viscous fluid forces, as those appearing in mechanical, aeronautical and ocean engineering, usually present terms of type $\dot{x}|\dot{x}|$. One possible approach to study the problems of static and dynamic equilibrium bifurcation related to this kind of system is the use of computer simulators, through exhaustive non-linear time domain simulations. Another possible one is based on some more recent techniques of Applied Mathematics, as the Central Manifold Theorem (CMT) and Integral Averaging Method (IAM), to quantify and qualify static and dynamic bifurcation scenarios in analytical form. The main advantages of these latter techniques are:

(I) The associated simplification of the dynamical system, near the bifurcation point, with no loss of relevant information on dynamics. This simplification may be seen in two ways. Firstly, by the reduction of dimensionality of the dynamical system. In most examples the analyst can reduce the dimensionality of the dynamical system near the bifurcation point from N to 1 or 2 dimensions. Secondly, this simplification implies the elimination of a number of non-linear terms of the dynamical system. With such a simplification the analyst can obtain, in a proper way, analytical post-critical scenarios to the static or dynamic bifurcation as well as can study the dependence of the system on the parameters that are relevant to the problem.

(II) These techniques present an intrinsic geometrical appeal, in the sense that they enable to re-write the system into new coordinates (on the Central Manifold) associated, in the linear part, with the eigenvalues with zero real part.

(III) Another important point is that the Central Manifold is an invariant manifold. So, once an orbit enters the Center Manifold, this orbit will not escape from it.

(IV) With this Technique it is possible to extend (in an analytical form) the results from Bernitsas (1999), including nonlinear terms in the analysis. The results we see in Bernitsas (1999) take into account information coming only from linear terms of Taylor expansion of the equations of motion. Considering only linear terms it would be eventually impossible to correctly qualify (analytically and *a priori*) any Dynamic Bifurcation and, certainly, it would be impossible to quantify the amplitude of the resulting cycle limits.

However, for systems with terms of type $\dot{x}|\dot{x}|$ the CMT is not directly applicable. Taylor expansions are necessary near the fixed point. Only after this is done one can re-write the system into new coordinates, on the Central Manifold. Therefore, theorems related to this technique demand differential equations exhibiting, at least, differentiability of order two at the bifurcation point; see Guckenheimer & Holmes, (1990). Papoulias, (1995) presents a strategy to treat this kind of non-smoothness in the study of submarine dynamic stability, by the utilization of the concept of the generalized gradient (Clarke, 1983). In this strategy, the gradient of the non-smooth function is approximated at the non-smooth point by a map equal to the convex closure of the limiting gradients near the discontinuity. The focus of the present work is to apply such a strategy to the fishtailing instability phenomenon, summarizing some results obtained by Bueno (2003).

2. Problem Formulation

2.1. The fish-tailing instabilities of a moored offshore vessel

The exploration of petroleum basins in deep water has gained more and more importance in the last twenty years. The development of floating production systems based on the use of moored vessels (Very Large Crude Carriers - VLCCs), known as Floating Production, Storage and Offloading (FPSO) systems, have been a matter of great importance for the oil industry. One type of mooring system for FPSOs is the Single Point Mooring (SPM), where the vessel is moored by just one point to a monobuoy, through a hawser; see figure 1.

Figure 1: A FPSO at Marlim 2 Basin (Petrobras/Brazil). Single Point Mooring at stern.

The dynamic instability of SPMs (single point mooring) is an important problem in Naval and Ocean Engineering. As well known, a vessel moored by a hawser under uniform current may develop self-sustained oscillations in the horizontal plane, with large amplitude periods. This type of dynamic instability is known, in Ocean Engineering, as fishtailing instability. See figure 2. Even chaotic motions could arise under special conditions; Jiang, Sharma (1993). Several authors studied the problem of dynamic instability of SPM systems. Following the work of Simos we mention Faltinsen (1979), Wichers (1987) and Jiang and Sharma (1993).

