A TIME DOMAIN BASED MECHANICAL SYSTEM IDENTIFICATION

Rogger Rodrigues Fonseca Oliveira Pereira
Solid Mechanics Laboratory
Universidade Federal do Rio de Janeiro, COPPE/PEM
PO Box 68503, Zip code 21945-970, Rio de Janeiro, RJ, Brasil
rpereira@mecsol.ufrj.br

Daniel Alves Castello
Solid Mechanics Laboratory
Universidade Federal do Rio de Janeiro, COPPE/PEM
PO Box 68503, Zip code 21945-970, Rio de Janeiro, RJ, Brasil
castello@mecsol.ufrj.br

Fernando Alves Rochinha
Solid Mechanics Laboratory
Universidade Federal do Rio de Janeiro, COPPE/PEM
PO Box 68503, Zip code 21945-970, Rio de Janeiro, RJ, Brasil
faro@serv.com.ufrj.br

Abstract. *The present work presents a time domain based identification technique within the inverse problems scope. Here the set of unknown parameters which characterizes the mechanical behaviour of the system is identified by means of the minimization of a suitable error function which includes time domain data from both the real system and its respective mathematical model. The technique takes into account the constraint associated to the system evolution equations as being part of an extended error function what naturally gives rise to the Lagrange multiplier variables which are obtained via solution of an adjoint problem. Mechanical modelling plays a crucial role here inasmuch as once one has decided which unknown parameters influence the system response, they are used to parameterize the mass, stiffness and damping matrices of the system. The effectiveness of the technique is assessed by using experimental data which was collected from a pinned-pinned steel beam instrumented with four piezoelectric accelerometers and an electro-mechanical shaker. The parameters which have been chosen to be identified were: localized damping at the bearings, the first three damping factors of the structure and a localized mass associated to the interaction between the structure and the shaker.*

Keywords: *system identification, time domain, inverse problems, vibrating structures*

1. Introduction

In the last years, System Identification [1] has emerged as an important discipline providing valuable tools within the Structural Dynamics field. The applications are very diverse, ranging from active control of vibrations [2] to model updating [3], passing through damage detection [4] and [5]. Identification is intended to improve the robustness and performance of the involved systems by helping on building reliable models which provide the ground of modern engineering. Those models are used to understand, to control involved phenomena and, probably their key feature, to predict future behavior.

Broadly speaking, identification consists on the process of developing a mathematical model for a real system by combining physical principles with experimental or field data. Therefore, identification uses a priori information, characterized by the model structure and characteristics, and a posteriori data, such as input-output observations. The main idea is to identify a set of parameters such that, over a desired range of operating conditions, the model outputs are close, in some well-defined sense, to the system outputs when both are submitted to the same inputs. Due to the incompleteness of available information and unavoidable measurement errors, system identification only achieves an approximation of the actual system [6], [1].

Structural dynamics deals, essentially, with three categories of identification, namely: modal parameter identification, model-based parameter identification and control-oriented identification. The first one, often referred to as modal testing, consists on obtaining modal parameters (e.g. damping, mode shapes, frequencies and modal participation factors) that, commonly, are taken as a basis to update analytical models or detect flaws in structures. The second category normally relies on a set of partial differential equations that express the physical principles that support the system's response. The sought parameters are physically meaning and their identification leads to both a reliable modelling and to a better understanding of the physics behind the processes [3], [7], [8]. The last approach, which is tailored to control and fault diagnosis applications, often uses the so called black-box models, whose parameters are, normally, physically meaningless [2], [9], [10] and [11] .

The capabilities and limitations of a myriad of identification methods included in each one of the aforementioned categories are investigated and reported in a vast literature . In the authors opinion, none of them alone can achieve all the goals defined in different applications. Therefore, there is a need of clearly understanding the performance of a determined method when applied to a specific class of problems. That gives rise to the present work, in which the use of a time domain system identification method, combining a Finite Element model of the mechanical system and an optimal control formulation, is introduced. In general terms, the situation is phrased as an inverse problem in which physical parameters or characteristics are sought once a set of system's responses, often obtained by experiments, are known.

2. Mathematical Formulation

As mentioned before, the system identification proposed here is formulated as an inverse problem that consists of a Finite Element model updating. Its numerical solution relies on the solution of two main subproblems, namely: the direct and the adjoint. Both, together with the inverse problem's formulation are addressed in this section.

2.1. Direct problem

The dynamics of a typical linear time-invariant multiple degree-of-freedom second order Finite Element model of a vibrating structure is governed by

