AN APPROXIMATE SOLUTION TO THE EXTERNAL COOLING OF A TWO DIMENSIONAL BLOCK SUBJECTED TO AN INTERNAL HEAT SOURCE

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Abstract. *In this contribution, a two-dimensional external conjugated heat transfer problem is studied. The physical problem consists of a rectangular block with an internal heat source, which is being cooled by an air stream at its upper surface. Various boundary conditions at the surfaces of the block are specified while the coupling between the solid and the fluid occurs at the top surface of the block where an unknown temperature distribution is sought. The mathematical model of this problem involves the steady non-homogeneous two-dimensional heat transfer equation together with the boundary layer formulation for the fluid flow. This model is solved throughout an approximate fashion by analytically solving the heat transfer problem and by employing various approximate polynomials for the velocity profile. Quantities of practical interest such as the block and fluid temperature distributions and the local and mean wall heat flux are presented with respect to the Reynolds number and the ratio of the fluid to the solid thermal conductivities.*

Keywords. *Cooling of electronic equipment, integral transform, external forced convection, conjugated heat transfer.*

1. Introduction

In engineering practice, heat transfer from a heated block subjected to convective cooling at its upper surface is the model for many applications including electronic equipment cooling, materials processing and heat exchangers.

A literature survey shows several relevant contributions that employ either analytic or approximate methods of solution for this type of problem. Luikov (1974) solved analytically the conjugate problem of forced convection over a heated semi-infinite thin flat plate by assuming constant velocities in the momentum boundary layer and a linear temperature profile for the thin plate. Karvinen (1978) presented an approximate solution for heat transfer from a finite thin plate subjected to forced convection by prescribing an initial estimate for the temperature distribution along the surface that allowed for the determination of a finite differences iterative scheme of solution. Rizk, Kleinstreuer and Özisik (1992) developed an analytic solution for the heat transfer problem of flow past a rectangular block by assuming constant axial and normal velocity components that considerably simplified the thermal boundary layer solution. Pop and Ingham (1993) considered forced convection over a one-dimensional finite plate therefore neglecting the axial conduction within the plate. Their finite differences numerical solution was based on asymptotic expansions. Vlassov (2002) solved a two-dimensional steady state heat conduction problem with multiple sources distributed on a rectangular region through the classical integral transform technique. The convective cooling process was represented by a convective boundary condition therefore avoiding the solution of the thermal boundary layer equation.

In this contribution, the physical situation studied by Rizk, Kleinstreuer and Özisik (1992) is revisited with a different and more realistic approach to the convection problem. Here, the assumption of constant axial and normal velocity components is replaced by the approximate solution of the boundary layer type flow past the block surface by means of the integral boundary layer method for the momentum equation as well as for the balancing of heat fluxes through the interface between solid and fluid.

2. Mathematical formulation

Steady conjugate heat transfer from a rectangular block with volumetric heat source to a free stream is considered. The coupling between the solid and fluid is through the unknown block surface temperature that varies in the axial direction. The boundary conditions are taken as constant temperature at the inlet side and either isothermal surface or constant wall heat flux at the exit side while the bottom is adiabatic. The mathematical formulation of this conjugated conduction-convection heat transfer problem is discussed in this section.

2.1. Conduction inside the block

The steady two-dimensional temperature field θ is governed by the following energy equation in dimensionless form

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+A^{-1}\varphi(x,y)=0 \quad , 0<x<1, \quad -A<y<0 \tag{1a}$$

subjected to the boundary conditions

$$\theta = 0, x = 0 \tag{1b}$$

$$\theta = 0 \text{ or } \frac{\partial\theta}{\partial x}=0, x=1 \tag{1c,d}$$

$$\frac{\partial\theta}{\partial y}=0, y=-A \tag{1e}$$

where various dimensionless groups are defined as

$$A = l/L,\ g=g_0\varphi\left(\hat{x}/L, \hat{y}/L\right),\ k^*=\frac{k_s}{k_f},\ x = \hat{x}/L,\ y = \hat{y}/L \text{ and } \theta=\frac{\psi-T_0}{g_0Ll/k_s} \tag{2a-f}$$

In the above parameters, L and l are the length and height of the block, respectively, g represents volumetric heat generation rate, ψ is the dimensional temperature field within the block, k_s and k_f represents the thermal conductivity for solid and fluid, respectively, and the symbol ^ denotes dimensional coordinate.

The boundary condition at the block-fluid interface is given in Section 2.3.