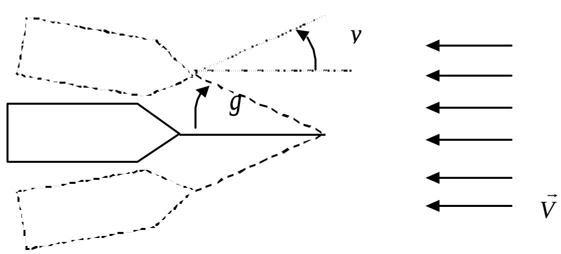

Figure 2: Schematic view of a fishtailing instability.

Simos (2001) conducted experiments to study the ability of prediction of the *Extended Heuristic Model* (EHM) (to be presented in section 2.2 of this article) to reproduce the dynamics of a SPM tanker moored by a hawser. The model tanker was under the action of a uniform maritime current. Besides, the experiments were conducted using a bar to emulate a hawser with infinity rigidity. In the following we are always considering a FPSO system SPM-moored with a rigid hawser.

2.2 The Mathematical Model

Hydrodynamic forces due to the action of current on the vessel are calculated through the *Extended Heuristic Model* (EHM) as proposed in Simos, (2001); see also Simos et al (2001). The *quasi-explicit* nature of this model is one of its main advantages. This model is summarized below.

The static forces on the hull are written in a standard form:

$$X_s(a,V) = \frac{1}{2} r\, TLC_{1s}(a)\left|V\right|^2$$

$$Y_s(a,V) = \frac{1}{2} r\, TLC_{2s}(a)\left|V\right|^2 \qquad (1)$$

$$N_s(a,V) = \frac{1}{2} r\, TL^2 C_{6s}(a)\left|V\right|^2$$

where, $\vec{V}$ is the relative velocity of the center of mass of the vessel with respect to the water and the hydrodynamic coefficients are given by:

$$C_{1s}(a) = \left[\frac{0.09375}{(\log(\mathrm{Re})-2)^2}\frac{S}{TL}\right]\cos(a) + \frac{1}{8}\frac{pT}{L}(\cos(3a) - \cos(a))$$

$$C_{2s}(a) = \left[C_Y - \frac{pT}{2L}\right] sin(a)\left|sin(a)\right| + \frac{pT}{2L} sin^3(a) + \frac{pT}{L}\left[1 + 0.4\frac{C_B B}{T}\right] sin(a)\left|\cos(a)\right| \qquad (2),$$

$$C_{6s}(a) = \frac{-lg}{L}\left[C_Y\right] sin(a)\left|sin(a)\right| - \frac{pT}{L} sin(a)\cos(a)$$

$$-\left[\frac{1+\left|\cos(a)\right|}{2}\right]^2 \frac{pT}{L}\left[\frac{1}{2} - 2.4\frac{T}{L}\right] sin(a)\left|\cos(a)\right|$$

being a the angle of attack and Re de Reynolds Number, calculated with respect to the length of the vessel.

The hydrodynamic forces, which are determined by the EHM, depend on the geometrical parameters of the ship. The geometry of the vessel is given by:

L: length of the ship on water line;

B: breadth;

T: draft;

S: wet surface area;

C_B : block coefficient

In the dynamic problem, the instantaneous value of a should be written in terms of the yaw angle y and the velocity components in the form:

$$a = p + a\tan\left(\frac{v}{u}\right) \qquad (3)$$

where

$$V^2 = \left[u^2 + v^2\right]^{1/2} \qquad (4),$$

being *u* and *v* the relative velocity components in the direction of *x* and *y*, respectively.

Dynamic Forces

Following Simos (2001) the current force components and related yaw moment, with respect to the center of gravity of the vessel, are given by:

$$F_{X,R}(u,v,r)=\frac{1}{2}rLTC_{1S}(a)U^2+\Delta F_{X,R}(u,v,r)$$
$$F_{Y,R}(u,v,r)=\frac{1}{2}rLTC_{2S}(a)U^2+\Delta F_{Y,R}(u,v,r) \quad (5)$$
$$N_{Z,R}(u,v,r)=\frac{1}{2}rL^2TC_{6S}(a)U^2+\Delta N_{Z,R}(u,v,r)$$

with the expressions for the dynamic corrections, $DF_{X,R}(u,v,r), DF_{Y,R}(u,v,r), DN_{Z,R}(u,v,r)$, given by:

