

FLOW IN ANNULI WITH VARING EXCENTRICITY

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Abstract. *Helical flow in annular space occurs in drilling operation of oil and gas wells. The correct predition of the flow of the drilling mud in the annular space between the wellbore wall the the drill pipe is essential to determine the variation in the mud pressure within the wellbore, the frictional pressure drop and the efficiency of the transport of the rock drill cuttings. A complete analysis of this situation is extremely complex; the inner cylinder is usualy rotating, the wellbore wall will depart significantly from cylindrical, during driling operation the drill pipe is eccentric, and the eccentricity varies with position along the well. A complete analysis of this situation would require the solution of the three-dimensional momentum equation and would be computationally expensive and complex. Models available in the literature to study this situation do consider the rotation of the inner cylinder but assume the position of the inner and outer cilinders fixed, i.e. they neglect the variation of the eccentricity along the length of the well, and assume the flow to be well developed. This approximation leads to a two-dimensional model to determine the three components of the velocity field in a cross-section of the annulus. The resulting differential equations have to be solved by some numerical method.*

The model presented in this work takes into account the variation of the eccentricity along the well; a more appropriate description of the geometric configuration of directional wells. As a consequence, the velocity field varies along the well length and the resulting flow model is three-dimensional. Lubrication theory is used to simplify the governing equations into a non-linear, two-dimensional Poisson Equation that describes the pressure field. The results show the effect of varying eccentricity on the friction factor, maximum and mininum axial velocity in each cross-section, and the presence of azimuthal flow even when the inner cylinder is not rotating.

Keywords. Drilling mud flow, eccentric annular space, lubrication approximation.

1. Introduction

Drilling muds are pumped through the annular space between the drillpipe and the wellbore wall during drilling operation. The main functions of the mud are to stabilize the well and prevent its wall to collapse, to transport rock cuttings, to cool the drillbit and to lubricate the drill pipe. In order to predict the pressure inside the wellbore, the frictional pressure drop of the flow and the efficiency of the transport of rock drill cutting, the flow field in the annular space has to be determined. A complete analysis of this situation is extremely complex, the drill pipe is eccentric and the eccentricity varies along the length of the well, it can be rotating and the well walls may depart significantly from cylindrical. The flow is three-dimensional and the solution of the differential equations that govern the flow is complex and computationally expensive.

Simplified analysis of this situation are available in the literature. The main goal is usualy to determine the frictional pressure loss as a function of flow rate and geometry of the annular space. Examples of such analysis include the work of AbdulMajeed (1996), Jensen and Sharma (1987) and Langlinais et al. (1985). Even the idealized models are far from simple. Escudier et al. (2002) presented an overview of all the work in this area to date. The first analysis were limited to flow inside concentric annulus without rotation of either cylindrical surfaces. The next step was the inclusion of the inner cylinder rotation in the models. Only recentely the axial flow through an eccentric annulus with inner cylinder rotation was analyzed. One limitation of the available models is that they assume the flow to be fully developed, i.e. they do not consider the variation of the eccentricity along the well, which always occurs during drilling operation. In the work of Escudier et al. (2000 and 2002), the resulting set of differential equations that describe the velocity components in the cross section of the flow was solved by the finite volume method.

In this work the three-dimensional differential equations that describe the flow of a Newtonian liquid inside an eccentric annular space with varying eccentriciy are simplified using lubrication theory. The solution of the flow field is much simpler and faster when compared to the fully developed models available in the literature. The lubrication model proposed here is able to determine the effect of the variation of the eccentricity along the well on the velocity profile and pressure drop. The limitation and accuracy of the lubrication model is tested by comparing the results of a constant eccentricity annulus to those presented by Escudier (2000) using a more complex and computationally expensive model. The lubrication model is then applied to study the flow through an annular space with a sinousoidal variation of the eccentricity of the inner cylinder.