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{q}}(t;\mathbf{p}) + \mathbf{D}(\mathbf{p})\dot{\mathbf{q}}(t;\mathbf{p}) + \mathbf{K}(p)\mathbf{q}(t;\mathbf{p}) = \mathbf{B}\mathbf{u}(t;\mathbf{p}) \tag{1}$$

with initial conditions

$$\dot{\mathbf{q}}(0;\mathbf{p}) = \dot{\mathbf{q}}_0(\mathbf{p})$$

$$\mathbf{q}(0;\mathbf{p}) = \mathbf{q}_0(\mathbf{p})$$

where t stands for time , $\mathbf{M}(\mathbf{p})$, $\mathbf{D}(\mathbf{p})$ and $\mathbf{K}(\mathbf{p})$ are n x n mass, damping and stiffness matrices which explicitly depends on p parameters organized in a vector referred to as $\mathbf{p}$; $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ are the acceleration, velocity and displacement vectors; $\mathbf{B}$ is the n x r input matrix; $\mathbf{u}$ is the load vector; $\dot{\mathbf{q}}_0(\mathbf{p})$ and $\mathbf{q}_0(\mathbf{p})$ are the $\mathbf{p}$ dependent initial velocity and displacement vectors;
The direct problem is concerned with the determination of $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$, given $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$, $\mathbf{B}$, $\mathbf{u}$, $\dot{\mathbf{q}}_0$, $\mathbf{q}_0$ and $\mathbf{p}$, $\forall\, t$ belonging to the interval of interest.

2.2. Inverse problem

In practical terms , only a subset of system's responses can be acquired. Thus, they are organized in a m-component output vector, denoted as $\mathbf{y}(t)$, which relates to the states by the expression

$$\mathbf{y}(t) = \mathbf{C}_a\ddot{\mathbf{q}}(t) + \mathbf{C}_v\dot{\mathbf{q}}(t) + \mathbf{C}_d\mathbf{q}(t) \tag{2}$$

where $\mathbf{C}_a$, $\mathbf{C}_v$ and $\mathbf{C}_d$ are the m x n output matrices, related to the kind and position of sensors used.
The actual response of an structure when excited by an arbitrary load is denoted as $\overline{\mathbf{y}}(t)$ and the corresponding measured input by $\overline{\mathbf{u}}(t)$, where bar superscripted variables mean experimentally obtained quantities.
The inverse problem consists of answering the following question:
Given an experimental system response $\overline{\mathbf{y}}(t)$ due to an external load $\overline{\mathbf{u}}(t)$, what is the value of $\mathbf{p}$ that makes the modelled system response $\mathbf{y}(t;\mathbf{p})$ closer (in some well precise sense) to $\overline{\mathbf{y}}(t)$? Closeness, in this work, is measured by the scalar objective functional $J(\mathbf{p})$ given by

$$J(\mathbf{p}) = \int_{t_0}^{t_f} \mathbf{e}(t;\mathbf{p})^T\,\mathbf{W}\,\mathbf{e}(t;\mathbf{p})\;dt \tag{3}$$

where the m-component residue vector $\mathbf{e}(t;\mathbf{p})$ is defined as

$$\mathbf{e}(t;\mathbf{p}) = \mathbf{y}(t;\mathbf{p}) - \overline{\mathbf{y}}(t) \tag{4}$$

with $\mathbf{W}$ being a m x m weighting matrix and t_0 and t_f are suitable initial and final instants of time defining the interval of observation.
The model's response $\mathbf{y}(t;\mathbf{p})$ is the output vector of the direct problem solution of evolution equation under the experimental load and, if it is the case, some unknown modelled load $\mathbf{f}(t;\mathbf{p})$, i.e.

$$\mathbf{u}(t;\mathbf{p}) = \overline{\mathbf{u}}(t) + \mathbf{f}(t;\mathbf{p}) \tag{5}$$

3. The adjoint problem

The inverse problem introduced before can be interpreted as a nonlinear constrained p-order optimization problem which reads as:

$$\min_{\mathbf{p}} J(\mathbf{p}) \tag{6}$$

with constraints

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}\mathbf{u} \tag{7}$$

In principle, any suitable optimization algorithm can be used to solve this problem. Here, the conjugate gradient method with adjoint problem is adopted. Another auxiliary problem is also necessary in the implementation of this method, the so called sensitivity problem. This section is devoted to the formulation of the adjoint and the sensitivity problems.

3.1. Adjoint Equation

The adjoint equation approach proposes the inclusion of the evolution equation constraint in the functional to be minimized making use of the adjoint variable, the n-component vector $\boldsymbol{\lambda}(t)$ (Lagrange multiplier):

$$J(\mathbf{p};\boldsymbol{\lambda}) = \int_{t_0}^{t_f} \mathbf{e}^T\mathbf{W}\,\mathbf{e}\,dt + \int_{t_0}^{t_f} \boldsymbol{\lambda}^T[\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} - \mathbf{B}\mathbf{u}]\,dt \tag{8}$$

Inverse problems usually lead to multiple solutions; some of them are totally unreasonable . Regularization techniques are usually adopted to handle this pitfall and so a p x 1 regularization term $\boldsymbol{\phi}(\mathbf{p})$ is also included in the functional expression, weighted by some p x 1 weighting regularization vector $\boldsymbol{\alpha}$.

$$J(\mathbf{p};\boldsymbol{\lambda}) = \int_{t_0}^{t_f} \mathbf{e}^T\mathbf{W}\mathbf{e}\,dt + \int_{t_0}^{t_f} \boldsymbol{\lambda}^T[\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} - \mathbf{B}\mathbf{u}]\,dt + \boldsymbol{\alpha}^T\boldsymbol{\phi} \tag{9}$$

