

COMPARATIVE STUDY OF COMPONENT MODE SYNTHESIS METHODS APPLIED TO STRUCTURE DYNAMICS

Felipe José Vinaud
Universidade de Brasília
Departamento de Engenharia Mecânica
70910-900 Brasília, DF, Brasil.
E-mail: fvinaud@unb.com

Alberto C.G.C. Diniz
Universidade de Brasília
Departamento de Engenharia Mecânica
70910-900 Brasília, DF, Brasil
E-mail: adiniz@unb.br

Abstract. *The modal synthesis methods are techniques used in the dynamic analysis of large structures comprising of substructures or components. These techniques are known to reduce model size, time and cost of required calculations without any loss of quality in the results.*

There are several modal synthesis methods, being each characterized through the way the modal subspace is constructed and boundary conditions are selected for the component modal analysis. These methods have been divided into three methodology fields: free interface methods, fixed interface methods and hybrid methods, which include branch mode methods. Regarding the structure complexity (geometry as well as the presence of damping, non-linearities, random features) the most suitable method must be applied considering numerical efficiency and the most faithfull representation of real interface behavior.

Applying a general formulation to the component mode synthesis, this work studies some existing component mode methods with different bonding conditions between the interfaces and compares each of them in the study of flat plates. Different comparison methods are employed with the plates. In this work the Craig & Bampton, MacNeal and Rubin methods are studied.

Keywords*. component mode synthesis, dynamic analysis, free interface, fixed interface, hybrid methods.*

1. Introduction

The component mode synthesis methods are variant forms of the Ritz method using the technique of transforming the substructure from the physical space to a modal subspace comprising of the first mode shapes of the structure and other complementary modes. This transformation results in the reduction of the dimension of the studied problem and also offers the advantage of reducing calculation costs and memory space required by problems of great size.

The structure is divided in components or substructures, which are analyzed separately for frequency and mode shape calculation (eigenvalues and eigenvectors). In the next step a reduced model of each substructure is achieved through numerical or experimental modal analysis techniques. Finally, the modal synthesis of the substructure is processed coupling the reduced modal equations of each substructure and calculating the frequencies and mode shapes of the global structure using the global reduced system of equations.

Another advantage of the component mode synthesis methods is the possibility of avoiding repeating calculations in the case of structures possessing many identical modules. Even with the performance of modern computers capable of solving large problems substructuring is widely used because it allows the separation and treatment of the different parts of the structure employing different tools that are more adequate to the properties of each partition. For instance, component mode synthesis allows the assessment or optimization of the structures with non-linear or non-deterministic parameters in a more economic basis than the finite element analysis (Diniz, 1999).

The classical component mode methods are grouped in free interface methods (Goldman, 1969; Hou, 1969 and Rubin, 1975) and fixed interface methods (Hurty, 1965; Craig & Bampton, 1968). A third method employs hybrid-coupling conditions of substructures (Gladwell, 1964; MacNeal, 1971, Hale & Meirovitch, 1982 and Diniz, 2001).

The fixed interface methods use vibration modes achieved clamping the substructure on its boundary with the neighboring substructures. The free interface methods use free vibration modes not clamped on its boundaries. Hybrid methods use a combination of free, fixed and other special vibration modes such as branch modes, loaded modes or residual flexibility. By choosing different boundary conditions in the interface between substructures, a different component mode synthesis method is characterized. Regarding the structure complexity (geometry as well as the presence of damping, non-linearities, random features) the most suitable method must be applied considering numerical efficiency and the most faithful representation of real interface behavior.

Applying a different formulation to the component mode synthesis, this paper shows the studies conducted with the Craig & Bampton, MacNeal and Rubin Methods. These classical methods are compared through the results achieved in the dynamic study of plates.

2. Modal synthesis equations

Every component mode method can be developed in three stages: substructuring, modal analysis of substructures and modal synthesis employing a reduced system of equations. In this section the general equations of each stage are presented.

The equations of motion of a finite element discrete structure can be written in matrix form:

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{f(t)\} \tag{1}$$

Where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $\{u(t)\}$ are $\{f(t)\}$ the dynamic displacement and loading vectors respectively.

2.1. Substructuring

Applying the substructuring technique, the structural matrices may be obtained from the assembly of the substructure matrices.

$$[K]=\sum_{s=1}^{m}[\alpha_s]^T[K_s][\alpha_s] \tag{2}$$

$$[M]=\sum_{s=1}^{m}[\alpha_s]^T[M_s][\alpha_s] \tag{3}$$

$$[C]=\sum_{s=1}^{m}[\alpha_s]^T[C_s][\alpha_s] \tag{4}$$

$$\{f(t)\}=\sum_{s=1}^{m}[\alpha_s]^T\{f(t)\} \tag{5}$$

Where the index "s" indicates structural matrices of the s-th substructure. $[\alpha_s]$ is the Boolean transformation matrix of size $(N_s\ x\ N)$ associated to the s-th substructure allowing the substructures assembly. N_s is the number of degrees of freedom of the s-th substructure and N is the total number of degrees of freedom of the structure. All elements in matrix $[\alpha_s]$ are zero except if the i-th local degree of freedom of the s-th substructure is the j-th global degree of freedom of the structure. In this case, the element α_{ij} is unity. "m" is the total number of substructures.