2.2. Forced convection over the block surface

By introducing the following dimensionless variables:

$$Pe=\frac{UL}{\alpha}, Re=\frac{UL}{\nu}, t=\frac{T-T_0}{g_0Ll/k_s}, u=\frac{\hat{u}}{U}, v=\frac{\hat{v}}{U}, x=\frac{\hat{x}}{L}, y=\frac{\hat{y}}{L}, \delta_H=\frac{\hat{\delta}_H}{L} \text{ and } \delta_T=\frac{\hat{\delta}_T}{L} \tag{3a-i}$$

the convection problem for laminar, steady, hydrodynamically fully developed flow over the block surface is given in dimensionless form as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4a}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \tag{4b}$$

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=\frac{1}{Pe}\frac{\partial^2 t}{\partial y^2} \tag{4c}$$

where $0<x<\infty$, $0<y<\infty$, and the following dimensionless boundary conditions apply:

$$u(x,y=0)=v(x,y=0)=0 \tag{4d}$$

$$u(x,y\to\infty)\to 1 \tag{4e}$$

$$t(x=0,y)=0 \tag{4f}$$

$$t(x,y=0)=\theta(x,y=0)=f(x) \tag{4g}$$

$$t(x,y\to\infty)\to 0 \tag{4h}$$

Here, U represents the mainstream velocity component, T is the dimensional fluid temperature, α and ν are respectively the fluid thermal diffusivity and kinematic viscosity, and the symbol ^ denotes dimensional coordinate.

2.3. Interfacial conditions

The conduction-convection problem defined in Sections 2.1 and 2.2 are coupled at the solid-fluid interface by the requirement of continuity of temperature and heat flux along the block surface. Thus,

$$\theta = t = f(x), \quad y = 0 \tag{5a}$$

$$k^* \frac{\partial \theta}{\partial y} = \frac{\partial t}{\partial y}, \quad y = 0 \tag{5b}$$

where a dimensionless function f (x) whose form is yet to be determined in the subsequent analysis was introduced in the form

$$f(x) = \frac{T_{wall}(\hat{x}) - T_0}{g_0 Ll / k_s} \tag{6}$$

The systems defined by Eqs. (1a-e) and Eqs. (4a-h), coupled through Eqs. (5a-b), constitute a conjugate heat transfer problem whose solution will be addressed in the following section.

3. Problem solution

3.1 Conduction problem

The steady, two-dimensional, conduction problem defined by Eqs. (1a-e) is solved through the classical integral transform technique (Özisik, 1980) as follows:
The temperature field is expressed in the form of an expansion in terms of eigenfunctions

$$\theta(x, y) = \sum_{m=1}^{\infty} A_{1m}(y) X_m(x) \tag{7}$$

where $X_m(x)$ is the solution of the following Sturm-Liouville problem, obtained through separation of variables applied to the homogeneous version of system (1a-e):

$$\frac{d^2 X_m}{dx^2} + \beta_m^2 X_m = 0, \; 0 < x < 1 \tag{8a}$$

$$X_m(0) = 0, \; X_m(1) = 0 \text{ or } \left.\frac{dX_m}{dx}\right|_{x=1} = 0 \tag{8b-d}$$

The eigenfunctions of the above eigenproblem are found to be

$$X_m = \sin(\beta_m x) \tag{9}$$

The eigenvalues β_m depend on the choice of the boundary condition at x =1, Eq. (1c) or Eq. (1d), and are given respectively by

$$\beta_m = m\pi \text{ (isothermal wall)}, \; \beta_m = m\pi - \pi/2 \text{ (insulated wall)} \tag{10a,b}$$

Equation (7) is now multiplied by the generic eigenfunction X_j and integrated to yield

$$\int_0^1 \theta(x, y) X_j dx = \sum_{m=1}^{\infty} A_{1m}(y) \int_0^1 X_m(x) X_j(x) dx \tag{11}$$

Recalling the orthogonality property of the eigenfunctions, which is written as:

$$\int_0^1 X_m(x)X_j(x)dx = \begin{cases} 0, m \neq j \\ N_m = \frac{1}{2}, m = j \end{cases} \tag{12}$$

Equation (11) becomes

$$A_{1m}(y) = \frac{1}{N_m}\int_0^1 \theta(x,y)X_m(x)dx \tag{13}$$

By substituting Eq. (13) into Eq. (7), the following integral transform pair is obtained:

$$\begin{cases} \theta(x,y) = \sum_{m=1}^{\infty} \frac{1}{N_m} X_m(x)\overline{\theta}_m(y) \\ \overline{\theta}_m(y) = \int_0^1 \theta(x,y)X_m(x)dx \end{cases} \tag{14a, b}$$

Equation (14b) is the transform of θ(x,y) whereas Eq. (14a) is its inversion. The original problem Eq. (1a) is then transformed as follows: Eq. (1a) is multiplied by the eigenfunctions X_m and the resulting expression is integrated which yields

$$\int_0^1 X_m(x)\frac{\partial^2\theta}{\partial x^2}dx + \int_0^1 X_m(x)\frac{\partial^2\theta}{\partial y^2}dx + \int_0^1 X_m(x)\left[A^{-1}\varphi(x,y)\right]dx = 0 \tag{15}$$