The first variation of $J(\mathbf{p};\boldsymbol{\lambda})$ relative to $\mathbf{p}$ can be written as

$$\begin{aligned}
\delta J(\mathbf{p};\boldsymbol{\lambda}) &= \int_{t_0}^{t_f} 2e^T\mathbf{W}[\mathbf{C}_a\delta\ddot{\mathbf{q}} + \mathbf{C}_v\delta\dot{\mathbf{q}} + \mathbf{C}_d\delta\mathbf{q}]\,dt + \\
&+ \int_{t_0}^{t_f} \lambda^T[\mathbf{M}\delta\ddot{\mathbf{q}} + \mathbf{D}\delta\dot{\mathbf{q}} + \mathbf{K}\delta\mathbf{q}]\,dt + \\
&+ \int_{t_0}^{t_f} \lambda^T[\delta\mathbf{M}\ddot{\mathbf{q}} + \delta\mathbf{D}\dot{\mathbf{q}} + \delta\mathbf{K}\mathbf{q} - \mathbf{B}\delta\mathbf{f}]\,dt + \\
&+ \boldsymbol{\alpha}^T\delta\boldsymbol{\phi}
\end{aligned}$$

since the first variation of the residue vector, system response and system load, respectively, are

$$\delta\mathbf{e}(t;\mathbf{p}) = \delta\mathbf{y}(t;\mathbf{p})$$

$$\delta\mathbf{y}(t;\mathbf{p}) = \mathbf{C}_a\delta\ddot{\mathbf{q}}(t;\mathbf{p}) + \mathbf{C}_v\delta\dot{\mathbf{q}}(t;\mathbf{p}) + \mathbf{C}_d\delta\mathbf{q}(t;\mathbf{p})$$

$$\delta\mathbf{u}(t;\mathbf{p}) = \delta\mathbf{f}(t;\mathbf{p})$$

Taking the following decomposition for the Lagrange multiplier

$$\boldsymbol{\lambda} = \ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d$$

and using integration by parts with some algebraic manipulation, the following expression comes up:

$$
\begin{aligned}
\delta J(\mathbf{p};\boldsymbol{\lambda}) &= \int_{t_0}^{t_f} [\ddot{\boldsymbol{\lambda}}_a^T \mathbf{M} - \dot{\boldsymbol{\lambda}}_a^T \mathbf{D} + \boldsymbol{\lambda}_a^T \mathbf{K} + 2e^T \mathbf{W}\mathbf{C}_a] \delta\ddot{\mathbf{q}}\, dt + \\
&+ [\dot{\boldsymbol{\lambda}}_a^T \mathbf{D}\delta\dot{\mathbf{q}} + \dot{\boldsymbol{\lambda}}_a^T \mathbf{K}\delta\mathbf{q} - \boldsymbol{\lambda}_a^T K \delta\dot{\mathbf{q}}]|_{t_0}^{t_f} + \\
&+ \int_{t_0}^{t_f} [-\ddot{\boldsymbol{\lambda}}_v^T \mathbf{M} + \dot{\boldsymbol{\lambda}}_v^T \mathbf{D} - \boldsymbol{\lambda}_v^T \mathbf{K} + 2e^T \mathbf{W}\mathbf{C}_v] \delta\dot{\mathbf{q}}\, dt + \\
&+ [\dot{\boldsymbol{\lambda}}_v^T \mathbf{M}\delta\dot{\mathbf{q}} + \boldsymbol{\lambda}_v^T \mathbf{K}\delta\mathbf{q}]|_{t_0}^{t_f} + \\
&+ \int_{t_0}^{t_f} [\ddot{\boldsymbol{\lambda}}_d^T \mathbf{M} - \dot{\boldsymbol{\lambda}}_d^T \mathbf{D} + \boldsymbol{\lambda}_d^T \mathbf{K} + 2e^T \mathbf{W}\mathbf{C}_d] \delta\mathbf{q}\, dt + \\
&+ [\boldsymbol{\lambda}_d^T \mathbf{M}\delta\dot{\mathbf{q}} - \dot{\boldsymbol{\lambda}}_d^T \mathbf{M}\delta\mathbf{q} + \boldsymbol{\lambda}_d^T \mathbf{D}\delta\mathbf{q}]|_{t_0}^{t_f} + \\
&+ \int_{t_0}^{t_f} (\ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d)^T [\delta\mathbf{M}\ddot{\mathbf{q}} + \delta\mathbf{D}\dot{\mathbf{q}} + \delta\mathbf{K}\mathbf{q} - \mathbf{B}\delta\mathbf{u}]\, dt + \\
&+ \boldsymbol{\alpha}^T \delta\boldsymbol{\phi}
\end{aligned}
$$