2. Mathematical formulation

The geometry of an annular space of varying eccentricity is sketched in Fig.1. The radius of the inner and outer cylinder are R_i and R_0, respectively. The coordinate system is attached to the center of the inner cylinder and the eccentricity in each cross section e varies along the length of the annulus. The radial gap R between the outer and inner cylinder walls is a function of the axial and azimuthal coordinates:

$$R(z,q) = e(z)\cos q + \sqrt{R_0^2 - e^2(z)\sin^2 q} \tag{1}$$

In this work, the function that describes the eccentricity along the length of the well is prescribed. In a more sofisticated model, it can be made a function of the dynamics of the rotation of the drillstring and liquid interaction.

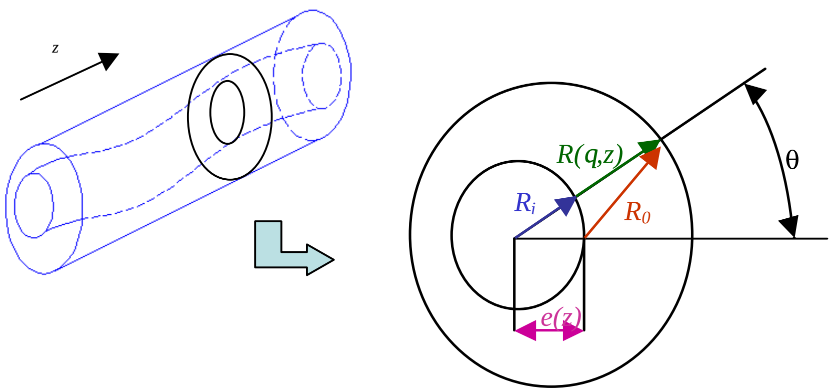

Figure 1: Configuration of annular space with eccentricity varying along the axial direction.

2.1. Governing Equations

The flow in an annular space with varying eccentricity is three-dimensional. If u, v and w are the axial, radial and tangential velocity components, respectively, the governing equations are written as:

$$\begin{aligned}
&r\left\{u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} + \frac{w}{r}\frac{\partial u}{\partial q}\right\} = -\frac{\partial p}{\partial z} + rg_z + m\left[\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial q^2}\right]\\
&r\left\{u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} + \frac{w}{r}\frac{\partial v}{\partial q} - \frac{w^2}{r}\right\} = -\frac{\partial p}{\partial r} + rg_r + m\left[\frac{\partial^2 v}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right) + \frac{1}{r^2}\frac{\partial^2 v}{\partial q^2} - \frac{2}{r^2}\frac{\partial w}{\partial q}\right]\\
&r\left\{u\frac{\partial w}{\partial z} + v\frac{\partial w}{\partial r} + \frac{w}{r}\frac{\partial w}{\partial q} + \frac{vw}{r}\right\} = -\frac{1}{r}\frac{\partial p}{\partial q} + rg_q + m\left[\frac{\partial^2 w}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rw)\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial q^2} + \frac{2}{r^2}\frac{\partial v}{\partial q}\right]
\end{aligned} \tag{2}$$

To avoid solving the system of coupled three-dimensional differential equations, dimensional analysis is used to eliminate some of the terms of the equations. The procedure is generaly known as lubrication approximation and it is summarized in the following section.

2.2. Lubrication Approximation

In the configuration analyzed here, the variation of the radial coordinate is much smaller than the other two coordinates and the axial and tangential components are greater than the radial component of the velocity vector, i.e.

$\Delta z, \Delta q >> \Delta r$ and $u, w >> v$.