The first term in the above expression is rewritten in terms of the transform (14b) as

$$\frac{d^2\overline{\theta}_m(y)}{dy^2} + \int_0^1 X_m(x)\frac{\partial^2\theta}{\partial x^2}dx + \int_0^1 X_m(x)\left[A^{-1}\varphi(x,y)\right]dx = 0 \tag{16}$$

Now, Eq. (8a) is multiplied by the temperature θ(x,y) and the resulting expression is integrated yielding

$$\int_0^1 \theta(x,y)\frac{d^2X_m}{dx^2}dx + \beta_m^2\int_0^1 X_m(x)\theta(x,y)dx = 0 \tag{17}$$

The second term of the above expression is rewritten in terms of the transform (14b) as

$$\int_0^1 \theta(x,y)\frac{d^2X_m}{dx^2}dx + \beta_m^2\overline{\theta}_m(y) = 0 \tag{18}$$

Equation (18) is subtracted from Eq. (16), resulting in

$$\frac{d^2\overline{\theta}_m(y)}{dy^2} - \beta_m^2\overline{\theta}_m(y) + \overline{g}_m(y) = \int_0^1 \left[\theta(x,y)\frac{d^2X_m}{dx^2} - X_m(x)\frac{\partial^2\theta}{\partial x^2}dx\right]dx \tag{19}$$

where $\overline{g}_m(y)$ is the transformed heat generation function, defined by

$$\overline{g}_m(y) = \frac{1}{A}\int_0^1 X_m(x)\varphi(x,y)dx \tag{20}$$

The integral in Eq. (19) is evaluated as

$$\int_0^1 \left[\theta(x,y)\frac{d^2X_m}{dx^2} - X_m(x)\frac{\partial^2\theta}{\partial x^2}dx\right]dx = \theta(x,y)\frac{dX_m}{dx}\bigg|_{x=0}^{x=1} - \int_0^1 \frac{dX_m}{dx}\frac{\partial\theta}{\partial x}dx - X_m(x)\frac{\partial\theta}{\partial x}\bigg|_{x=0}^{x=1} + \int_0^1 \frac{\partial\theta}{\partial x}\frac{dX_m}{dx}dx \tag{21}$$

Clearly, the second and fourth terms on the right hand side of Eq. (21) cancel. Furthermore, by substituting the boundary conditions at x = 0 and x = 1, the first and third terms on the right hand side vanish. Thus,

$$\frac{d^2\overline{\theta}_m(y)}{dy^2} - \beta_m^2\overline{\theta}_m(y) + \overline{g}_m(y) = 0 \qquad (22)$$

The original partial differential equation is therefore transformed into a set of m decoupled ordinary differential equations for the transformed temperature $\overline{\theta}_m(y)$. The boundary conditions for system (22) are obtained from transformation of Eq. (5a) and Eq. (1e), respectively, as

$$\begin{cases} \overline{\theta}_m(0) = \int_0^1 X_m(x)f(x)dx \\ \left.\dfrac{d\overline{\theta}_m}{dy}\right|_{y=-A} = 0 \end{cases} \qquad (23a, b)$$

For simplicity in the analysis, heat generation is taken as a constant and therefore

$$\overline{g}_m = \frac{1}{A}\int_0^1 X_m(x)dx = \frac{1}{A}\frac{1-\cos\beta_m}{\beta_m} \qquad (24)$$

The solution of system (22) subjected to Eqs. (23a,b) is

$$\overline{\theta}_m(y) = [tgh(\beta_m A)\sinh(\beta_m y) + \cosh(\beta_m y)]\left[\int_0^1 X_m(x)f(x)dx - \frac{\overline{g}_m}{\beta_m^2}\right] + \frac{\overline{g}_m}{\beta_m^2} \qquad (25)$$

Finally, the inversion (14a) is applied to Eq. (25) to furnish an expression for the temperature within the solid:

$$\theta(x,y) = \sum_{m=1}^{\infty}\frac{X_m(x)}{N_m}\left\{[tgh(\beta_m A)\sinh(\beta_m y) + \cosh(\beta_m y)]\left[\int_0^1 X_m(x)f(x)dx - \frac{\overline{g}_m}{\beta_m^2}\right] + \frac{\overline{g}_m}{\beta_m^2}\right\} \qquad (26)$$

At this point, function f (x) remains unknown and must be determined by matching the solid and fluid solutions at the interface.