The adjoint equations come from the imposition of the following restrictions (in the form of evolution equations with final conditions) to the $\boldsymbol{\lambda}$ variables:

$$
\begin{aligned}
\ddot{\boldsymbol{\lambda}}_a^T \mathbf{M} - \dot{\boldsymbol{\lambda}}_a^T \mathbf{D} + \boldsymbol{\lambda}_a^T \mathbf{K} &= -2e^T \mathbf{W}\mathbf{C}_a \\
\dot{\boldsymbol{\lambda}}_a(t_f)^T \mathbf{D} - \boldsymbol{\lambda}_a(t_f)^T \mathbf{K} &= 0 \\
\dot{\boldsymbol{\lambda}}_a(t_f)^T \mathbf{K} &= 0
\end{aligned}
$$

$$
\begin{aligned}
-\ddot{\boldsymbol{\lambda}}_v^T \mathbf{M} + \dot{\boldsymbol{\lambda}}_v^T \mathbf{D} - \boldsymbol{\lambda}_v^T \mathbf{K} &= -2e^T \mathbf{W}\mathbf{C}_v \\
\dot{\boldsymbol{\lambda}}_v(t_f)^T \mathbf{M} &= 0 \\
\lambda_v(t_f)^T \mathbf{K} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\ddot{\boldsymbol{\lambda}}_d^T \mathbf{M} - \dot{\boldsymbol{\lambda}}_d^T \mathbf{D} + \boldsymbol{\lambda}_d^T \mathbf{K} &= -2e^T \mathbf{W}\mathbf{C}_d \\
\boldsymbol{\lambda}_d(t_f)^T \mathbf{M} &= 0 \\
-\dot{\boldsymbol{\lambda}}_d(t_f)^T \mathbf{M} + \boldsymbol{\lambda}_d(t_f)^T \mathbf{D} &= 0
\end{aligned}
$$

3.2. Sensitivity equation

The sensitivity problem, introduced bellow, is needed in the computational procedure of optimization. It is in the form of an evolution equation with initial conditions and relates the sensitivities of states $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ to variations in $\mathbf{p}$.

$$
\begin{aligned}
\mathbf{M}\delta\ddot{\mathbf{q}} + \mathbf{D}\delta\dot{\mathbf{q}} + \mathbf{K}\delta\mathbf{q} &= \mathbf{B}\delta\mathbf{u} - \delta\mathbf{M}\ddot{\mathbf{q}} - \delta\mathbf{D}\dot{\mathbf{q}} - \delta\mathbf{K}\delta\mathbf{q} \\
\delta\dot{\mathbf{q}}(0) &= \delta\dot{\mathbf{q}}_0 \\
\delta\mathbf{q}(0) &= \delta\mathbf{q}_0
\end{aligned}
$$

Sensitivities quantities are also important for understanding and assessing the identifiability associated to the proposed inverse problem.

4. Numerical Algorithm

An iterative computational procedure is adopted in this work. At each iteration step, the functional J diminishes following some descent direction relying on its gradient and eventually $\mathbf{p}$ converges to some reasonable value. The adjoint equation, in conjunction with the sensitivity problem, is helpful in the determination of the search direction and search step size.

4.1. Gradient Computation

If $\boldsymbol{\lambda}$ satisfy the adjoint equations, then δJ is simplified and becomes

$$\begin{aligned}
\delta J(\mathbf{p}) &= [\dot{\boldsymbol{\lambda}}_a(t_0)^T\mathbf{K} + \boldsymbol{\lambda}_v(t_0)^T\mathbf{K} + \boldsymbol{\lambda}_d(t_0)^T\mathbf{D} - \dot{\boldsymbol{\lambda}}_d(t_0)^T\mathbf{M}]\delta\mathbf{q}_0 + \\
&+ [\dot{\boldsymbol{\lambda}}_a(t_0)^T\mathbf{D} - \boldsymbol{\lambda}_a(t_0)^T\mathbf{K} + \dot{\boldsymbol{\lambda}}_v(t_0)^T\mathbf{M} + \boldsymbol{\lambda}_d(t_0)^T\mathbf{M}]\delta\dot{\mathbf{q}}_0 + \\
&+ \int_{t_0}^{t_f} (\ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d)^T[\delta\mathbf{M}\ddot{\mathbf{q}} + \delta\mathbf{D}\dot{\mathbf{q}} + \delta\mathbf{K}\mathbf{q} - \mathbf{B}\delta\mathbf{u}]\,dt + \\
&+ \boldsymbol{\alpha}^T\delta\boldsymbol{\phi}
\end{aligned}$$

The right side of the above equation contains the needed terms in the gradient optimization algorithm: the first and second ones are related to the estimation of the possibly unknown initial states $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$, the third term is used in the estimation of the possibly unknown dynamic system properties $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ and system load $\mathbf{u}$ and the last term takes into account the regularization factor $\boldsymbol{\phi}$. These terms are used in the evaluation of an approximation to $\nabla J(\mathbf{p})$, the gradient of the functional $J(\mathbf{p})$ in $\mathbf{p}$, viz.:

$$\delta J(\mathbf{p}) \approx \nabla J(\mathbf{p})^T \Delta\mathbf{p}$$

If for each δX quantity is possible to write

$$\delta X \approx \sum_{i=1}^{p} \frac{\partial X}{\partial p_i}\Delta p_i$$

then

$$\begin{aligned}
[\nabla J(\mathbf{p})]_i &\approx [\dot{\boldsymbol{\lambda}}_a(t_0)^T\mathbf{K} + \boldsymbol{\lambda}_v(t_0)^T\mathbf{K} + \boldsymbol{\lambda}_d(t_0)^T\mathbf{D} - \dot{\boldsymbol{\lambda}}_d(t_0)^T\mathbf{M}]\frac{\partial\mathbf{q}_0}{\partial\mathbf{p}_i} + \\
&+ [\dot{\boldsymbol{\lambda}}_a(t_0)^T\mathbf{D} - \boldsymbol{\lambda}_a(t_0)^T\mathbf{K} + \dot{\boldsymbol{\lambda}}_v(t_0)^T\mathbf{M} + \boldsymbol{\lambda}_d(t_0)^T\mathbf{M}]\frac{\partial\dot{\mathbf{q}}_0}{\partial\mathbf{p}_i} + \\
&+ \int_{t_0}^{t_f} (\ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d)^T\frac{\partial\mathbf{M}}{\partial\mathbf{p}_i}\ddot{\mathbf{q}}\,dt + \\
&+ \int_{t_0}^{t_f} (\ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d)^T\frac{\partial\mathbf{D}}{\partial\mathbf{p}_i}\dot{\mathbf{q}}\,dt + \\
&+ \int_{t_0}^{t_f} (\ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d)^T\frac{\partial\mathbf{K}}{\partial\mathbf{p}_i}\mathbf{q}\,dt + \\
&+ \int_{t_0}^{t_f} -(\ddot{\boldsymbol{\lambda}}_a + \dot{\boldsymbol{\lambda}}_v + \boldsymbol{\lambda}_d)^T\mathbf{B}\frac{\partial\mathbf{u}}{\partial\mathbf{p}_i}\,dt + \\
&+ \boldsymbol{\alpha}^T\frac{\partial\boldsymbol{\phi}}{\partial\mathbf{p}_i}
\end{aligned}$$

4.2. Algorithm

Similarly to other gradient based methods, the conjugate gradient method updates the p-component parameter vector iteratively according to the relation bellow:

$$\mathbf{p}^{k+1} = \mathbf{p}^k - \boldsymbol{\beta}^k\mathbf{d}^k$$

where β is the search step size, found from the solution of the linear system

$$\mathbf{A}\boldsymbol{\beta} = \mathbf{b} \tag{10}$$

where

$$\begin{aligned}
A_{ij} &= \int_{t_0}^{t_f} \delta e_i^T\delta e_j\,dt \\
b_i &= \int_{t_0}^{t_f} e_i^T\delta e_i\,dt
\end{aligned}$$

and $\mathbf{d}$ is the conjugate direction of search, given by

$$\mathbf{d}^k = \nabla J^k + \gamma^k\mathbf{d}^{k-1}$$

where the conjugation factor γ is given by one of the following forms, the Polak-Ribiere form

$$\gamma^k = \frac{(\nabla J^k)^T(\nabla J^k - \nabla J^{k-1})}{(\nabla J^{k-1})^T(\nabla J^{k-1})}$$

or the Fletcher-Reeves form

$$\gamma^k = \frac{(\nabla J^k)^T(\nabla J^k)}{(\nabla J^{k-1})^T(\nabla J^{k-1})}$$

Further details about the gradient computation, sensitivity equation and conjugation factor may be found in [13].

A pseudocode for the iterative procedure implemented is in the sequence bellow:

1. Give $\mathbf{p}$ an initial guess $\mathbf{p}^0$.
2. Initialize all the necessary variables (∇J^0, $\mathbf{d}^0$, etc), zeroing them.
3. With $\mathbf{p}$, evaluate J.
4. Check termination criteria. For instance, check if $J \leq \epsilon$, where ϵ is a tolerance factor, or check for excessive number of iterations. If ok, then end. Else, continue.
5. Using the gradient equation, find ∇J. The adjoint problem is a pre-requisite for gradient equation.
6. With ∇J and ∇J^0, find γ.
7. Knowing ∇J, γ and $\mathbf{d}^0$, find the search direction $\mathbf{d}$.
8. After solving sensitivity problem making use of $\mathbf{d}$, find $\boldsymbol{\beta}$.
9. Update $\mathbf{p}$ using $\mathbf{p}^0$, $\boldsymbol{\beta}$ and $\mathbf{d}$.
10. Update last iteration variables ($\nabla J^0 \longleftarrow \nabla J$, etc).
11. Go to step 3.

This code is a basic one. Of course, difficulties associated with convergence may arise and appropriate changes must be made. For instance, the search step size $\boldsymbol{\beta}$ may be overestimated by the relation given. One possible modification is halving it until a satisfactory value is achieved.
With the numerical procedure implemented and a set of experimental data obtained, next section contains some results and considerations about the method.

5. Results and discussion

5.1. Experimental setup

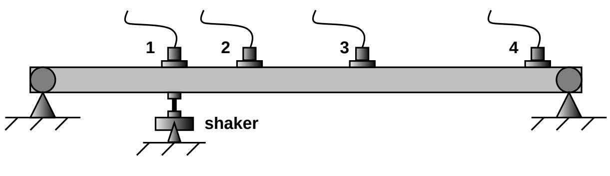

Figure 1: Testbed sketch of the second experiment.

The testbed for the experiment is depicted in Figure (1). The pinned-pinned beam is made of steel and its dimensions are: 1467x76.2x7.9 mm. It has been used four piezoelectric accelerometers: 33B52-PCB-PIEZOTRONICS. Starting from the left end of the beam, the accelerometers are located at 1/4, 1/3, 1/2 and 5/6 of the beam length.