$$DF_{X,R}(u,v,r)=-\frac{1}{4}rpT^2Lvr-\frac{1}{16}rpT^2L^2\cos(a_a)r^2$$
$$DF_{Y,R}(u,v,r)=-\frac{1}{4}rpT^2L(1-4.4B/L+0.160\,B/T)ur+M_{11}ur$$
$$-\frac{1}{2}rTL\left[I_0(v,r)v^2-C_y v|v|\right]-rTL^2I_1(v,r)-\frac{1}{2}rTL^3I_2(v,r)r^2$$
$$DN_{Z,R}(u,v,r)=-\frac{1}{8}rpT^2L^2\left(1+0.16\frac{B}{T}-2.2\frac{B}{L}\right)u|r+M_{26}ur \quad (6)$$
$$-\frac{1}{2}rTL^2\left[I_1(v,r)v^2+l_pC_y v|v|\right]-rTL^3I_2(v,r)vr-\frac{1}{2}rTL^4I_3(v,r)r^2$$
$$-\frac{1}{2}rTL^4\left[\frac{C_y}{16}r|r|-I_3(0,r)r^2\right]$$

with

$$a_a=a\tan\left(\frac{v}{u}+\frac{rL}{u}\right),\quad I_j(v,r)=\frac{1}{L^{j+1}}\int_{-L/2}^{L/2}Cd(x)(v+rx)x^j\,dx \quad (7)$$

where $C_d(x)$ is the drag coefficient for each transversal section and x is its longitudinal coordinate and $r=\dot{y}$.

3. Bifurcation Analysis and the Technique of Central Manifold

From the mathematical point of view, fishtailing instabilities correspond to the well-known Hopf Bifurcation. In the Hopf Bifurcation a pair of complex conjugate eigenvalues, of the associated eigenvalue problem, crosses transversally the imaginary axis. When this type of Bifurcation appears the system presents an oscillatory dynamics. The post-critical Hopf Bifurcation scenario is, usually, of two types: a super-critical or a sub-critical one. In the case of a super-critical Hopf Bifurcation, stable limit cycles are generated after the system looses its stability around a fixed point. The amplitude of the limit cycles grows in a continuous manner as the control parameter that regulates the phenomenon increases its distance from the critical point. In the case of a sub-critical Hopf Bifurcation, unstable limit cycles are generated before the system looses its stability. The identification of these two qualitatively different types of bifurcation is important in Engineering. However, in general, this is not an easy computational task, demanding Taylor Series expansions up to second or even third order.

One should remember that Hopf Bifurcation is of local nature, in opposition to Global Bifurcations. In the study of local bifurcations an important conceptual framework is that of the Central Manifold. References in this subject are Guckenheimer and Holmes (1990), Wiggins (1990), or Carr (1981).

We consider the following dimensionless variables

$$\bar{t}=\frac{Vt}{L},\quad \left(\bar{u};\bar{v};\bar{r}\right)=\left(\frac{u}{V};\frac{v}{V};\frac{rL}{V}\right),\quad \bar{V}^2=\bar{u}^2+\bar{v}^2 \quad (8)$$

$$A_{11}=\frac{M+M_{11}}{M},\quad A_{22}=\frac{M+M_{22}}{M},\quad A_{26}=\frac{Mx_{CG}+M_{26}}{ML},\quad A_{66}=\frac{I_Z+M_{66}}{ML^2} \quad (9)$$

where M_{ij} is the added mass tensor in the horizontal plane and

$$\left(\bar{F}_X\right)=\frac{L}{MV^2}(F_X),\quad \left(\bar{F}_Y\right)=\frac{L}{MV^2}(F_Y),\quad \left(\bar{N}_Z\right)=\frac{L}{MV^2}(N_Z/L) \quad (10)$$

are the forces and moment imposed by the rigid hawser.