3.2 Convection problem

Here, an alternative solution to the one addressed by Rizk, Kleinstreuer and Özisik (1992) for the hydrodynamic problem is sought. A simple model that allows for approximate results but still retains some insight in the physics of the problem is the classical integral method of solution for boundary layer equations for the flat plate situation.

Thus, following the basic steps in the analysis with this method, Eq. (4b) is integrated with respect to y over the boundary layer thickness $\delta_H(x)$ and the velocity component v(x,y) appearing in this equation is eliminated by means of the continuity equation, Eq. (4a). As a result, the so-called momentum integral equation is obtained but an additional relationship between $\delta_H(x)$ and u(x,y) is needed. At this point, an approximation is introduced in the analysis in the form of an assumption regarding the functional form of the velocity profile u(x,y). Such an approximation is chosen to be a first, second, third or fourth-degree polynomial, in the form:

$$u(x, y) = a_0(x) + a_1(x)\,y + a_2(x)\,y^2 + a_3(x)\,y^3 + a_4(x)\,y^4 \qquad (27)$$

Constants a_0 to a_4 are defined according to the desired polynomial degree and follow the constraints for the physical problem. Thus, for a first-degree approximation two conditions are needed and for every additional degree a new condition is necessary up to the limiting situation of a fourth-degree polynomial defined by five constraints, as follows

$$u(x, 0) = 0,\ u(x, \delta_H) = 1,\ \left.\frac{\partial u}{\partial y}\right|_{y=\delta_H} = 0,\ \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = 0,\ \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=\delta_H} = 0 \qquad (28a\text{-}e)$$

The application of the first two conditions given by Eq. (28a) and Eq. (28b) results in a velocity profile in the form of a first-degree polynomial, whereas application of Eqs. (28a-c) results in a second-degree polynomial approximation and so forth. Table (1) summarizes the resulting polynomial velocity profiles.

For every different polynomial approximation for u(x,y) the corresponding momentum boundary layer thickness δ_H is needed. This task is accomplished by introducing the chosen polynomial approximation into the momentum integral equation and by performing the integration over y to obtain an ordinary differential equation for the determination of δ_H (x). Once this equation is solved, one obtains

$$\delta_H(x) = \left(c_{vel} \frac{x}{Re} \right)^{1/2} \tag{29}$$

The constant c_{vel} depends on the degree of the polynomial approximation and is summarized on Tab. (1).

In a similar fashion, the thermal problem is also solved by the integral method and a polynomial representation for the temperature profile within the thermal boundary layer is adopted. Such an approximation is chosen among polynomials up to the fourth-degree:

$$t(x, y) = b_0(x) + b_1(x)\, y + b_2(x)\, y^2 + b_3(x)\, y^3 + b_4(x)\, y^4 \tag{30}$$

Coefficients b above are determined by constraining Eq. (30) to the following constraints:

$$t(x, 0) = f(x),\ t(x, \delta_T) = 0,\ \left.\frac{\partial t}{\partial y}\right|_{y=\delta_T} = 0,\ \left.\frac{\partial^2 t}{\partial y^2}\right|_{y=0} = 0,\ \left.\frac{\partial^3 t}{\partial y^3}\right|_{y=0} = 0 \tag{31a-e}$$

The resulting temperature profiles are also summarized on Tab. (1).

Table 1. Polynomial approximations and values for constants in Eq. (29) and Eq. (32)

Velocity profile	c_{vel}	c_q	Temperature profile
$u = \left(y/\delta_H \right)$	12	1	$t = f(x)\left(1 - \frac{y}{\delta_T}\right)$
$u = -\left(y/\delta_H \right)^2 + 2\left(y/\delta_H \right)$	30	2	$t = f(x)\left(\frac{y}{\delta_T} - 1\right)^2$
$u = -\frac{1}{2}\left(y/\delta_H \right)^3 + \frac{3}{2}\left(y/\delta_H \right)$	280 / 13	3/2	$t = f(x)\left[\frac{1}{2}\left(\frac{y}{\delta_T}\right)^3 - \frac{3}{2}\left(\frac{y}{\delta_T}\right) + 1\right]$
$u = \left(y/\delta_H \right)^4 - 2\left(y/\delta_H \right)^3 + 2\left(y/\delta_H \right)$	1260 / 37	4/3	$t = f(x)\left[\frac{1}{3}\left(\frac{y}{\delta_T}\right)^4 - \frac{4}{3}\left(\frac{y}{\delta_T}\right) + 1\right]$

At this point, the temperature profile within the thermal boundary layer depends on f(x) and $\delta_T(x)$, e.g. the temperature at the solid-fluid interface and thermal boundary layer thickness, respectively. An expression for the determination of f(x) is obtained by matching the solid and fluid heat flux expressions at the interface as follows:

The derivative of the fluid temperature profile at the interface along y = 0 is

$$\left.\frac{\partial t}{\partial y}\right|_{y=0} = f(x) \times \left(-c_q\right) \frac{1}{\delta_T} \tag{32}$$

where constant c_q depends on the degree of the polynomial approximation considered as reported on Tab. (1).

