

COMPARISON BETWEEN MULTIPLE VIBRATION ABSORBERS AND SINGLE VIBRATION ABSORBERS SYSTEMS

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Abstract. The advance of structural analysis has been allowing design of more economic and consequently more flexible structures. Due to this advance, Civil Engineering began to deal more frequently with new kind of problems, such as structures more susceptible to vibration. Several solutions were conceived to reduce vibrations: structural modification, stiffness increasing and finally the use of vibration absorbers.

This paper has purpose of comparing multiple vibration absorbers with single vibration absorber systems, and showing the advantages of using multiple vibration absorbers. One of these advantages is the possibility of evaluating errors of calibration and including these errors in the process of calibration. Another advantage is that multiple vibration absorbers require low damping ratios. The analysis will be performed by a numerical tool implemented with an optimization method that will calculate optimal parameters for absorbers calibration.

The system uses, for comparison, the discretized model of a platform located at Structural Laboratory of COPPE/UFRJ (Federal University of Rio de Janeiro-Brazil), designed for experimental tests to describe loads induced by human activities.

Keywords. *Tuned Vibration Absorbers, Structural Vibration, Structural Dynamics*

1. Introduction

Tuned vibration absorbers (TVA's for simply) are passive absorbers connected with the main structure, which has the objective to reduce the vibration level. They are formed by a mass, a spring element and a damping element. The loss of efficiency of TVA's is due to uncertainties introduced by mathematical and physical approaches as well as variability of the physical properties of structures and systems (Cabral et al, 1998).

The present paper uses TVA's systems in a structure discretized by Finite Elements Method, regarding a non-proportional damping matrix, where this non-proportionality was introduced by the absorbers. Several cases disposition and amounts of TVA's and its respective efficiencies were analyzed. It will be demonstrate the advantages of using the multiple TVA's instead of an only one. The calibration parameters were determined by an optimization technique. Parametric studies available in the literature relate the effects of the mass, damping and frequencies ratios of the structure and TVA's in the efficiency of the absorbers systems (Xu and Igusa, 1992; Kareem and Kline, 1995). The model used in the analysis is a structure submitted to loads induced by human activities such as walking, jumping and dancing, etc.

2. Mathematical Model

The methodology proposed by Magluta (1993) obtains the appropriate calibrations for the TVA's systems through an optimization algorithm known as "Goal Programming" (Ignizio, 1979). This optimization technique implicates in an interactive process that requests, in each one of the iterations, the solution of an eigensystem problem with non-proportional damping, which is the bottleneck process. The generalized eigensystem problem assumes the form:

$$(\underline{A}\lambda + \underline{B})\phi = \underline{0} \quad (1)$$

being,

$$\underline{A} = \begin{bmatrix} \underline{C} & \underline{M} \\ \underline{M} & \underline{0} \end{bmatrix} \quad (2)$$

$$\underline{B} = \begin{bmatrix} \underline{K} & \underline{0} \\ \underline{0} & -\underline{M} \end{bmatrix} \quad (3)$$

$$\lambda = \xi \cdot \omega + \omega \sqrt{1 - \xi^2} i \quad (4)$$

being,

M - Mass Matrix

K - Stiffness Matrix

C - Damping Matrix

ϕ - Complex Eigenvector

λ - Complex Eigenvalue

ω - Natural Frequency

ξ - Damping Ratio

i - The complex Number

The eigensystem problem showed in eq. (1) has dimension of $2n$, the QZ transformation (Moler and Stewart (1973)) was applied to solving this eigensystem. A substructuring technique was used to increase system efficiency, this technique reduce the order of the matrices reordering the degrees of freedom, ranking the nodes like internal or external. The internal nodes are eliminated, however its influence in the others degrees of freedom remains, the external nodes remain without any modification. The global displacements vector becomes:

$$\underline{X} = \begin{bmatrix} \underline{X}_i \\ \underline{X}_e \end{bmatrix} \quad (5)$$

where \underline{X}_i is the internal nodes displacement vector and \underline{X}_e is the external nodes displacement vector.

The reduction of the matrix dimension is obtained through eq. (6), taking ϕ_i for $i < n$:

$$\underline{U} = \underline{T} \underline{X} \quad (6)$$

being,

$$\underline{T} = \begin{bmatrix} \underline{\phi}_i & \underline{\phi}_e \\ 0 & \underline{I} \end{bmatrix} \quad (7)$$

the eq. (7) shows the transformation matrix to the new vector of displacements \underline{U} . $\underline{\phi}_i$ is the matrix with some vibration modes associated with the internal nodes, $\underline{\phi}_e$ is the matrix with constrained modes obtained when unitary displacements are applied in each external degree of freedom. \underline{I} is the identity matrix.