2.2. Modal analysis of substructures

Applying the Rayleigh-Ritz method only a few low frequency mode shapes are utilized as a base for the generalized modal space. The mode shapes of the substructure are defined as the solution of the eigenproblem of the undamped substructure. Thus, for the s-th substructure the following modal equations are achieved:

$$([K]-\lambda_i[M])\{\phi_i\}=\{0\} \qquad i=1,2,\ldots,N_s \tag{6}$$

The displacement of the s-th substructure is expressed in terms of the modal matrix $[\phi]$ formed by the first lower mode shapes:

$$\{u_s(t)\}=[\phi_s]\{\eta_s\} \qquad \text{with} \qquad [T_s]=[\phi_s] \tag{7}$$

Each matrix $[T_s]$ is constructed with few modes and its size is given by $dim[T_s] = N_s\ x\ R_s$, $R_s < N_s$. Where R_s is the number of retained modes of the s-th substructure and $\{\eta_s\}$ is the generalized coordinates vector of the system s-th substructure.

Thus, the displacement vector of the assembled structure is given by:

$$\{u\}=[T]\{\eta\} \qquad \text{with} \qquad [T]=\sum_{s=1}^{m}[\alpha_s^*]^T[\phi_s][\alpha_s^*] \tag{8}$$

where $\{\eta\}$ is the generalized coordinated vector of the global structure ($dim\{\eta\} = R = \Sigma R_s$). The matrices $[\alpha_s^*]$ have analogous form and function as matrix $[\alpha]$ to represent the relationship between the global generalized coordinates and the local generalized coordinates of the s-th substructure. $[T]$ ($dim[T] = R\ x\ N,\ R < N$) is the transformation matrix of the structure from physical coordinates $\{u(t)\}$ to generalized coordinates $\{\eta\}$.

2.3. Reduced system of equations

Plugging Eq. (8) into Eq. (1) and premultiplying by $[T^T]$ yields the governing dynamic equations for the whole structure:

$$\left[M^*\right]\{\ddot{\eta}(t)\}+\left[C^*\right]\{\dot{\eta}(t)\}+\left[K^*\right]\{\eta(t)\}=\left\{f^*(t)\right\} \tag{9}$$

where the reduced global matrices of the complete structure are achieved through:

$$\left[K^*\right]=\sum\left[T^T\right][\alpha_s]^T[K_s][\alpha_s][T] \tag{10}$$

$$\left[M^*\right]=\sum\left[T^T\right][\alpha_s]^T[M_s][\alpha_s][T] \tag{11}$$

$$\left[C^*\right]=\sum\left[T^T\right][\alpha_s]^T[C_s][\alpha_s][T] \tag{12}$$

$$\left\{f^*(t)\right\}=\sum[\alpha_s]^T\{f(t)\} \tag{13}$$

The transformation matrix $[T]$ yields to the reduction of the number of equations in the problem. Reduced matrices $[M^*]$, $[C^*]$ and $[K^*]$ have dimensions $(R\ x\ R)$.

The transformation matrix $[T]$, in the component mode synthesis methods, contains a selection of few lower frequency mode shapes and other supplementary modes. The several modal synthesis methods differ in the form the transformation matrix $[T]$ can be defined. The Craig & Bampton method uses a transformation matrix constructed with fixed-interface modes and the constraint modes (Craig & Bampton, 1968). The MacNeal method uses the free-interface modes and the residual flexibility modes (MacNeal, 1971). Rubin’s method uses free interface and residual flexibility modes as well as residual mass modes (Rubin, 1975).

3. Modal synthesis methods

3.1. Fixed-interface methods: the Craig & Bampton´s method

From the enhancement of Hurty´s method, Craig & Bampton proposed a simple and practical method that does not need a special treatment of rigid body modes, which are represented as a linear combination jof static deformations. This method has good precision and is widely used when substructure data is attained through finite element models.

The dynamic behavior of the structure in Craig & Bampton´s method is discribed by means of its vibration modes, achieved by clamping the substructure ti its neighboring substructures, and its static modes condensed on the interfaces. The static modes are achieved apllying unitary displacements to the interface degrees of freedom being all other degrees of freedom clamped.

In the absence of inertia forces, one can define for each substructure:

$$\begin{Bmatrix} u_F \\ u_I \end{Bmatrix}=\begin{bmatrix} I_{FF} & 0_{FR} \\ \Phi_{IF} & \Psi_{IR} \end{bmatrix}\begin{Bmatrix} u_F \\ \eta_R \end{Bmatrix} \tag{14}$$

Where:

$[\Phi_{IF}]$ is the interface static mode matrix defined as $[\Phi_{IF}]=-[K_{II}]^{-1}[K_{IF}]$;

$[\Psi_{IR}]$ is the matrix of the “R” first modes with fixed interface, solution of $\left([K_{II}]-\lambda_{I_i}[M_{II}]\right)\{\psi_{II_i}\}=\{0\}$;

$\{\eta_R\}$is the modal participation vector reduced to the “R” first modes.