On the other hand, the derivative of the solid temperature profile at the interface yields

$$\left.\frac{\partial \theta(x, y)}{\partial y}\right|_{y=0} = \sum_{m=1}^{\infty} \frac{X_m(x)}{N_m} \left[\int_0^1 X_m(x)\, f(x)\, dx - \frac{\bar{g}_m}{\beta_m^{\ 2}} \right] tgh(\beta_m A)\beta_m \tag{33}$$

Recalling the interfacial condition, Eq. (5b), and substituting for Eq. (32) and Eq. (33), an expression for f(x) is obtained in the form

$$f(x) = -\frac{k^* \delta_T}{c_q} \sum_{m=1}^{\infty} \frac{X_m(x)}{N_m} \beta_m \left(C_m - \frac{\bar{g}_m}{\beta_m^2} \right) tgh(\beta_m A) \tag{34}$$

where C_m is defined as

$$C_m = \int_0^1 X_m(x)\, f(x)\, dx = \bar{\theta}_m(0) \tag{35}$$

Now, Eq. (34) is multiplied by X_j (x) on both sides and the resulting expression is integrated over x:

$$C_j = -\frac{k^*}{c_q} \sum_{m=1}^{\infty} \frac{\beta_m}{N_m} \left(C_m - \frac{\bar{g}_m}{\beta_m^2} \right) tgh(\beta_m A) \int_0^1 X_j(x) X_m(x) \delta_T(x) dx \tag{36}$$

The thermal boundary layer thickness δ_T is related to hydrodynamic boundary layer thickness δ_H through the well-known expression

$$\Delta(x) = \frac{\delta_T(x)}{\delta_H(x)} \tag{37}$$

and Eq. (36) can be rewritten in the form

$$\sum_{m=1}^{\infty} \delta_{jm} C_m + \frac{k^*}{c_q} \sum_{m=1}^{\infty} D_{jm} E_m C_m = \frac{k^*}{c_q} \sum_{m=1}^{\infty} \frac{\bar{g}_m}{\beta_m^2} D_{jm} E_m \tag{38}$$

where,

$$D_{jm} = \int_0^1 X_j(x) X_m(x) \Delta(x) \delta_H(x) dx \,, \quad E_m = \frac{\beta_m}{N_m} tgh(\beta_m A) \tag{39a,b}$$

and δ_{jm} is the Kroenecker delta.

In order to evaluate the fluid temperature distribution, the integral energy equation would have to be determined and a temperature approximation would have to be chosen. At this point, a coupled system for the two unknowns, namely $\Delta(x)$ and f(x), would be reached. Moreover, this system exhibits an interesting mixed feature as it is represented by an ordinary differential equation for $\Delta(x)$ and an algebraic equation for f(x). While solutions to this problem are feasible through standard mathematical subroutines, such as routine DASPG from the IMSL package, such solutions usually have poor convergence behavior due to their mixed nature and a discussion to the related solution strategies can be found at Santos (2003).

At this point, a further hypothesis is adopted. Actually, from the well know thermal boundary layer problem for the flat plate geometry, it can be shown that for this specific case, $\Delta(x)$ is independent of x and varies only with the Prandtl number. A literature review suggests that this conclusion can also be applied to other situations with a reasonably degree of accuracy (Lienhard, 2003). Of course, one would expect a strong axial dependency at regions close to the leading and trailing edges and also in situations involving blocks with aspect ratio around a unitary value. Keeping in mind that the main focus of this work is to predict the thermal behavior of electronic circuit boards, whose aspect ratios are usually smaller than 1/8, it would appear that taking $\Delta(x)$ to be independent of x is a reasonable assumption. Therefore, in the present analysis it is assumed that:

$$\Delta(x) = 0.9747\, Pr^{-1/3} \tag{40}$$

Now system (38) can be truncated to a sufficiently high order, N, and the desired coefficients C_m can be determined by solving this algebraic problem with the aid of subroutine LSARG, from the IMSL package. Since coefficients C_m are identical to the expression for $\bar{\theta}_m(0)$, Eq. (35), an expression for the interfacial temperature f(x) is obtained in terms of the following fast converging series expansion

$$f(x) = \sum_{m=1}^{\infty} \frac{X_m(x)}{N_m} C_m \tag{41}$$

Moreover, the temperature within the block, Eq. (26), and the fluid temperature field for the various polynomial orders given in Tab. (1) can also be obtained. Also of interest are the dimensionless local and average heat transfer rates at the interface, which are computed by:

$$\dot{q}(x) = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = \frac{c_q}{k^*} \frac{f(x)}{\Delta\delta_H(x)}, \quad \bar{\dot{q}} = \int_0^1 \dot{q}(x)dx = \frac{c_q}{k^*} \sum_{m=1}^{\infty} \frac{C_m}{N_m} \int_0^1 \frac{X_m(x)}{\Delta\delta_H(x)} dx \tag{42, 43}$$