Tables 1 and 2 present the first three natural frequencies obtained experimentally and by a Finite Element (FE) analysis of the system and the data acquisition parameters utilized for all experiments.

Table 1: Natural frequencies (Hz) for the pinned-pinned beam.

Mode no.	Experimental	FEM
1	8.6	8.6
2	33.6	34.4
3	75.8	77.3

Table 2: Experiment setup.

Experiment no.	Excitation type	sampling frequency (Hz)	f_{cut} (Hz)	t_f (s)
1	white-noise	400	100	55
2	sine-chirp (0-100) Hz	800	100	8
3	$\sin(68\pi)$	800	100	4
4	$\sin(150\pi)$	800	100	4

5.2. Case 1

As modelling is inherent to the identification process, the first step to be considered is to determine the physical phenomena associated to the system under study. For the first case ($C1$), the authors have decided to identify the structural damping factors associated to the first three modes of the structure ζ_1, ζ_2 and ζ_3 and the bending stiffness EI. Moreover, it was considered the existence of local dissipation at the bearings. Such a local dissipation was modelled by means of rotatory dampers c_0 and c_L located at the end of the bearings.
The parameter vector $\mathbf{p}$ has six components, and as an initial guess, all of them were considered equal to zero, but the one associated to the bending stiffness. The initial guess for EI was obtained considering the Young's modulus of the steel and the beam dimensions. For the identification process it was used the data from experiment number two (see table 2). Despite the fact that there were four available accelerometers, it was chosen the collocated one, accelerometer 1, for the identification step. The data from the other accelerometers will be used at the validation step. Figure (2) depicts, on the left, the residue e associated to the first accelerometer,

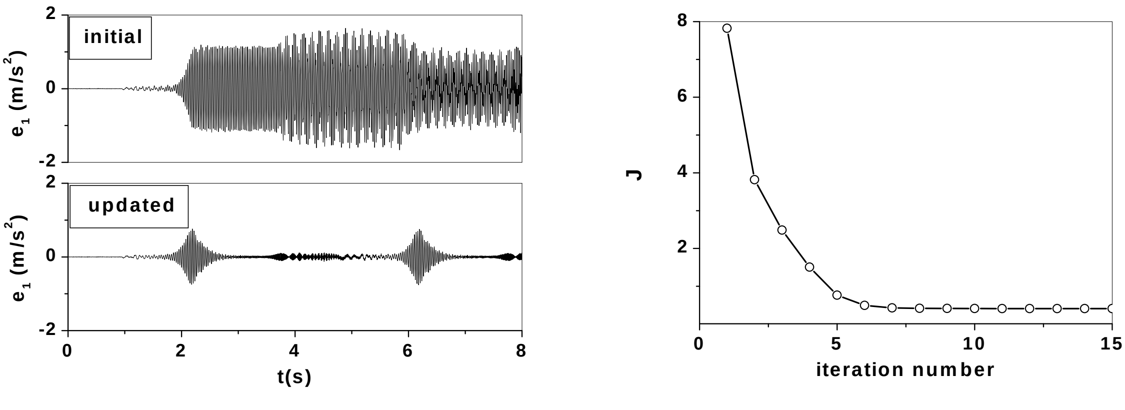

Figure 2: Residue of the first accelerometer and evolution of the functional J.

before and after the updating and, on the right, the evolution of the respective functional J. Figure (2) clearly indicates that the functional J remained almost steady from iteration number ten to iteration number fifteen. It is also clear from figure (2) that the magnitude of the residue e has substantially decreased in almost every region of the observation period. The intervals encompassing $(1.8, 2.5)\,s$ and $(5.8, 6.5)\,s$ are the ones where the main frequency of the excitation force is around the second natural frequency of the system. Figure (3) shows the evolution of the damping parameters of the system. Here, two points should be emphasized. The first one is concerned with the values of the damping factors ζ_1 and ζ_2, which have achieved meaningless physical values for a steel structure. The identification provided $\zeta_1 = 0.0206$, $\zeta_2 = 0.0254$ and $\zeta_3 = 0.0021$, which are extremely high. Nevertheless, an identification using the Eigensystem Realization Algorithm (ERA) and the Common Based Normalized System Identification ($CBSI$) [1] provided $\zeta_1^{ERA} = 0.034$, $\zeta_2^{ERA} = 0.0035$ and $\zeta_3^{ERA} = 0.0031$. The second point is concerned with the evolution of the rotatory dampers, their evolution, which does not get steady, although the functional J does, clearly indicates that one is facing a problem of