Then the system of dynamic equations may be written as follows:

$$
\begin{aligned}
&A_{11}\dot{\bar{u}} - A_{22}\,\bar{v}\bar{r} - A_{26}\,\dot{\bar{r}} = \bar{F}_{X,R}(\bar{u},\bar{v},\bar{r}) + \bar{F}_X(\bar{u},\bar{v},\bar{r})\\
&A_{22}\dot{\bar{v}} + A_{26}\,\dot{\bar{r}} + A_{11}\,\bar{u}\bar{r} = \bar{F}_{Y,R}(\bar{u},\bar{v},\bar{r}) + \bar{F}_Y(\bar{u},\bar{v},\bar{r})\\
&A_{66}\dot{\bar{r}} + A_{26}\,\dot{\bar{v}} + A_{26}\,\bar{u}\bar{r} = \bar{F}_{Z,R}(\bar{u},\bar{v},\bar{r}) + \bar{F}_Z(\bar{u},\bar{v},\bar{r})
\end{aligned}
\qquad (11)
$$

The hawser rigidity implies the following kinematics restrictions:

$$
\begin{aligned}
&\bar{u}\left(\bar{t}\right) = \cos\left(y\left(\bar{t}\right)\right) + \frac{l}{L}\frac{dg}{d\bar{t}}sen\left(g\left(\bar{t}\right) + y\left(\bar{t}\right)\right)\\
&\bar{v}\left(\bar{t}\right) = -sen\left(y\left(\bar{t}\right)\right) + \frac{l}{L}\frac{dg}{d\bar{t}}\cos\left(g\left(\bar{t}\right) + y\left(\bar{t}\right)\right) - a\frac{dy}{d\bar{t}}\\
&\bar{r}\left(\bar{t}\right) = \frac{dy}{d\bar{t}}
\end{aligned}
\qquad (12)
$$

For simplicity we consider all variables to be dimensionless, from now on.

So, using (11) and (12) it is possible to re-write equations (11) in the compact form:

$$\dot{z} = Az + g(z) \qquad (13)$$

where $z = \begin{bmatrix} g \\ y \\ g_1 \\ y_1 \end{bmatrix}$, (14)

is the state vector, being $g_1 = \frac{dg}{dt}$, $y_1 = \frac{dy}{dt}$

where A is the Jacobian matrix related to eq. (8), determined after using (9) and (10) in (8). To apply the Central Manifold Theorems we need the Taylor Series expansion of (13), which requires the Taylor expansion of the cross-flow integrals given in (7) and expressions involving terms of form v|v| e r|r|. For this, we use a strategy found in Papoulias (1995), where the concept of generalized gradient due to Clarke (1983) is applied. This concept consists in the approximation, at a discontinuity point, of the gradient of a non-smooth function by a map equal to the convex closure of the limiting gradient near the discontinuity. In our case we have that the sign function can be approximated by,

$$sign(v_0) = \lim_{b\to 0}\tanh\left(\frac{v_0}{b}\right) \qquad (15)$$

with b , a small regularization parameter.

Whence, from (10), the function $f(v) = v|v|$ can be approximated for $v_0 = 0$ by

$$f(v) = v|v| \approx \frac{1}{b}v^3 \qquad (16)$$

Applying these previous results to the expression of type v|v|, in (13), and using Taylor expansions of second and third order, (13) is re-written as:

$$\dot{x} = Ax + g_2(x) + g_3(x) \qquad (17)$$

Using a matrix M of eigenvectors of A at the critical point x_c , we have the linear change of coordinates:

$$x = My, \quad y = M^{-1}x \qquad (18)$$

which gives, for (17):

$$\dot{y} = M^{-1}AMy + M^{-1}g_2(My) + M^{-1}g_3(My) \qquad (19)$$

The matrix $M^{-1}AM$ at the Hopf bifurcation point is of the form:

$$M^{-1}AM = \begin{bmatrix} p_1 + p'_1 e & 0 & 0 & 0 \\ 0 & p_2 + p'_2 e & 0 & 0 \\ 0 & 0 & a'e & -(w_0 + w'e) \\ 0 & 0 & (w_0 + w'e) & a'e \end{bmatrix} \quad (20)$$

where $e = x - x_c$ is the criticality distance, w_0 is the imaginary part of the critical pair of eigenvalues, p_1, p_2 are the real negative eigenvalues, p'_1, p'_2 are the derivatives of p_1, p_2 with respect to e, w' is the derivative of the imaginary part with respect to e and a' is the derivative of the real part with respect to e.