3.2. Free-interface methods: MacNeal´s method

The method proprosed by MacNeal in 1971 introduced the residual flexibility correction, which conducts to a satisfatory precision and usage simplicity. This method allows the usage of modes with hybrid conditions.

In general, free interface methods bring a transformation from physical coordinates “$\{u\}$” to modal coordinates “$\{\eta\}$” using matrix “$[\Theta_{\tilde{N}\tilde{N}}]$” of the substructure with free interfaces. The high frequency truncated modes contribution is approximated through the residual flexibility matrix of truncated modes “$[GT_{\tilde{N}\tilde{N}}]$” (MacNeal, 1971). For a discrete substructure with “$\tilde{N}$” degrees of freedom the displacement equation is expressed as:

$$\{u_{\tilde{N}}\}=[\Theta_{\tilde{N}\tilde{N}}]\ \{\eta_{\tilde{N}}\}=[\Theta_{\tilde{N}R}]\{\eta_{\tilde{N}}\}+[\Theta_{\tilde{N}T}]\{\eta_T\} \tag{15}$$

$[\Theta_{\tilde{N}\tilde{N}}]$ is the free interface mode matrix, $[\Theta_{\tilde{N}R}]$ is the retained mode matrix and $[\Theta_{\tilde{N}T}]$ is the truncated mode matrix. MacNeal approximates the effect of truncated modes using the flexibility matrix of the structure and the interface forces: $[\Theta_{\tilde{N}T}]\ \{\eta_T\}= [GT_{\tilde{N}\tilde{N}}]\{f_{\tilde{N}}.\}$. Thus, in MacNeal's method Eq. (15) becomes:

$$\{u_{\tilde{N}}\}=[\Theta_{\tilde{N}R}]\ \{\eta_{\tilde{N}}\}+[G_{T_{\tilde{N}\tilde{N}}}]\{f_{\tilde{N}}\} \tag{16}$$

where:

$$[G_{T_{\tilde{N}\tilde{N}}}]\ =[G_{\tilde{N}\tilde{N}}]-[G_{RET}] \tag{17}$$

$$[G_{RET}]=[\Theta_{\tilde{N}R}][K_{\tilde{N}R}]^{-1}[\Theta_{\tilde{N}R}]^T \tag{18}$$

Where $[G_{T\tilde{N}\tilde{N}}]$ is the truncated flexibility matrix and $[G_{RET}]$ is the retained flexibility matrix. $[\Theta_{\tilde{N}R}]$ and $[K_{\tilde{N}R}]$ are the mode shape and stiffness generalized matrices for the retained modes respectively. The truncated and retained flexibility matrices of the substructure are calculated using the classical definition of flexibility matrix (Craig Jr., 1981).

Partitioning the substructure degrees of freedom in interface (index F) and internal (index I) degrees of freedom, Eq. (16) becomes:

$$\begin{Bmatrix} u_F \\ u_I \end{Bmatrix}=\begin{bmatrix} \Theta_{FR} \\ \Theta_{IR} \end{bmatrix}\{\eta_R\}+\begin{bmatrix} G_{T_{FF}} & G_{T_{FI}} \\ G_{T_{IF}} & G_{T_{II}} \end{bmatrix}\begin{Bmatrix} f_F \\ 0_I \end{Bmatrix} \tag{19}$$

Writing the interface forces "f_F" as function of displacements "u_F" and generalized displacements "η" the physical displacements can be written as function of interface degrees of freedom and free interface modes of each substructure:

$$\begin{Bmatrix} u_F \\ u_I \end{Bmatrix}=\begin{bmatrix} \mathbf{I}_{FF} & \mathbf{0}_{FR} \\ \mathbf{B}_{IF} & \mathbf{C}_{IR} \end{bmatrix}\begin{Bmatrix} u_F \\ \eta_R \end{Bmatrix} \tag{20}$$

Where matrices $\boldsymbol{B_{IF}}$ and $\boldsymbol{C_{IR}}$ represent the residual flexibility correction terms:

$$[\mathbf{B}_{IF}]=[G_{T_{IF}}]\ [G_{T_{FF}}]^{-1} \tag{21}$$

$$[\mathbf{C}_{IR}]=[\Theta_{IR}]-[G_{T_{IF}}]\ [G_{T_{FF}}]^{-1}[\Theta_{FR}] \tag{22}$$