Consequentely, the inertial terms are negligible and the derivatives with respect to the radial direction are much larger than the others:

$$\frac{\partial^2 u}{\partial r^2} >> \frac{\partial^2 u}{\partial z^2}, \frac{\partial^2 u}{\partial q^2} \quad \text{e} \quad \frac{\partial^2 w}{\partial r^2} >> \frac{\partial^2 w}{\partial z^2}, \frac{\partial^2 w}{\partial q^2} \ .$$

If the appropriate terms of the Navier-Stokes equation are neglected, the system of differential equations becomes:

$$\begin{aligned} 0 &= -\frac{\partial p}{\partial z} + rg_z + m\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \\ 0 &= -\frac{\partial p}{\partial r} + rg_r \\ 0 &= -\frac{1}{r}\frac{\partial p}{\partial q} + rg_q + m\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rw)\right)\right] \end{aligned} \quad . \qquad (3)$$

The boundary conditions are of zero-velocity at outer wall and prescribed tangential velocity at the inner rotating cylinder wall. If the gravity terms are neglected, the radial momentum balance is reduced to zero pressure gradient on that direction, e.g.

$$p = p(z,q)$$

The other two equations can be integrated with the appropriate boundary conditions to yield the velocity profile in the axial and tangential direction. Each velocity component varies with the axial, tangential and radial direction, as shown in the formulas. The dependence on the axial and tangential coordinates is implicitly defined through the expression for the radial gap, eq.(1), and the pressure gradient:

$$u(z,r,q) = \frac{R_i^2}{4m}\left(-\frac{\partial p}{\partial z}\right)\left[1 - \left(\frac{r}{R_i}\right)^2 + \frac{\left(R(z,q)/R_i\right)^2 - 1}{\ln\left(R(z,q)/R_i\right)}\ln\left(\frac{r}{R_i}\right)\right] \qquad (4)$$

$$\begin{aligned} w(z,r,q) = {} & \frac{\Omega_i R_i^2}{R^2 - R_i^2}\left(\frac{R^2}{r} - r\right) + \\ & \frac{R_i^2}{2m}\left(\frac{\partial p}{\partial q}\right)\left\{\frac{r}{R_i}\left(\ln r - 1/2\right) - \frac{R_i}{r}\left(\ln R_i - 1/2\right) + \left(\frac{r/R_i - R_i/r}{R^2 - R_i^2}\right)\left[R_i^2\left(\ln R_i - 1/2\right) - R^2\left(\ln R - 1/2\right)\right]\right\} \end{aligned} \qquad (5)$$

Ω_i is the angular velocity of the inner cylinder.

Up to this point, the pressure field $p(z,q)$ is still unknow. In order to evaluate it, the integral of the continuity equation is used, as shown bellow:

$$\int_{Ri}^{R}\left\{\frac{1}{r}\frac{\partial w}{\partial q} + \frac{\partial u}{\partial z}\right\}dr = 0 \cdot$$

Each term of the equation can be evaluated as a function of a derivative of the integral along the radial direction of the velocity profiles presented in eqs. (4) and (5).

$$\int_{Ri}^{R}\frac{\partial u}{\partial z}dr = \frac{d}{dz}\int_{Ri}^{R}u\,dr - \underbrace{u(R_0)}_{0}\frac{\partial R}{\partial z} + \underbrace{u(R_i)}_{0}\frac{\partial R_i}{\partial z} = \frac{d}{dz}\int_{Ri}^{R}u(z,r,q)\,dr$$

$$\int_{Ri}^{R}\frac{1}{r}\frac{\partial w}{\partial q}dr=\int_{Ri}^{R}\frac{\partial}{\partial q}\left[\frac{w}{r}\right]dr=\frac{d}{dq}\int_{Ri}^{R}\frac{w}{r}dr-\frac{1}{r}\underbrace{w(R_0)}_{0}\frac{\partial R}{\partial z}+\frac{1}{r}w(R_i)\underbrace{\frac{\partial R_i}{\partial z}}_{0}=\frac{d}{dq}\int_{Ri}^{R}\frac{w(z,r,q)}{r}dr$$