4. Results and Discussion

In order to explore the mathematical solution of the conjugated heat transfer problem subjected to a cooling stream as outlined above, this section presents some results based on the evaluation of quantities of practical interested such as local and average heat transfer flux at the interfacial surface together with its temperature distribution. Also desirable are the contour plots of the temperature distribution inside the heated block and of the fluid flow.

It is clear from the previous analysis that the above-mentioned quantities depend upon various parameters such as the thermal conductivity ratio, aspect ratio, Reynolds and Prandtl numbers, and the thermal boundary conditions at the exit wall. Therefore, it is interesting to establish a discussion based on typical situations related to cooling of electronic equipment. Experimental values of k* range between 38 and 384 when one considers the flow of air at 25°C and the solid block made of either epoxy or ceramic materials, respectively (Wang and Saulnier, 1993). So, in order to explore a broad interval, we considered a k* raging from 40 to 500 in our computations. As for the Reynolds number in this application, typical values for the air stream are in the vicinity of 1.5 m/s and as far as the kinematic viscosity is concerned the air temperature ranges from 25°C to 90°C. The characteristic length for a single processor is about 10 mm and for a whole motherboard is typically around 244 mm. These values suggest that realistic values for Reynolds numbers are found to be between 1.0×10^3 (processor) and 2.5×10^4 (motherboard). In all cases here reported, the Prandtl number was fixed at 0.72. As for the aspect ratio, a reasonable value in this case was found to be A = 1/128.

In order to adhere to the space limitations, results reported here are concerned only to the fourth order approximation for both the temperature and velocity profiles. A full account of the other polynomial approximations can be found in Santos (2003).

Table (2) and Tab. (3) present the wall temperature and heat fluxes at the interface for two values of the Reynolds number and k* and for the two boundary conditions at the exit wall. As for the isothermal situation depicted on Tab. (2), these results indicate that f(x) starts at its prescribed value, attains a peak value, and decreases toward the imposed isothermal boundary condition. Accordingly, the wall heat flux exhibits a similar distribution. This behavior is expected because the heat generation inside the block is taken to be constant. When the Reynolds number is shifted from 1.0×10^3 to 2.5×10^4 the temperature field exhibits a decreasing pattern while the wall heat flux is noticeably higher. Also, the peak value of the interfacial temperature distribution is displaced to the right. Of course, this trend is also expected due to the more accentuated convection effects over the board.

Table 2 - Dimensionless interfacial temperature and heat flux distributions for a fourth-degree polynomial temperature and velocity approximation. Isothermal surface at x = 1, A= 1/128.

x	$Re = 1.0 \times 10^3$				$Re = 2.5 \times 10^4$			
	k* = 50		k* = 500		k* = 50		k* = 500	
	f(x)	$\dot{q}(x)$	f(x)	$\dot{q}(x)$	f(x)	$\dot{q}(x)$	f(x)	$\dot{q}(x)$
0.1	0.155E+01	0.651E+00	0.452E+01	0.190E+00	0.418E+00	0.878E+00	0.243E+01	0.511E+00
0.2	0.262E+01	0.779E+00	0.800E+01	0.238E+00	0.643E+00	0.955E+00	0.421E+01	0.626E+00
0.3	0.340E+01	0.826E+00	0.105E+02	0.255E+00	0.805E+00	0.977E+00	0.551E+01	0.668E+00
0.4	0.396E+01	0.832E+00	0.120E+02	0.253E+00	0.936E+00	0.983E+00	0.638E+01	0.670E+00
0.5	0.430E+01	0.808E+00	0.126E+02	0.237E+00	0.105E+01	0.982E+00	0.682E+01	0.641E+00
0.6	0.439E+01	0.753E+00	0.122E+02	0.210E+00	0.113E+01	0.969E+00	0.680E+01	0.583E+00
0.7	0.416E+01	0.660E+00	0.108E+02	0.172E+00	0.117E+01	0.931E+00	0.624E+01	0.496E+00
0.8	0.349E+01	0.518E+00	0.837E+01	0.124E+00	0.112E+01	0.833E+00	0.504E+01	0.374E+00
0.9	0.219E+01	0.307E+00	0.478E+01	0.670E-01	0.841E+00	0.589E+00	0.303E+01	0.212E+00
1.0	0.108E-14	0.144E-15	0.212E-14	0.281E-16	0.520E-15	0.346E-15	0.142E-14	0.942E-16
	$\bar{\dot{q}}$ = 0.633E+00		$\bar{\dot{q}}$ = 0.180E+00		$\bar{\dot{q}}$ = 0.843E+00		$\bar{\dot{q}}$ = 0.492E+00	

The effect of solid to fluid conductivity ratio in the cooling process can also be inferred from Tab.(2). As k* increases from 50 to 500 for a fixed Reynolds number, the interfacial temperature distribution exhibits a significant increase with its peak value leaning toward the left. Due to the higher conduction process within the solid, the convective cooling process is markedly reduced as shown by the values of $\dot{q}(x)$ and $\bar{q}$.