The transformation expressed by eq. (6) cause imprecisions in the evaluation of the modal ordinates associated to the internal degrees of freedom. These imprecisions hardly influence the evaluate, however the same doesn't happen with the internal forces calculation. This way the system efficiency was analyzed using the displacements, allowing a great reduction of the matrix dimensions without significantly influence in the final results.

The reduction of the main structure vibration or efficiency was simply defined as being:

$$e(\%) = \left(1 - \frac{D_1}{D_0} \right) \times 100 \quad (8)$$

being e the absorbers efficiency, D_0 the displacement before the absorber connection and D_1 the displacement after absorber connection.

Equation (8) has been used like an objective function in the search for more appropriated calibration parameters. It must be defined constrains functions which force the search pattern respects a consistent solution space.

The "Goal Programming" technique is based in minimizing a scalar function starting from x variables defined as below:

$$f(\bar{x}); \bar{x} \in \mathfrak{R}^n \quad (9)$$

This function is submitted to a set of objectives (g) and restrictions (h),

$$g_j(\bar{x}) \geq 0; j = 1, 2, \dots, J; \quad (10)$$

$$h_k(\bar{x}) = 0; k = 1, 2, \dots, K \quad (11)$$

There are two design variables in the problem of using TVA's for reducing vibration level: stiffness and damping factor. The mass of TVA must be set like input data, because the system efficiency increases by adding mass to the system. If the TVA's mass would be used as a variable in the optimization process, it should always tend to the maximum value set in the restriction function showed in eq. (11). Optimizing TVA's stiffness and damping factor, it obtains least displacements amplitudes (highest efficiencies values).

3. Simulated Examples

The chosen model used for analysis was a steel-concrete composite simple supported platform designed by Faisca (2000) for experimental tests to describe dynamics loads induced by human activities. The platform dimensions are 2m x 12m. Due to its dimensions the platform was discretized in frame finite elements and the absorbers were discretized in discrete spring and damping elements. Table (1) compares the dynamics characteristics obtained from the experimental tests and from the numerical tool.

Table 1-Experimental and numerical results

Vibration Mode	ω (experimental)	ξ (experimental)	ω (numerical)
1°	3,2 hz	1%	3,18 hz
2°	11,60 hz	-	12,69 hz

As it can be seen in the tab. (1), the numerical model is well adjusted to the experimental results, mainly for the first vibration mode. This is due to the fact that during the adjustment of the numerical model a certain preference was given to the first natural frequency, once the load used in the numeric model has excitation frequency varying in the range of 1.8-3.4 Hz. The used load represents the activity of jumping and it could be characterized by the first three Fourier's Series coefficients, which values, according CEB (1991), are respectively 1.8, 1.3 and 0.7. These coefficients are normalized in relation to an individual mean weight (700 N).

In order to reduce the vibration levels of this structure the absorbers systems were inserted as described as follow:

- A single TVA system located at the middle of the spam (case I), the same system assuming a frequency calibration error of $\pm 10\%$ (case I-a and I-b) and assuming an damping ratio error of $\pm 10\%$ (case I-c and I-d);
- Three TVA's systems located at the middle of the spam, calibrated with the same optimized parameters (case II), distinctly optimized calibrations (case II-a), assuming an error in order of $\pm 10\%$ in frequency calibration (case II-b and case II-c);
- Three TVA's systems, with one of the TVA's located at $\frac{1}{4}$ of the spam, another one situated at the middle of the spam and a last one situated at $\frac{3}{4}$ of the spam (the distances were referred to the first support), with identically optimized calibrations (case III) and different calibrations (case III-a).

Figure (1) compares the results of the cases I and II, in terms of efficiency versus mass ratio M_a/M_p (M_a – mass of the absorber system and M_p – mass of the main structure), as it can be observed in this figure the efficiency of the TVA system increases when the mass relationship grows. It also can be observed that a single TVA system has similar efficiency (considering that they were installed in the same point of the case I, each one of them had a third of the mass of the case I and the three absorbers had the same calibration). It should be emphasized that the calibration parameters of the cases I and II are identical. This figures also presents the results obtained with three TVA systems with calibrations distinctly (case II-a). It's verified that this last case has more efficiency for the smallest mass ratio.