After evaluating the integrals of the velocity profiles, the integral continuity equation becomes a non-linear Poison equation for the pressure field:

$$\frac{\partial}{\partial q}\left\{F_1(z,q)\frac{\partial p}{\partial q}\right\}+\frac{\partial}{\partial r}\left\{F_2(z,q)\frac{\partial p}{\partial r}\right\}=\frac{\partial}{\partial q}F_0(z,q)\cdot \tag{6}$$

The dependence of $F_{0,}\ F_1$ and F_2 on the axial and tangential coordinates is through the definition of the radial gap between the inner and outer cylinder surface R, given by eq.(1):

$$F_0(z,q)=-\frac{\Omega_i R_i^2}{R^2-R_i^2}\left[R\left(2-\frac{R}{R_i}\right)-R_i\right]$$

$$F_1(z,q)=\frac{R_i}{2m}\left\{\frac{R(z,q)}{R_i}\left(\ln R-3/2\right)-\left(\ln R_i-3/2\right)+\left(\ln R_i-1/2\right)\left(R_i/R-1\right)+\frac{R_i^2\left(\ln R_i-1/2\right)-R^2\left(\ln R-1/2\right)}{R^2-R_i^2}\left(R/R_i+R_i/R-2\right)\right\}$$

$$F_2(z,q)=\left(-\frac{R_i^2}{4m}\right)\left\{R-R_i-\frac{R^3-R_i^3}{3R_i^2}+\frac{\left(R/R_i\right)^2-1}{\ln\left(R/R_i\right)}\left[R_i+R\left(\ln\left(R/R_i\right)-1\right)\right]\right\}$$

The domain of integration is $0\le z\le L$ and $0\le q\le 2p$. The appropriate boundary conditions are:

$p(z=0)=P_{IN}$; $p(z=L)=P_{OUT}$; $p(q=0)=p(q=2p)$; and $\frac{\partial p}{\partial q}(q=0)=0$.

The variables are written in dimensionless form as:

$u^*=\frac{u}{\overline{U}}$; $v^*=\frac{v}{\overline{U}}$; $w^*=\frac{w}{\overline{U}}$ and $p^*=\frac{p}{\frac{m\overline{U}}{2(R_0-R_i)}}$. Where $\overline{U}$ is the average axial velocity.

The relevant dimensionless parameters are:

- Reynolds Number: $\mathrm{Re}=\frac{r\overline{U}\,2(R_0-R_i)}{m}$.
- Taylor Number: $Ta=\left(\frac{R_0}{R_i}-1\right)\left(\frac{r\Omega_i R_i(R_0-R_i)}{m}\right)^2$
- Friction Factor: $f=\frac{\Delta p}{L}\frac{R_0-R_i}{r\overline{U}^2}$
- Radius ratio: $k=\frac{R_i}{R_0}$
- Eccentricity parameter: $e=\frac{e}{R_0-R_i}$

3. Numerical approach

The differential equation (6) was solved by the finite difference method (central difference). The functions F_0, F_1 and F_2 were evaluated at each node position in order to calculate the appropriate derivatives. The resulting system of equation is sparse. The matrix was stored in Compress Sparse Row Format to save memory and processing time.

Figure 2 shows the structure of the matrix. For the cases presented here, the number of nodes was 4100. The banded diagonal structure of the matrix is lost due to the periodic boundary condition on the tangential direction.

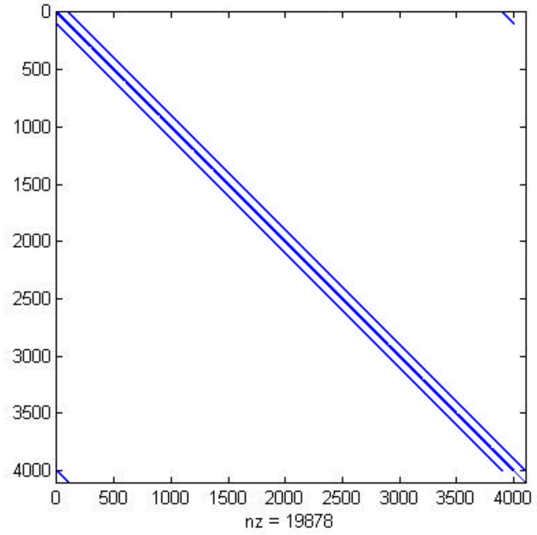

Figure 2: Structure of the sparse matrix.