3.3. Free interface methods: Rubin´s method

Rubin has extended MacNeal's method to include inertial and damping (when applicable) effects of the higher frequency modes using a second order MacLaurin series expansion. MacNeal approximates the high frequency truncated modes contribution by means of the residual flexibility. This approximation is performed with Eq. (15) by Rubin who also includes the residual mass and damping second order effects leading to Eq. (23) (cf. eq. (16)):

$$\{u_{\tilde{N}}\}=[\Theta_{\tilde{N}R}]\{\eta_R\}+[G_{T_{\tilde{N}\tilde{N}}}]\{f_{\tilde{N}}\}-[H_{T_{\tilde{N}\tilde{N}}}]\{\ddot{f}_{\tilde{N}}\}-[B_{T_{\tilde{N}\tilde{N}}}]\{\dot{f}_{\tilde{N}}\} \tag{23}$$

where:

$$\begin{aligned} &[H_{T\tilde{N}\tilde{N}}]=[H_{\tilde{N}\tilde{N}}]-[H_{RET}], \quad [H_{RET}]=[G_{RET}]^T[M_{\tilde{N}\tilde{N}}]\ [G_{RET}] \\ &[B_{T\tilde{N}\tilde{N}}]=[B_{\tilde{N}\tilde{N}}]-[B_{RET}], \quad [B_{RET}]=[G_{RET}]^T[C_{\tilde{N}\tilde{N}}]\ [G_{RET}] \\ &[H_{\tilde{N}\tilde{N}}]=[G_{\tilde{N}\tilde{N}}]^T\ [M_{\tilde{N}\tilde{N}}][G_{\tilde{N}\tilde{N}}] \\ &[B_{\tilde{N}\tilde{N}}]=[G_{\tilde{N}\tilde{N}}]^T\ [C_{\tilde{N}\tilde{N}}][G_{\tilde{N}\tilde{N}}] \end{aligned} \tag{24}$$

Inertial and damping correction matrices [$H_{\tilde{N}\tilde{N}}$] and [$B_{\tilde{N}\tilde{N}}$] are only a transformation of mass and stiffness matrices respectively by pre and post-multiplying by [$G_{\tilde{N}\tilde{N}}$]. The corresponding residual matrices [$G_{T\tilde{N}\tilde{N}}$,], [$H_{T\tilde{N}\tilde{N}}$] and [$B_{T\tilde{N}\tilde{N}}$] are obtained from the complete matrices [$G_{\tilde{N}\tilde{N}}$], [$H_{\tilde{N}\tilde{N}}$] and [$B_{\tilde{N}\tilde{N}}$] through the elimination of retained modes.

Thus the correction terms produced by the second order approximation involve only the physical damping and mass matrices pre and post-multiplied by the flexibility matrix [$G_{\tilde{N}\tilde{N}}$].

Making use of the modal orthogonality as matrizes truncadas H e B can also be obtained by:

$$
\begin{aligned}
\left[H_{T\tilde{N}\tilde{N}}\right] &= \left[G_{T\tilde{N}\tilde{N}}\right]\left[M_{\tilde{N}\tilde{N}}\right]\left[G_{T\tilde{N}\tilde{N}}\right], \\
\left[B_{T\tilde{N}\tilde{N}}\right] &= \left[G_{T\tilde{N}\tilde{N}}\right]\left[C_{\tilde{N}\tilde{N}}\right]\left[G_{T\tilde{N}\tilde{N}}\right]
\end{aligned}
\tag{25}
$$

Ignoring damping terms of Eq.(23) leads to:

$$
\{u_{\tilde{N}}\} = \left[\Theta_{\tilde{N}R}\right]\{\eta\} + \left(G_{T\tilde{N}\tilde{N}} - H_{T\tilde{N}\tilde{N}}\Theta_{\tilde{N}\tilde{N}}\right)\{F\}
\tag{26}
$$

Where the residual flexibility correction matrix can be expressed by an [R] matrix:

$$
\left[R_{T\tilde{N}\tilde{N}}\right] = \left[G_{T\tilde{N}\tilde{N}}\right] - \left[H_{T\tilde{N}\tilde{N}}\right]\left[\Theta_{\tilde{N}\tilde{N}}\right]
\tag{27}
$$

As in MacNeal's method the displacement equation can be written in terms of internal and interface degrees of freedom:

$$
\begin{Bmatrix} u_J \\ u_I \end{Bmatrix} = \begin{bmatrix} \Theta_{JR} \\ \Theta_{IR} \end{bmatrix}\{\eta_R\} + \begin{bmatrix} R_{T_{JJ}} & R_{T_{JI}} \\ R_{T_{IJ}} & R_{T_{II}} \end{bmatrix}\begin{Bmatrix} f_J \\ 0_I \end{Bmatrix}
\tag{28}
$$

Expressing interface forces in terms of interface displacements one can write physical displacements as a function of interface degrees of freedom the free interface modes of the substructure with residual flexibility correction and Rubin´s second order inertial correction:

$$
\begin{Bmatrix} u_J \\ u_I \end{Bmatrix} = \begin{bmatrix} I_{JJ} & 0_{JR} \\ B_{IJ} & C_{IR} \end{bmatrix}\begin{Bmatrix} u_J \\ \eta_R \end{Bmatrix}
\tag{29}
$$

Where [B] and [C] are given by:

$$
\left[B_{IJ}\right] = \left[R_{T_{IJ}}\right]\left[R_{T_{JJ}}\right]^{-1}
\tag{30}
$$

$$
\left[C_{IR}\right] = \left[\Theta_{IR}\right] - \left[R_{T_{IF}}\right]\left[R_{T_{FF}}\right]^{-1}\left[\Theta_{FR}\right]
\tag{31}
$$

4. Comparison of component mode methods

Aiming the comparison of different component mode methods of a linear conservative system, the results attained using the methods discussed before are presented in the modeling of a simple structure.