From the Central Manifold Theory we know that the following relations is valid in a vicinity of the bifurcation point:

$$\begin{aligned} y_1 &= a_1 y_3^2 + a_2 y_3 y_4 + a_3 y_4^2 \\ y_2 &= b_1 y_3^2 + b_2 y_3 y_4 + b_3 y_4^2 \end{aligned} \quad (21)$$

Where the coefficients $a_1, a_2, a_3, b_1, b_2, b_3$ in the quadratic center manifold expansion (19) are to be determinate in the following manner. We derive (21) with respect to y_3, y_4, use that $\dot{y}_3 = -w_0 y_4, \dot{y}_4 = w_0 y_3$ and substitute the above relations (21) in

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + T^{-1} g_2(Tz) \quad (22)$$

Then we obtain an algebraic system in ($a_1, a_2, a_3, b_1, b_2, b_3$). Solving this system we obtain $a_1 = 0,\ a_2 = 0,\ a_3 = 0,\ b_1 = 0, b_2 = 0, b_3 = 0$.

Whence, the coordinates (y_1, y_2), in the Central Manifold, are written as third order polynomials:

$$\begin{aligned} y_1 &= a_{30} y_3^3 + a_{21} y_3^2 y_4 + a_{12} y_3 y_4^2 + a_{03} y_4^3 \\ y_2 &= b_{30} y_3^3 + b_{21} y_3^2 y_4 + b_{12} y_3 y_4^2 + b_{03} y_4^3 \end{aligned} \quad (23)$$

Where the coefficients $a_{30}, a_{21}, a_{12}, a_{03}, b_{30}, b_{21}, b_{12}, b_{03}$ are to be determined. However, remembering that the expressions for the Taylor Series, in the case FPSO-SPM-rigid hawser, presents 3rd order terms in y_1, y_2, y_3, y_4 (they are combinations of the original coordinates of the system), we eliminate the terms in y_1, y_2 by changing coordinates

$$x = My \Leftrightarrow \begin{bmatrix} g \\ y \\ g_1 \\ y_1 \end{bmatrix} = M^{-1} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}.$$

So, we can write the reduced two-dimension system as:

$$\begin{aligned} \dot{y}_3 &= a'e\ \ y_3 - (w'e + w_0) y_4 + q_{11} y_3^3 + q_{12} y_3^2 y_4 + q_{13} y_3 y_4^2 + q_{14} y_4^3 \\ \dot{y}_4 &= (w_0 + w'e) y_3 + a'e\ \ y_3 + q_{21} y_3^3 + q_{22} y_3^2 y_4 + q_{23} y_3 y_4^2 + q_{24} y_4^3 \end{aligned} \quad (24)$$

Where the coefficients ($q_{11}, q_{12}, q_{13}, q_{14}, q_{21}, q_{22}, q_{23}, q_{24}$) comes from collecting and reducing the terms of equal order in y_3, y_4. To obtain a', w' we use a classical perturbation method. Writing $l/L = (l/L)_{critical} + e$ and $x = a'e\ \ 3 + I(w'e + w_0)$, where I represents the imaginary unity, we substitute these expressions in the characteristic polynomial, separate the real and imaginary parts and finally solve the resulting equations for a', w'.

At this point we start the application of the Integral Averaging Technique. With the introduction of the polar coordinates, (R, q) such that,

$$y_3 = R\cos(q),\ \ y_4 = R\sin(q) \quad (25)$$

Eq. (24) turns to be

$$\dot{R} = a'e\ \ R + P(q)R^3 + Q(q)R^2 \quad (26)$$

Integrating over one cycle (in q) and observing that $\int_0^{2p} Q(q)dq = 0$, the above equation is written:

$$\dot{R} = a'e \;\; R + KR^3 \tag{27}$$

Resulting, finally, in the equation:

$$\dot{R} = a'e \;\; R + KR^3 \tag{28}$$

where

$$K = \frac{1}{8}\left(3q_{11} + q_{13} + q_{22} + 3q_{24}\right) \tag{29}$$

Equation (28) presents only two possible solutions, $R = 0$ (the trivial equilibrium) or

$$R_0 = \sqrt{-\frac{a'}{K}e} \tag{30}$$

So, depending on the signals of a' and K, equation (28) will present limit cycles for either, $e > 0$ or $e < 0$. Firstly, consider the case $a' > 0$ and $K > 0$. Then (see the signal of the eigenvalue of (28) for $e < 0$) the stable equilibrium point coexists with unstable limit cycles for $e < 0$. For $e > 0$ the trivial equilibrium point is unstable. Repeating this analysis for the other cases we deduce the following post-critical Hopf Bifurcation scenario, depending on the signals of a' and K :