Figure (2) compares the results of cases I, III and III-a. Cases III and III-a. It can be seen that the results of case III and III-a are less efficient than the case I. It's due to the fact that the cases III and III-a have two absorbers located out of the middle of the spam, where the maximum amplitude of the first mode occurs. These results show the fact of using multiple absorbers systems should cause a loss of efficiency, if couldn't be possible to install the systems near to the higher amplitude of vibration mode.

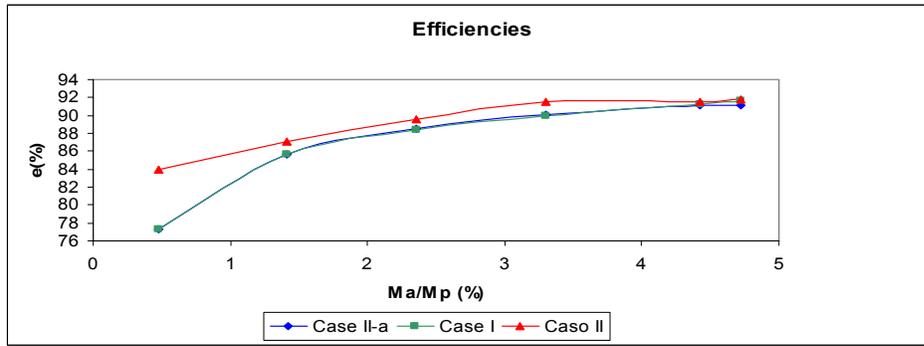


Figure 1. Results of Cases I, II and II-a

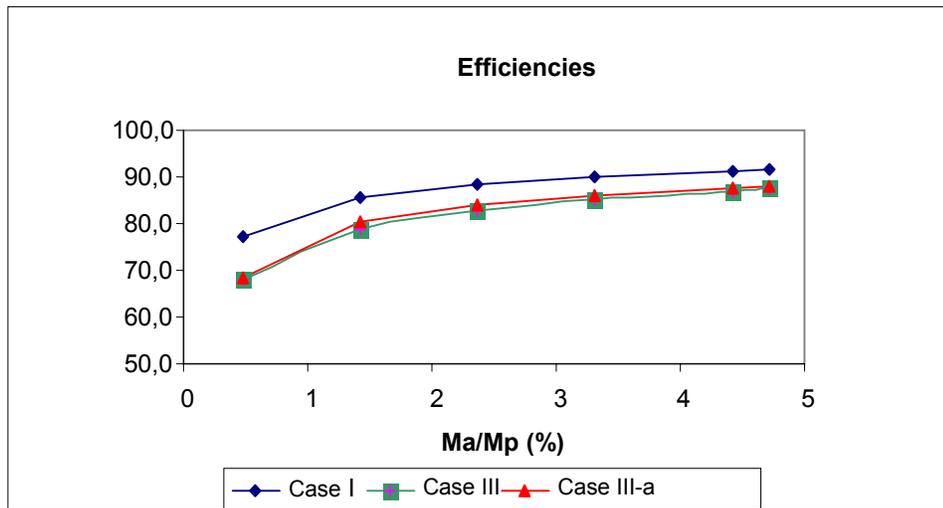


Figure 2. Results of cases I, III and III-a

According to Brownjohn (2001), people acting at a structure could modify the natural frequency and increase the damping ratio. Besides, the change of the ideal calibration could also be modified during the installation of the TVA. In order to verify the influence of these changes in the efficiency a parametric study was performed. Figure (3) shows the obtained results for the ideal calibration (case I) and with changes of $\pm 10\%$ both in frequency (cases I-a: $+10\%$; I-b: -10%) and damping ratio (cases I-c: $+10\%$; I-d: -10%). One can see in this figure that the efficiency is more sensible in relation of frequency changes. In both cases was observed higher loss of efficiency when the both parameters were super estimated.