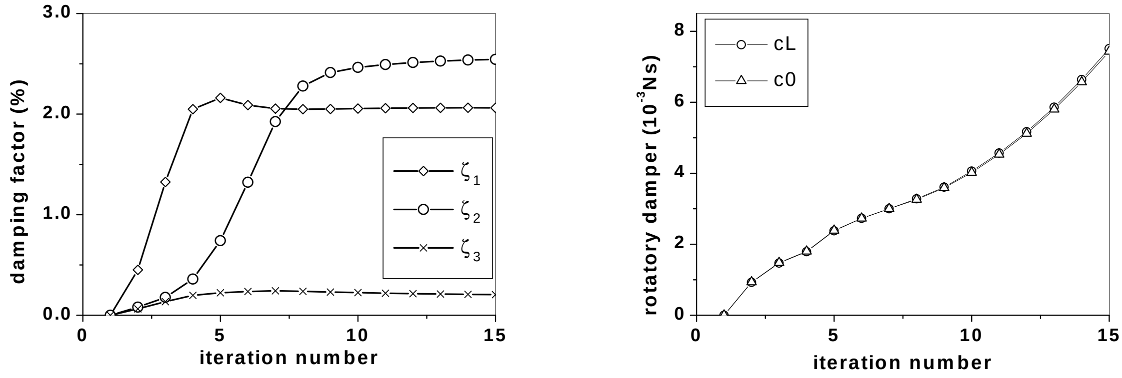

Figure 3: Evolution of the damping parameters.

non-modelled dynamics [7]. Here, a new set of experimental data would be worthy inasmuch as the analysis domain would get larger. The ideal updating scenario would be a sequential estimate of the parameters, which demands different experimental data to be used in sequence, such that, each updating vector would work as a regularization for the next updating [7].

Figure (4) show the evolution of the normalized bending stiffness which has been defined as the ratio between the current bending stiffness and its initial value.

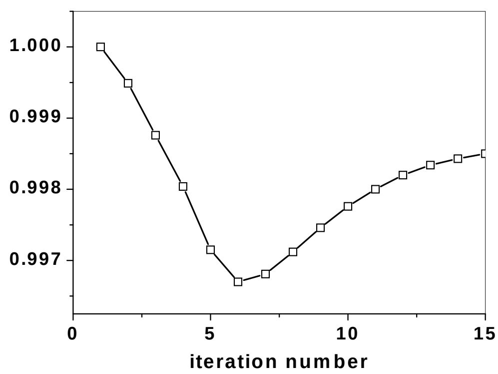

Figure 4: Evolution of normalized bending stiffness.

As it has been previously said, the experimental data from the accelerometers 2 and 3 are used at the validation step. Figure (5) depicts the residue e associated to the accelerometer 2, on the left, and the one associated to the accelerometer 3, on the right. Figure (5) shows a good agreement between the experimental data and the response provided by the identified system. It should be remarked that, once again, the response provided by the identified system was not quite accurate for the second mode of the structure. As a last assessment, one may use experiments number three and four aiming at validating the identified system. It has been chosen the collocated accelerometer for the analyses. Figures (6) and (7) show the validation based on the third and fourth experiment respectively. Based on the residues, one may state that the identified system represents a betterment of the mathematical model of the system, although, as it has already been cited, we are probably facing a problem of non-modelled dynamics. Hence, it would be necessary to acquire new experimental data and try to include in the model some physical phenomena that had nor been considered.

It should be remarked that the evaluation of the identified system must be based on a set o error indicators [5]. Therefore, once one is in front of such error indicators, it can be possible to state if the model is accurate for the kind of applications it has been designed for. Nevertheless, if the analysis leads to the conclusion that the model does not fill the required conditions to be considered accurate, new strategies must be adopted. Such strategies may consider staggered identification or sophisticate the physical model for example [7].

6. Concluding remarks

The present work presented a time domain based identification technique within the inverse problems scope. The technique minimizes an error function which is defined as the difference between the responses of the model

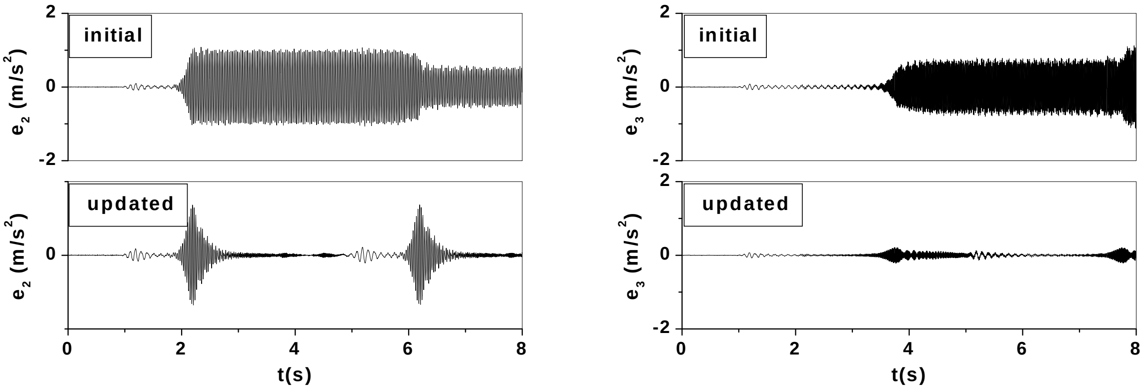

Figure 5: Residue of the second and third accelerometers.