1. $a' > 0$ and $K > 0$: the origin is an unstable fixed point for $e > 0$. The origin is an asymptotically stable fixed point with a coexisting unstable limit cycle for $e < 0$;
2. $a' > 0$ and $K < 0$: the origin is an asymptotically stable fixed point for $e < 0$ and unstable fixed point for $e > 0$. An asymptotically stable limit cycle exists for $e > 0$;
3. $a' < 0$ and $K > 0$: there is an unstable limit cycle for $e > 0$ coexisting with an asymptotically stable fixed point. There is an unstable fixed point for $e < 0$;
4. $a' < 0$ and $K < 0$: for $e > 0$ the origin is an asymptotically stable fixed point; for $e < 0$ it is an unstable fixed point. There is an asymptotically stable limit cycle for $e < 0$.

When $K < 0$ (cases 2 and 4) we call the Hopf Bifurcation as Supercritical Hopf Bifurcation (stable limit-cycle). Otherwise, when $K > 0$ (cases 1 and 3) there is a Subcritical Hopf Bifurcation (unstable limit-cycle). Also, the amplitude of the limit cycles depends on the magnitude of K in the form shown in (30).

4. Results

In this section we present some results obtained with the application of CMT to the problem of fishtailing instabilities of a moored vessel. The case study is the VLCC Vidal de Negreiros. Values of a' are given in Tab. (1) below:

Table 1: values of a' for the VLCC Vidal de Negreiros. Draft: 100%.

l/L	0,1	0,2	0,3	0,4	0,5	0,6	0,8	1,0	1,2	1,3	1,4	1,5
a'	1,21	1,32	0,26	0,15	0,09	0,04	0,03	0,01	0,005	0,002	0,0004	-0,001

Where l/L is the bifurcation parameter and indicates the relation between the hawser and vessel lengths.

For this particular vessel, the values of Kb are given by the graph:

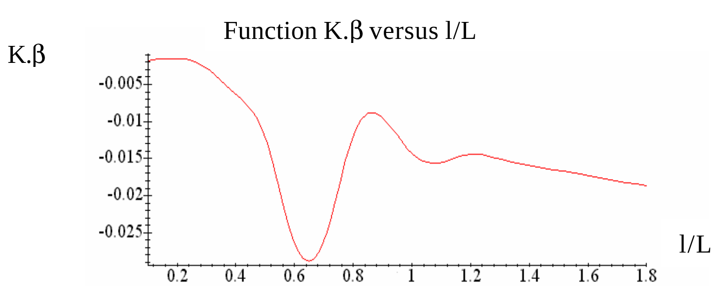

Figure 3: Values of $K.b$ as function of l/L for the VLCC Vidal de Negreiros.

Using the values of a' and K, given above in equation (27), we obtain the following three graphics, showing the amplitude of limit cycles (ψ, γ), as function of l/L. These analytical results, of a local nature, are confronted with the time domain simulation results by Simos (2001), with a rather good agreement.