As an example it was used a non-symmetric plate clamped on one of its end, Fig.(1a). The same structure was divided in two substructures as depicted in Fig.(1b).

The reference results were obtained using the Finite Element Analysis for the plate shown in Fig.(1a) modeled as one complete structure.

For the finite element analysis shell elements were used with 3 degrees of freedom by node. The modeled plate has 193 nodes and 11 of them are restrained. There are 8 interface nodes between the 2 substructures. The complete finite element model has a total of 546 degrees of freedom, 24 interface degrees of freedom and 522 internal degrees of freedom.

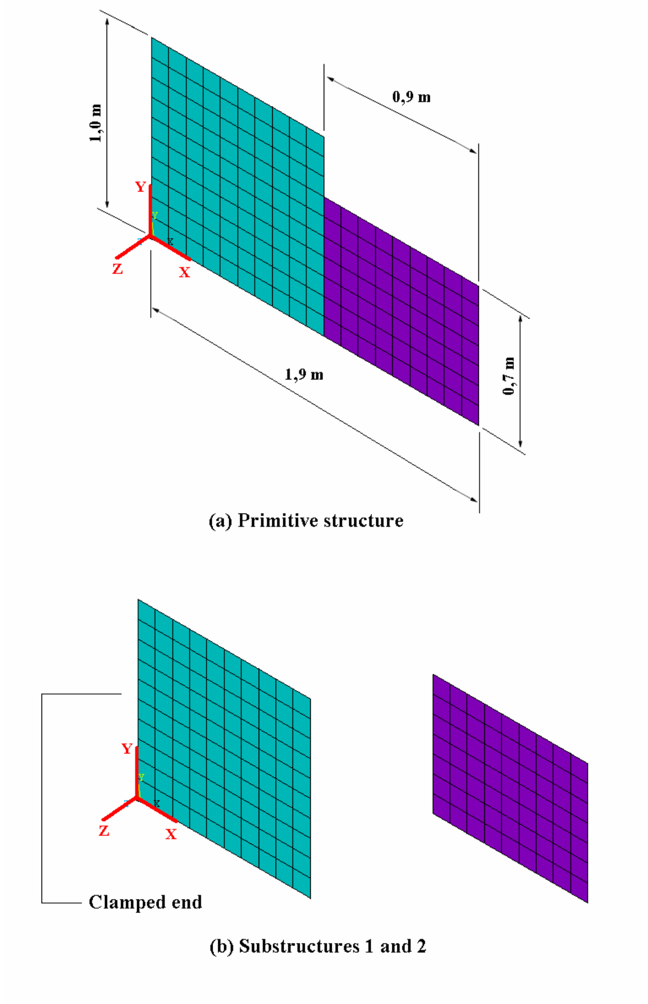

Figure 1. Finite element modeling and division into substructures.

As a next step, the comparison of frequencies and mode shapes was performed. For mode shape comparison the Modal Assurance Criterion (MAC), which is an indicator parameter of correlation between the i-th mode of the first test and the j-th mode of the second test (Friswell, 1985), was employed. In the discussed problem, the comparison was performed between the reference results, attained with the finite element analysis, and the component mode synthesis results.

$$MAC_{i,j} = \frac{\phi_{c_i}^T \phi_{m_j}}{\sqrt{\phi_{c_i}^T \phi_{c_i} \phi_{m_j}^T \phi_{m_j}}} \tag{32}$$

The MAC coefficient varies between 0 and 1. A value next to 1 indicates a good correlation between the two compared mode shapes, while a value next to zero indicates poor correlation.

In the comparison of experimental results and analytic answers a MAC above 0,75 indicates that two mode shapes are correlated. For numerical simulations the MAC coefficient should be above 0,9.

4.1. Results of substructuring methods

For comparison of frequencies and mode shapes, the fixed interface method used 24 interface static modes (one for each interface degree of freedom) and 20,10 and 5 fixed interface modes for each substructure. Figure (2) shows the comparison between mode shapes of both simulations (substructuring and finite element analysis) using the MAC coefficient for the Craig & Bampton and MacNeal methods with 5 and 10 modes. Figure (3) makes the same comparison, but with Craig & Bampton and Rubin methods.

The comparison of mode shapes for 20 modes is performed in Fig. (4). Rubin´s method presented the same results and the MAC comparison was omitted.