Table (3) carry out the same study but now taking into account an adiabatic exit wall condition. As for the insulated situation here presented, the results indicate that f(x) increases monotonically toward a peak value at x = 1. On the other hand, the local heat flux exhibits a pattern similar to the previously studied case but here the magnitude of both local and average heat flux are higher. As a matter of fact, the insulated wall at boundary x = 1 inhibits the conduction process within the solid and thus a higher interfacial heat flux is naturally expected. Also, because of the present insulated condition, higher values for the temperature field are predictable when compared to the isothermal boundary condition.

Table 3 - Dimensionless interfacial temperature and heat flux distributions for a fourth-degree polynomial temperature and velocity approximation. Insulated surface at x = 1; A = 1/128.

x	$Re = 1.0 \times 10^3$				$Re = 2.5 \times 10^4$			
	k* = 50		k* = 500		k* = 50		k* = 500	
	f(x)	$\dot{q}(x)$	f(x)	$\dot{q}(x)$	f(x)	$\dot{q}(x)$	f(x)	$\dot{q}(x)$
0.1	0.159E+01	0.668E+00	0.679E+01	0.285E+00	0.418E+00	0.878E+00	0.266E+01	0.559E+00
0.2	0.272E+01	0.809E+00	0.127E+02	0.376E+00	0.643E+00	0.955E+00	0.473E+01	0.703E+00
0.3	0.361E+01	0.875E+00	0.177E+02	0.430E+00	0.806E+00	0.978E+00	0.641E+01	0.778E+00
0.4	0.433E+01	0.909E+00	0.221E+02	0.463E+00	0.939E+00	0.986E+00	0.781E+01	0.820E+00
0.5	0.493E+01	0.926E+00	0.257E+02	0.483E+00	0.105E+01	0.990E+00	0.897E+01	0.843E+00
0.6	0.543E+01	0.932E+00	0.287E+02	0.492E+00	0.116E+01	0.992E+00	0.993E+01	0.852E+00
0.7	0.585E+01	0.929E+00	0.310E+02	0.493E+00	0.125E+01	0.991E+00	0.107E+02	0.850E+00
0.8	0.617E+01	0.917E+00	0.327E+02	0.486E+00	0.133E+01	0.987E+00	0.113E+02	0.838E+00
0.9	0.639E+01	0.895E+00	0.337E+02	0.472E+00	0.139E+01	0.974E+00	0.116E+02	0.816E+00
1.0	0.647E+01	0.860E+00	0.341E+02	0.453E+00	0.142E+01	0.943E+00	0.118E+02	0.783E+00
	$\bar{q}$ = 0.846E+00		$\bar{q}$ = 0.427E+00		$\bar{q}$ = 0.947E+00		$\bar{q}$ = 0.758E+00	

Figure (1) and Fig.(2) depict the temperature contours for the case of isothermal and adiabatic boundary conditions, respectively.