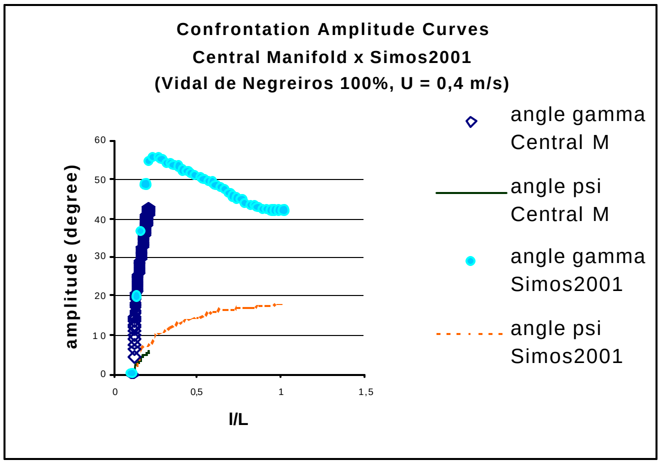

Figure 4a: Confrontation of Amplitude Curves, Central Manifold vs. Simos (2001), VLCC Vidal de Negreiros. Draft 100% and current U = 0,4 m/s.

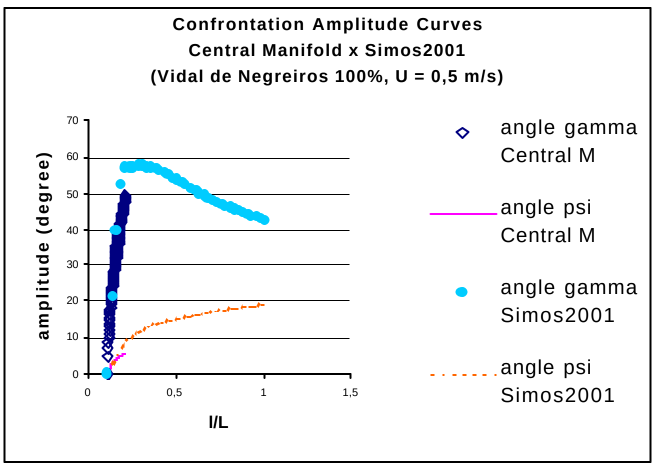

Figure 4b: Confrontation of Amplitude Curves, Central Manifold vs. Simos (2001), VLCC Vidal de Negreiros. Draft 100% and current U = 0,5 m/s.

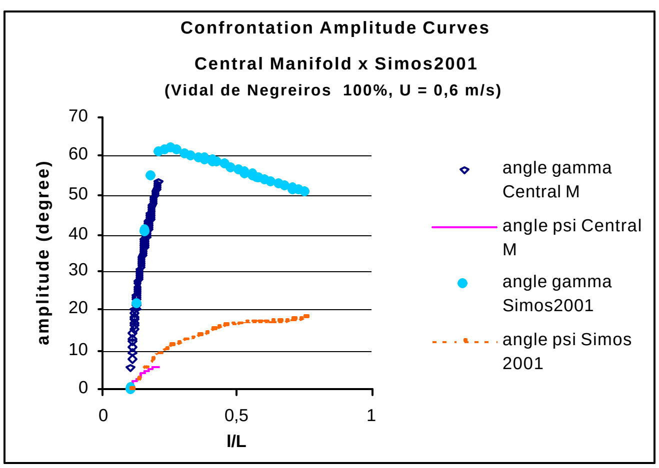

Figure 4c: Confrontation of Amplitude Curves, Central Manifold vs. Simos (2001), VLCC Vidal de Negreiros. Draft 100% and current U = 0,6 m/s.

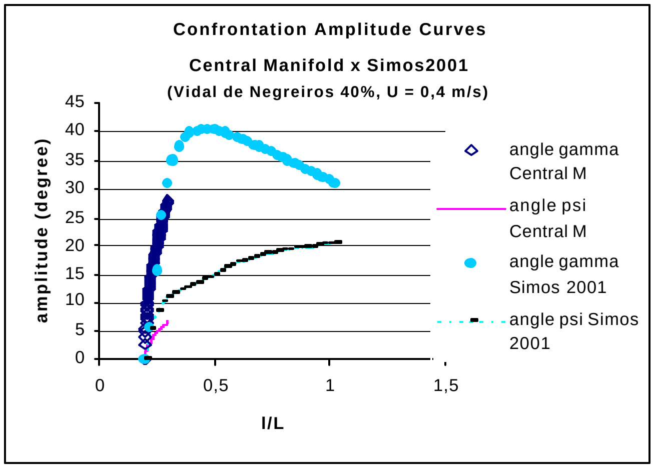

Figure 5a: Confrontation of Amplitude Curves, Central Manifold vs. Simos (2001), VLCC Vidal de Negreiros. Draft 40% and current U = 0,4 m/s.