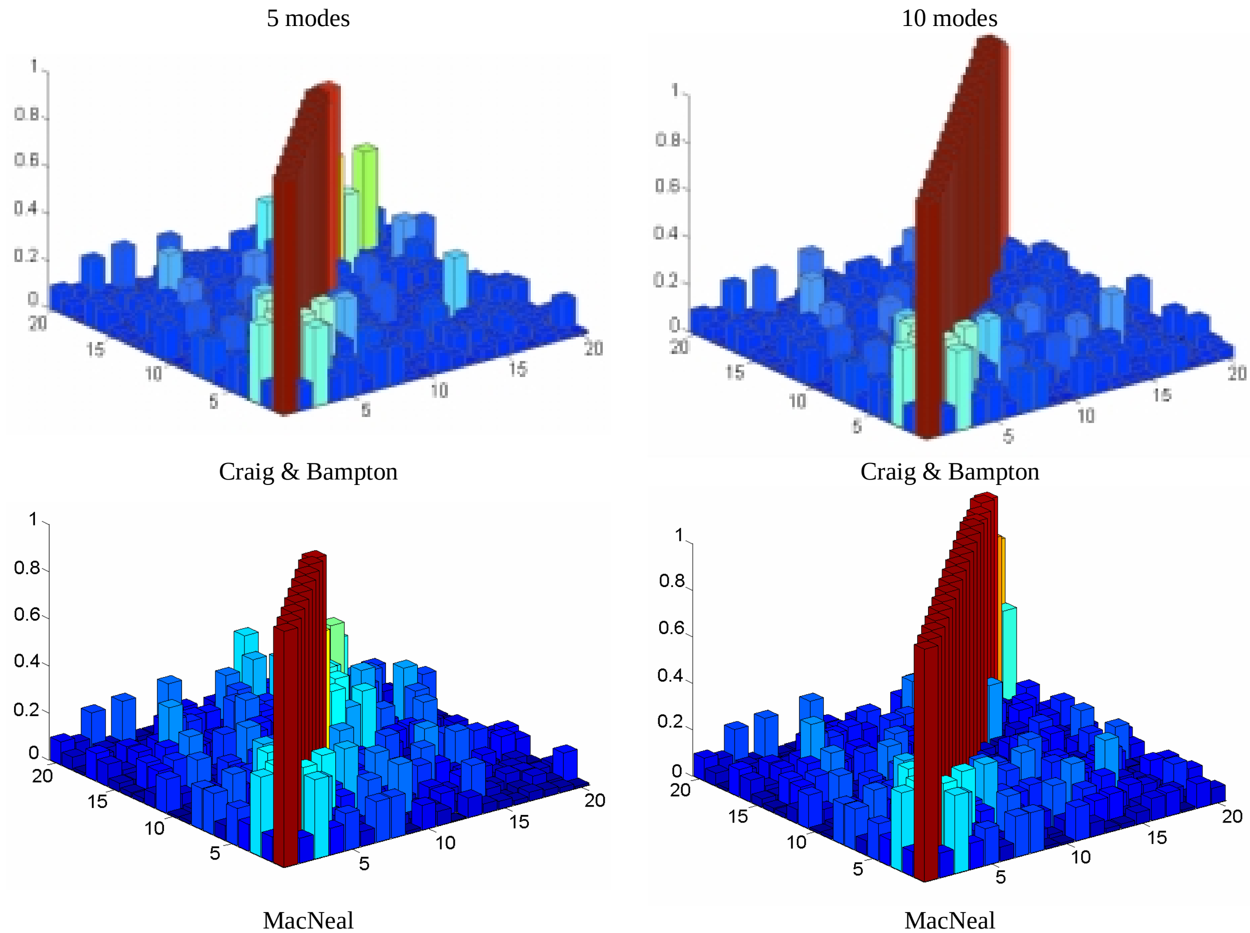

Figure 2. MAC coefficient comparison between Craig & Bampton and MacNeal with 05 and 10 modes for each substructure.

For the three cases, the increase of modes used for substructuring increases the quality of results. In the case of 5 modes for each substructure, one can observe that only the 10 first modes are achieved with quality, regarding the size of the modal space employed.

Observing Fig.(1), the MAC matrices achieved with MacNeal's and Rubin's methods are identical. This also occurs for the mode frequency values shown in Tab.(1). Thus, one can verify that the order 2 correction applied in Rubin's method increases the difficulties of calculation, but in the studied case it does not enhance the quality of results. This same pattern is repeated when 20 modes are used in each substructure and for that reason, only MAC matrices for Craig & Bampton's and MacNeal's methods was shown.

In the case with 20 modes, the presented results achieved high quality regarding that the representing modal space of the reduced problem has a highly superior dimension than the 20 modes shown in Fig.(3)

In the majority of industrial applications only few lower frequency modes are important. The achieved results show, for instance, that for achieving the first 5 structure vibration modes the component mode synthesis technique with only 5 modes for each substructure leads to results really near those attained with finite element analysis and with a reduced computational cost (memory and time).

In general, the studied free interface methods, lead to better results than the fixed interface method. Due to a better representation of the interface dynamic behavior, the free interface modes with residual flexibility and second order inertial corrections approximated eigenvalues of high frequencies with a smaller error than fixed interface modes with static interface modes.