The solution algorithm was implemented in MatLab enviroment. The computation of each flow state took less than 50 seconds in a Pentium III, 532 MHz PC.

4. Results

The results presented here are restricted to the case of $\Omega_i = 0$, i.e. the inner cylinder is not rotating.

4.1. Constant Eccentricity

The first results were obtained for annular spaces with constant eccentricity. The goal was to evaluate the accuracy of the lubrication approximation used in this work. The computational time required to obtain each flow state with the lubrication model was orders of magnitude smaller than that required to compute the flow field using the models available in the literature.

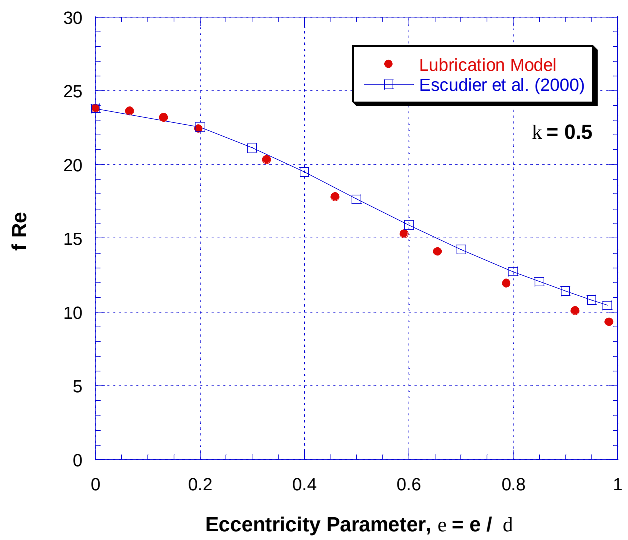

Figure 3: $f \times \mathrm{Re}$ as a function of eccentricity parameter predicted by the lubrication model is complete two-dimensional solution.

The product of the friction factor and Reynolds number $f \times \mathrm{Re}$ predicted by the lubrication model is presented as a function of the eccentricity in Fig.3 at $k = 0.5$. The plot also shows the results presented by Escudier et al. (2000).

The agreement is excellent over the entire range of eccentricity. The maximum discrepancy occurs at high eccentricity, and it is less than 10%.

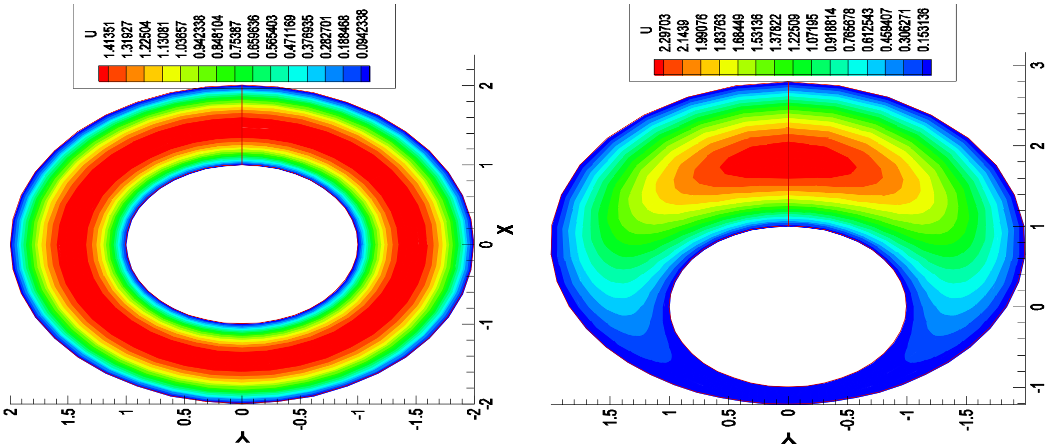

Figure 4: Axial velocity distribution in an annular space at different eccentricity. $\mathrm{Re} = 100$. (a) $e = 0$; (b) $e = 0.787$.