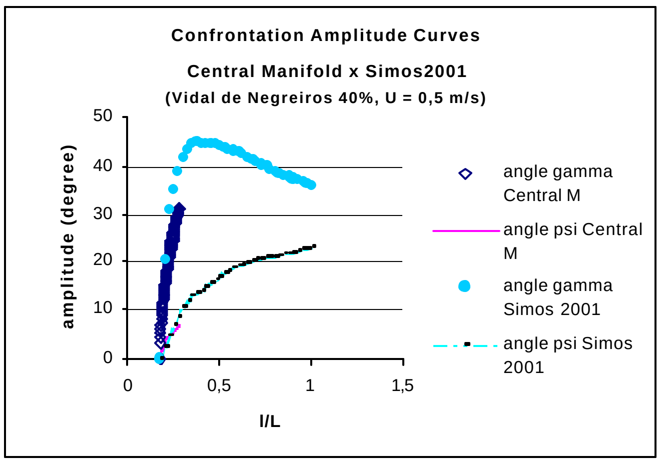

Figure 5b: Confrontation of Amplitude Curves, Central Manifold vs. Simos (2001), VLCC Vidal de Negreiros. Draft 40% and current U = 0,5 m/s.

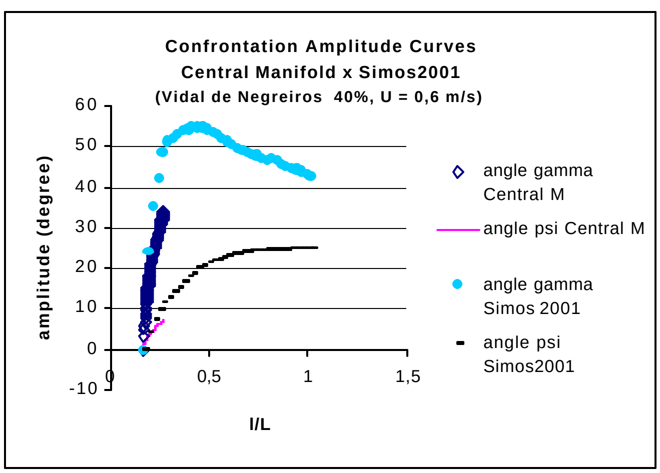

Figure 5c: Confrontation of Amplitude Curves, Central Manifold vs. Simos (2001), VLCC Vidal de Negreiros. Draft 40% and current U = 0,6 m/s.

An important result, in the cases treated above, is the qualification (in analytical and *a priori* form) of the post-critical scenario of Hopf Bifurcation as being of the supercritical type. Note, from Figure 3 and table 1, that all figures 4 and 5 pertain to case number 2, when $a' > 0$ and $K < 0$. This is very important from the Engineering point of view as, in the case of occurrence of a sub-critical Hopf Bifurcation, an unstable limit cycle would appear, leading, in some situations, to very large amplitude of oscillations (eventually catastrophic ones).

5. Conclusions

In this work the Techniques of Central Manifold and Integral Averaging were applied to a problem originated in Naval and Ocean Engineering. This sort of problem, concerning the dynamics of a FPSO with SPM-hawser mooring, usually exhibits self-sustained oscillations related to terms of type v|v| which appear in the governing differential equations. The analytical results so obtained, including the amplitude of limit cycles in the post-critical scenario, were compared to results obtained through numerical simulations. Those numerical simulations had been previously confronted by Simos (2001), within a very good degree of accuracy, with experimental results, obtained through reduced scale models, in a towing tank. With the technique of Central Manifold Theory it is possible to conduct, *a priori*, a search of bifurcation points in an easy way. Also this technique enabled to qualify the emerging Hopf Bifurcations as supercritical ones.

In addition, with this Technique we extended (in an analytical form) the results from Bernitsas et al (1999) including nonlinear terms in the analysis. The eigenvalue technique used by those authors is not able to qualify, strictly speaking, the Hopf Bifurcation (if Super or Subcritical) or to quantify the amplitude of the associated limit cycles.

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