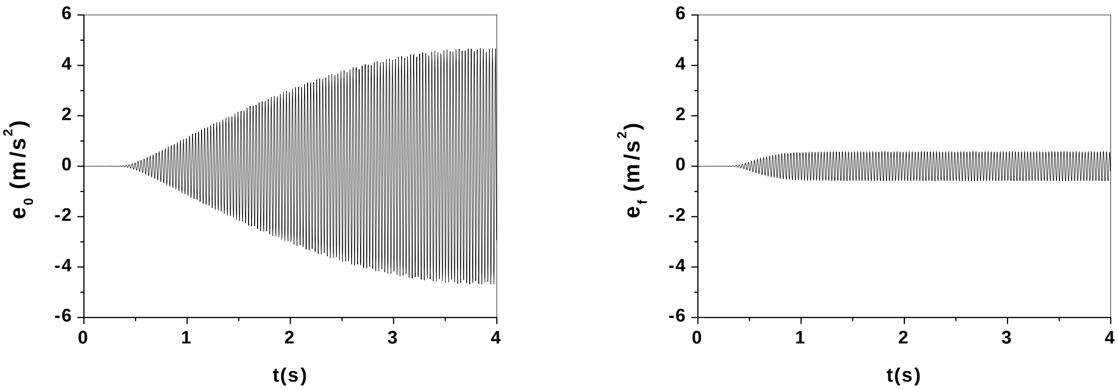

Figure 6: Residue of the first accelerometer (validation based on experiment 3).

and the real structure. It also considers the evolution equation of the system as being part of the functional to be minimized, which naturally gives rise to the Lagrange multipliers. An important feature associated to this technique is the fact that it enables one to sophisticate the physical model used to describe the system. The parameters of the system can be distributed over the body for example, or a non-liner model used. The effectiveness of the technique was assessed using experimental data coming out of a pinned-pinned steel beam instrumented with four accelerometers an an electromechanical shaker. The results can be considered compelling and the adopted error indicators showed positive results concerning the identified system.

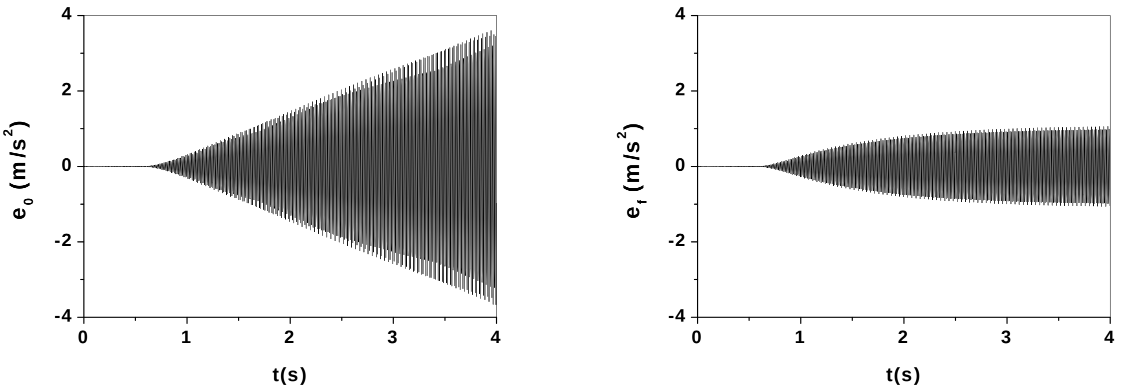

Figure 7: Residue of the first accelerometer (validation based on experiment 4).

References

[1] Juang Jer-Nan 1994 *Applied System Identification* Prentice Hall, New Jersey, U.S.A.

[2] Clark R.L., Saunders W.R., and Gibbs G.P. 1998 *Adaptive Structures.* John Wiley and Sons Inc., U.S.A.

[3] Friswell M.I. and Mottershead J.E. 1995 *Finite Element Model Updating in Structural Dynamics.* Kluwer.

[4] Natke H.G. and Cempel C. 1997 *Model-Aided Diagnosis of Mechanical Systems - Fundamentals, Detection, Localization, Assessment*, Springer Verlag, Germany.

[5] Castello D.A., Stutz L.T., and Rochinha F.A. 2002 *A Structural Defect Identification Based on a Continuum Damage Model*, Computers & Structures,**80** 417-436 .

[6] Ljung L. 1987 *System Identification - Theory for the User*, Prentice Hall ,USA.

[7] Woodbury K.A. "Editor" 2003 *Inverse Engineering Handbook*, CRC Press.

[8] Castello D.A., Stutz L.T., and Rochinha F.A. 2002 *A Time Domain Technique for Defect Identification Based on a Continuous Damage Model*, ASME International Mechanical Enginering Congress & Exposition, New Orleans, Louisiana, USA.

[9] Kuo S.M. and Morgan D.R. 1996 *Active Noise Control Systems*, John Wiley and Sons Inc., U.S.A.

[10] Diniz P.S.R. 1997 *Adaptive Filtering*, Kluwer Academic Publishers, U.S.A.

[11] Castello D.A. and Rochinha F.A. 2001 *Adaptive Filter Theory Applied on Mechanical System Identification*, 17^{th} Brazilian Congress of Mechanical Engineering, Uberlândia, M.G., Brasil.

[12] Huang C.-H. 2001 *An Inverse Non-Linear Force Vibration Problem of Estimating the External Forces in a Damped System with Time-Dependent System Parameters*, Journal of Sound and Vibration,**242(5)**749-765.

[13] Özisic M. N. and Orlande H.R.B. 2000 *Inverse Heat Transfer: Fundamentals and Applications*, Taylor and Francis.