The value of $f \times \mathrm{Re}$ falls as eccentricity rises, i.e. at a fixed pressure gradient the flow rate increases as the inner cylinder moves away from the center of the outer cylinder. This can be explained by analyzing the axial velocity distribution at two values of eccentricity ($e = 0$ and $e = 0.787$), presented in Fig.4. When the two cylinders are concentric, the flow is axisymmetric, as expected. The maximum velocity is approximately 40% larger than the average velocity. As the inner cylinder moves away from the center, most of the flow occurs in the region of larger gap between the cylinder surfaces. The flow in the narrow gap region is very slow and the shear stress at the wall is small. The axial velocity distribution presented in Fig.4 agrees well to those reported by Escudier et al. (2000). The main conclusion from Figs. 3 and 4 is that the lubrication approximation can be used to obtain accurate predictions of the flow field at a much smaller computational cost.

4.2. Eccentricity varing along the flow direction

The effect of the variation of the eccentricity along the length of the well is analyzed by studying the flow through an annular space in which the variation of the position of the inner cylinder center along the axial direction is described by a sinousoidal function, as sketched in Fig.5. The cases reported here were at $k = 0.5$, $l / R_0 = 16$, $\mathrm{Re} = 100$, and different amplitude of the eccentricity function A. l is the wavelength of the sinousoidal function that describes the eccentricity of the inner cylinder. When the amplitude of the variation vanishes, the concentric annulus situation is recovered. In this particular case, the pressure only depends on the axial coordinate and the pressure gradient on the flow directin is constant,as shown in Fig.6.

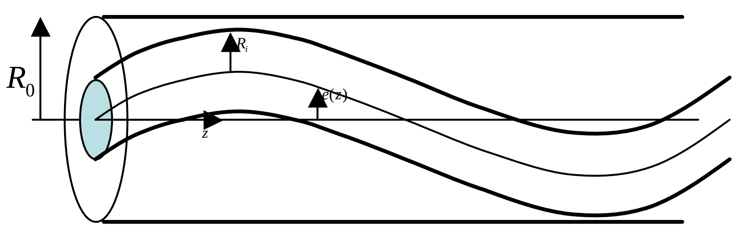

Figure 5: Sketch of annular space with the position of the center of the inner cylinder following a sinousoidal function.

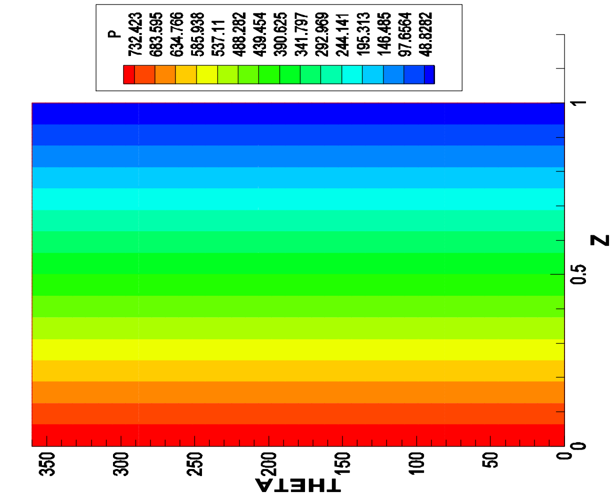

Figure 6: Pressure distribution for concentric annular space.