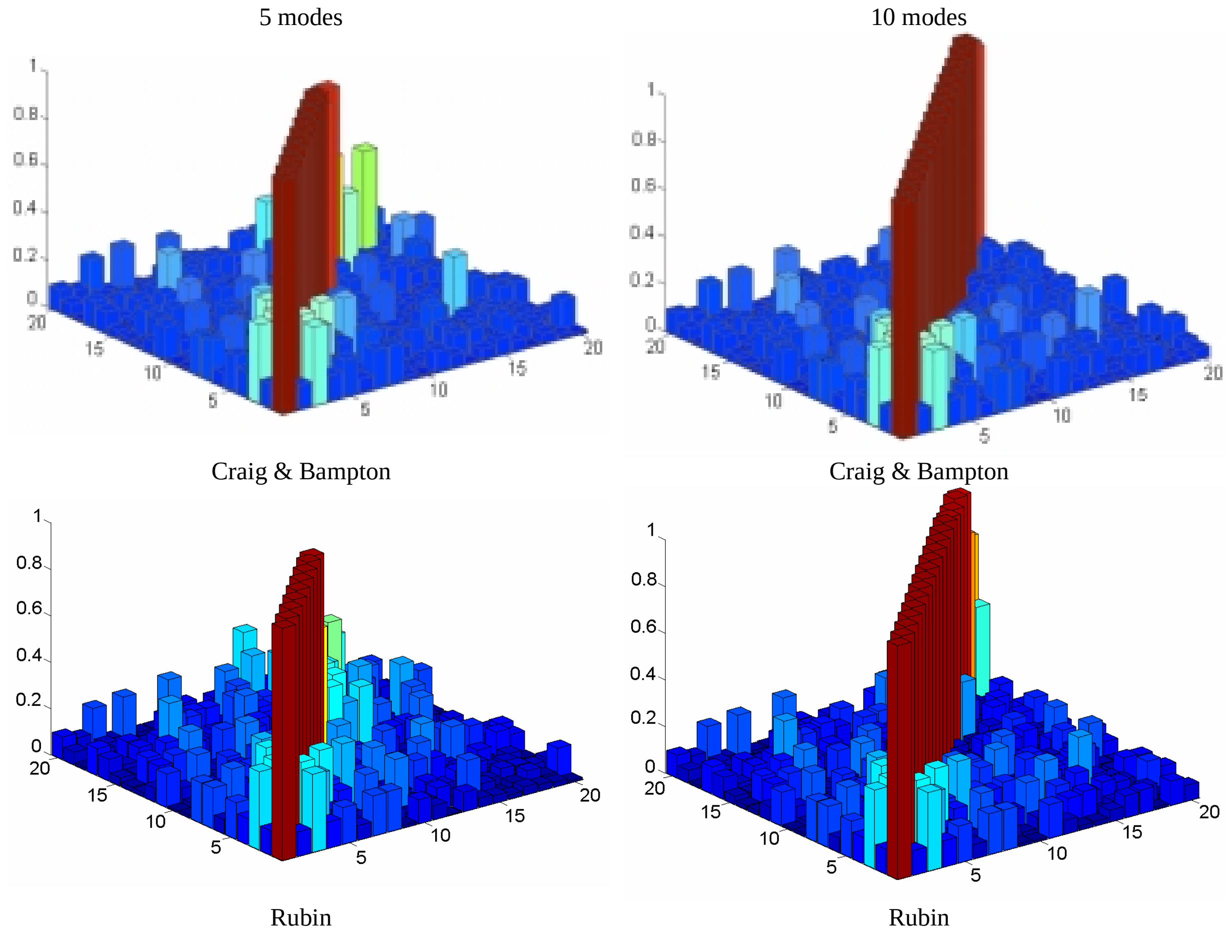

Figure 3. MAC coefficient comparison between Craig & Bampton and Rubin with 05 and 10 modes for each substructure.

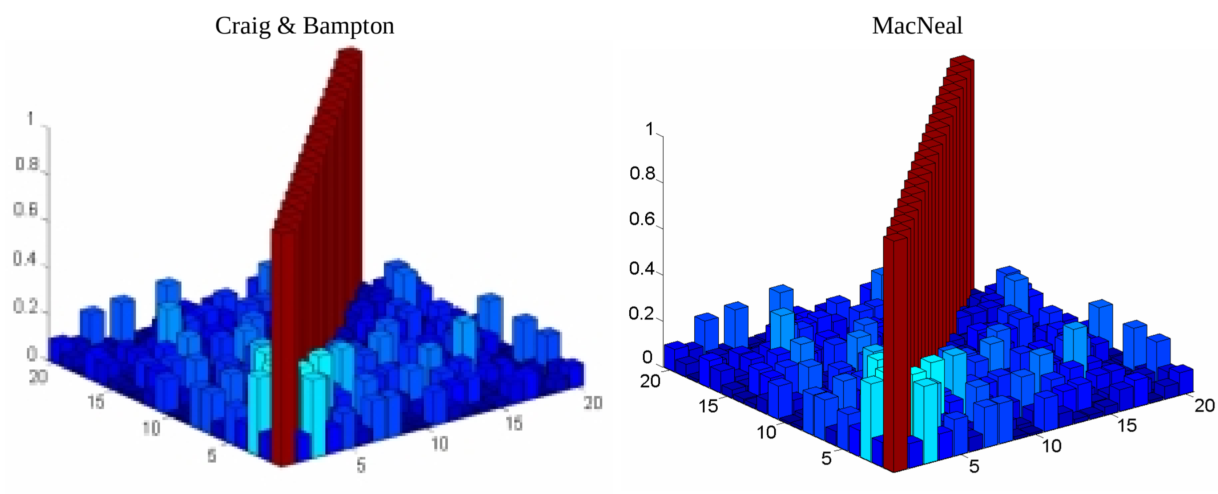

Figure 4. MAC coefficient comparison for Rubin and MacNeal’s methods with 20 modes for each substructure.