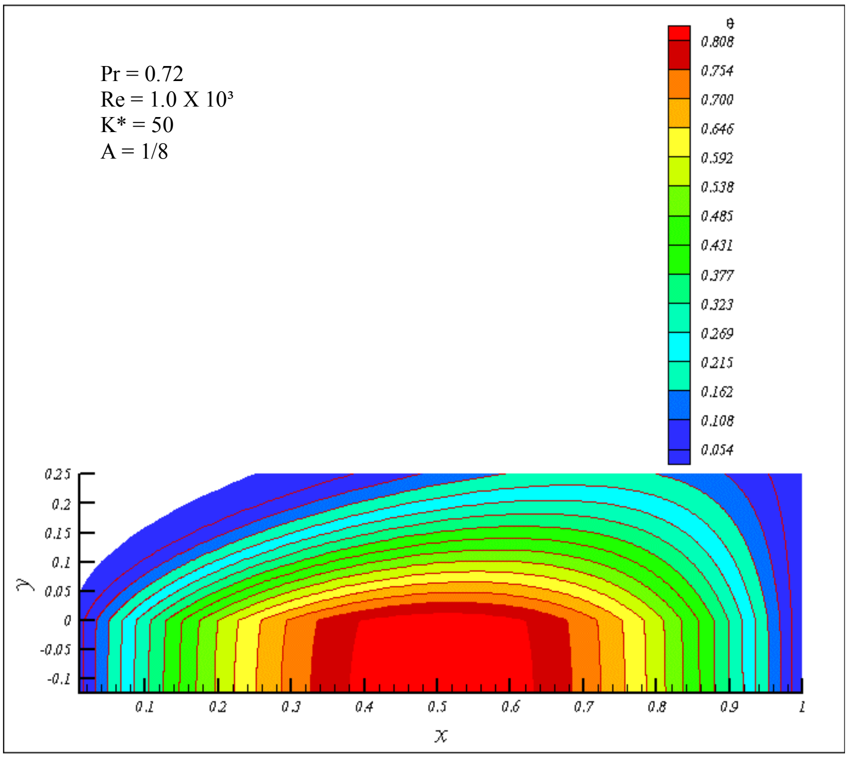

Figure 1 – Block and fluid temperature contours for a fourth-degree polynomial temperature and velocity approximations. Isothermal surface condition at x = 1, k* = 50, A = 1/8, $Re = 1.0 \times 10^3$.

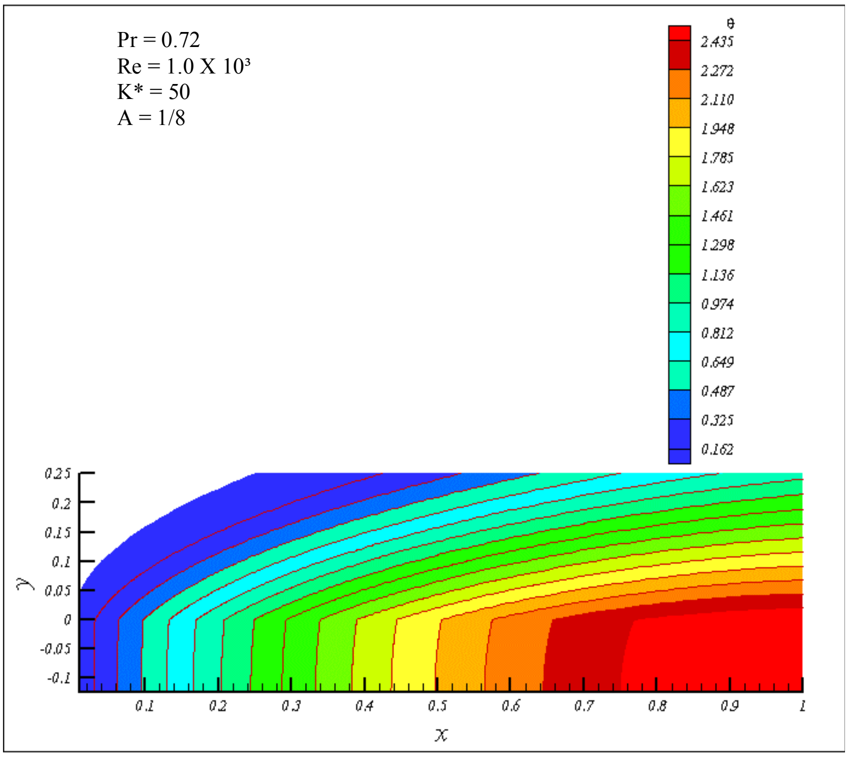

Figure 2 – Block and fluid temperature contours for a fourth-degree polynomial temperature and velocity approximations. Insulated surface condition at x = 1, k* = 50, A = 1/8, Re = 1.0×10^3.

A comparison between these two results indicate the influence of the boundary condition in the context of this problem, as higher temperature levels are again noticeably for the insulated situation. Moreover, for the case of Re = 1.0×10^3 the temperature contours appear to be slightly distorted due to the not so efficient cooling effect associated to a low Reynolds number. Also for this specific case, one notices that the "hot spots" are located in the center of the block. In contrast, the insulated wall in Fig. (2) displaces these "hot spots" to the right of the block while the isotherms are markedly affected by the more efficient heat removal process due to a higher Reynolds number.

As a conclusion, an approximate but yet accurate methodology for the assessment of temperature levels and heat transfer removal in electronic equipment was here advanced. This methodology is based on the analysis of a conjugate problem that represents the interaction between the circuit board or the processor and a cooling air stream. The mathematical formulation employs a two dimensional differential equation for the solid material together with polynomial approximations for the fluid flow and thus an analytical solution is obtained. Numerical simulations show that the worst-case scenario for cooling of electronic equipment is represented by an insulated exit wall condition where the levels of temperature are higher than the isothermal situation and a markedly higher wall heat flux to be dissipated to the cooling fluid is demanded.

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