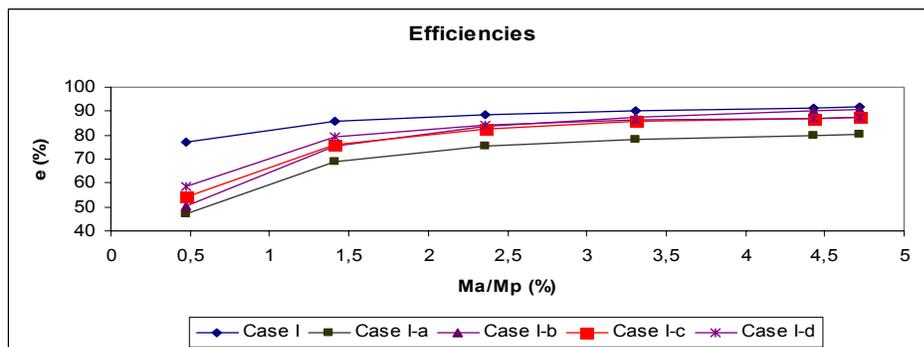


Figure 3. Results of case I and variations.

The same procedure shown previously it was used in case II. Figure (4) shows a comparison between the cases I-a and II-a, in case of errors at calibration the multiple absorbers systems present a better performance than an only one system (the same error patterns were used for the comparison). Possible mistakes at the calibration parameters are better dissipated when multiple TVA's have been used.

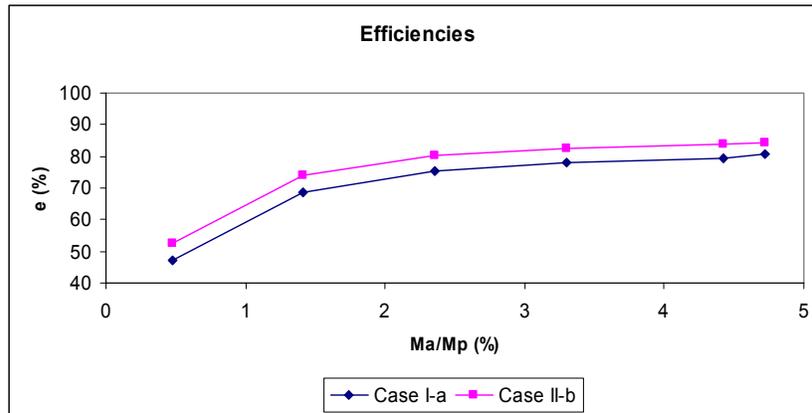


Figure 4. Comparison between case I and II with errors in calibration.

4. Conclusions

The ideal calibration led to high damping ratios, in order of 15 a 20 %. In practice this values are very difficult to be reached, and so the absorbers systems would have high loss of efficiency. Multiple absorbers systems presented less loss of efficiency than the single absorber, because they could distribute better this discrepancy.

Otherwise, the variations in the main structure in the absorbers calibration parameters, could be estimated in the design of multiple absorbers systems, making the loss of efficiency minimized.

5. References

- Brownjohn, J., 2001, "Energy dissipation in one-way slabs with human participation", *Journal of Shock and Vibration* Vol. 8 no. 6, pp. 315-323.
- Cabral, S. V. S. de, Riera, J. D. and Rocha, M. M., 1998, "Reliability-based desing of multiple mass dampers", *Proceedings of Icosar'97 – The 7th International Conference on Structural Safety and Reliability*, Kyoto, Japan, pp. 1015-1022.
- CEB – Comitê Euro-Internacional do Beton, 1991, "Vibration Problems in Structures – Pratical Guidelines, Bulletin d'information," N° 209.
- Faisca, R. G., 2000, "Descrição de Cargas Dinâmicas Geradas por Atividades Humanas", *Proposta de Tese do Exame de Qualificação para Doutorado* (in Portuguese), COPPE/UFRJ, RJ, Brazil.
- Ignizio, J. P., 1979, "Goal Programming and Extensions", *Lexington Books*, New York., N.Y., U.S.A..
- Kareem, A., Kline, S., 1995, "Performance of multiple mass dampers under random loading", *Journal of Structural Engineering*, Vol. 121, pp. 2766-2772.
- Magluta, C., 1993, "Sistemas Dinâmicos Passivos para Absorção de Vibrações Estruturais", *Doctor Science Thesis* (in Portuguese), COPPE-UFRJ, RJ, Brazil.
- Moler C. B., Stewart, G. W., 1973, "An algorithm for generalized eigenvalue problems", *SIAM Journal of Numerical Analysis*, Vol. 10, pp. 241-256.
- Xu, K., Igusa, T., 1992, "Dynamic characteristics of multiple substructures with closely spaced frequencies", *Earthquake Engineering and Structural Dynamics*, Vol. 21, n. 12, pp. 1059-1070.