Table 1. Frequencies attained with component mode methods and finite element analysis.

Reference F.E.A. Freq. [Hz]	Craig & Bampton - values in [Hz] 24 static modes			MacNeal - values in [Hz] 24 flexibility components			Rubin - values in [Hz] 24 flexibility components		
	number of modes			number of modes			number of modes		
	5 modes	10 modes	20 modes	5 modes	10 modes	20 modes	5 modes	10 modes	20 modes
2,6763	2,6763	2,6763	2,6763	2,6763	2,6763	2,6763	2,6763	2,6763	2,6763
13,15	13,1517	13,1506	13,1504	13,1504	13,1504	13,1503	13,1504	13,1504	13,1503
14,56	14,5633	14,5604	14,5599	14,56	14,5599	14,5598	14,56	14,5599	14,5598
32,642	32,6526	32,6441	32,6425	32,6433	32,6427	32,6425	32,6433	32,6427	32,6425
41,468	41,4981	41,471	41,4681	41,4683	41,4681	41,468	41,4683	41,4681	41,468
65,808	66,1499	73,4828	65,8156	65,8147	65,8114	65,8093	65,8147	65,8114	65,8093
73,237	73,4516	73,2663	73,2419	73,2486	73,2417	73,2391	73,2486	73,2417	73,2391
81,261	81,8107	81,3521	81,2755	81,297	81,277	81,267	81,297	81,277	81,267
103,29	103,9116	103,374	103,3041	103,332	103,3142	103,3001	103,332	103,3142	103,3001
117,85	118,4899	117,9308	117,8654	117,8681	117,8589	117,8547	117,8681	117,8589	117,8547
134,73	144,7446	135,1136	134,7583	134,7654	134,75	134,7363	134,7654	134,75	134,7363
140,69	149,8788	141,3486	140,7576	140,7445	140,724	140,6988	140,7445	140,724	140,6988
159,85	184,4957	160,6491	160,0415	159,9088	159,8751	159,8572	159,9088	159,8751	159,8572
168,74	298,1045	169,1028	168,7739	168,8258	168,7708	168,7533	168,8258	168,7708	168,7533
193,54	320,5442	193,9325	193,6106	193,8974	193,6818	193,5913	193,8974	193,6818	193,5913
195,83	369,3196	196,6802	195,9484	196,2956	196,0294	195,8791	196,2956	196,0294	195,8791
214,76	448,5822	216,6528	214,9512	215,343	214,9635	214,8261	215,343	214,9635	214,8261
241,61	615,589	244,207	242,261	242,0703	241,7743	241,6773	242,0703	241,7743	241,6773
257,56	957,5654	269,4902	258,0741	258,3147	258,0555	257,6664	258,3147	258,0555	257,6664
281,43	1002,142	296,0545	281,6961	281,8283	281,6867	281,506	281,8283	281,6867	281,506

5. Conclusions

In this work, three component mode synthesis methods were implemented. In the formulation of the methods and in their implementation, a generalizing notation was proposed, following the work of Aziz et all (1993), intending the comprehension of a similar procedure for all studied methods. In this procedure, the main feature is the presence of a coordinate transformation matrix from a modal space to a physical space.

The studied methods were Craig & Bampton’s method, which uses fixed interface modes with static interface modes, MacNeal’s method, using free interface modes with flexibility correction modes and Rubin’s method, which applies free interface modes with flexibility correction terms and second order inertial correction terms. The three methodologies were applied to the study of vibration frequencies and mode shapes of two attached flat plates, each one of them representing one substructure.

The comparison of results was performed using the MAC criterion, which compares different mode shapes achieved with the substructuring methodologies and the respective mode shapes achieved with a finite element analysis. Frequencies were compared as well.

The results showed that free interface methods achieved better accuracy than fixed-interface methods due to better representation of interface behavior between the two substructures. Rubin’s method, which applied second order inertial correction terms, presented the same results than MacNeal’s method. Thus, for the flat plate case, the inclusion of second order corrections was not necessary. A study of different linking patterns between substructures is necessary to confirm the need of second order corrections.

This study also emphasized that the use of component mode synthesis methods with a reduced modal space requires lower computer costs compared to finite element analysis if the user is interested only in lower frequency results. The first five frequencies achieved with all methodologies were achieved with a small error showing the advantage in the use of fewer modes for building a modal space to represent the dynamic behavior of the structure.

6. References

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