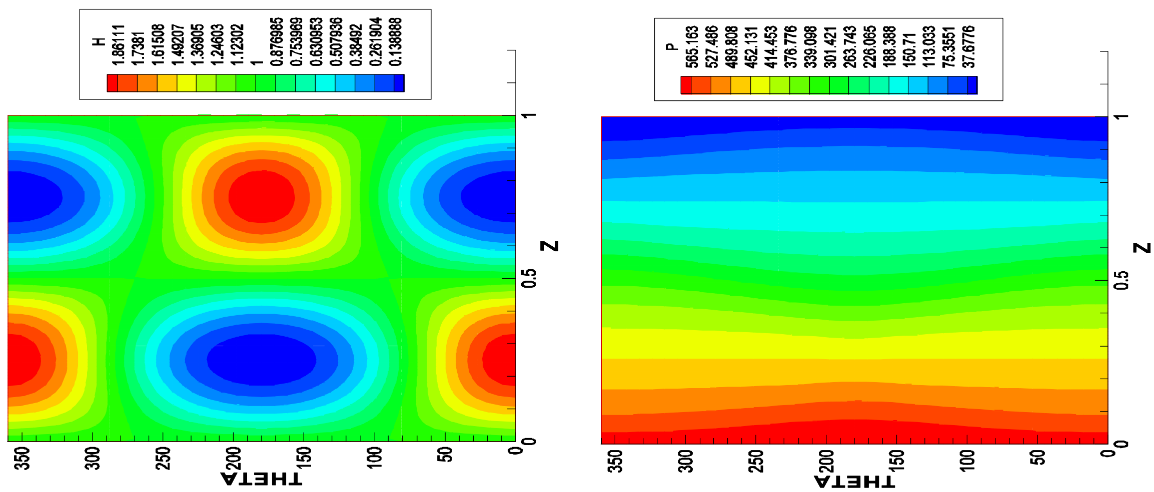

Figure 7: (a) radial gap and (b) pressure distribution for a sinousoidal variation of the eccentricity along the axial direction. $A/R_0 = 0.49$

The dimensionless gap between the two cylinders, $R(z,q)/R_0 - R_i$, is shown in Fig.7(a) at $A/R_0 = 0.49$. The corresponding pressure field is presented in Fig.7(b). The gap between the two cylinders varies both in the axial and tangential direction. As expected, the flow is not axisymmetric and the pressure gradient in the flow direction is not constant. Because there is a pressure gradient in the azimuthal direction at a a fixed axial position, there is a pressure driven tangential flow. The axial and tangential velocity distribution at different cross sections of the annular space are shown in Figures 8, 9 and 10.

Figure 8 shows a cross section of the annular space at $z/L = 0.25$. This cross section is where the minimum distance between the two walls occurs. The local eccentricity is $e = 0.98$, the walls are almost touching each other in one side of the annular space. The maximum axial velocity is approximately $u_{\max} = 2.75 \times \overline{U}$. The pressure distribution in the cross section is shown in Fig.8(b). There is a very weak pressure gradient in the azimuthal direction, but enough to drive liquid from the narrow gap to the wide gap region. At $z/L = 0.35$, the azimuthal flow changes direction, as shown in Fig.9. The higher pressure in the cross section occurs in the wide gap region and drives the flow towards the narrow gap side. This occurs because the wide gap region is converging in the flow direction.

Figure 10 shows the cross section at $z/L = 0.5$. The local eccentricity is zero, i.e. the cylinders are concentric. Because of the variation in the main flow direction, the flow is not axisymmetric, i.e. $u = u(z,q)$, and there is a tangential flow. The maximum axial velocity is approximately $u_{\max} = 1.54 \times \overline{U}$ (in well developed flow, it is approximately $u_{\max} = 1.4 \times \overline{U}$. The flow fields presented in Figs. 8, 9 and 10 illustrate the importance of including eccentricity variation in the model to describe flows in wells.

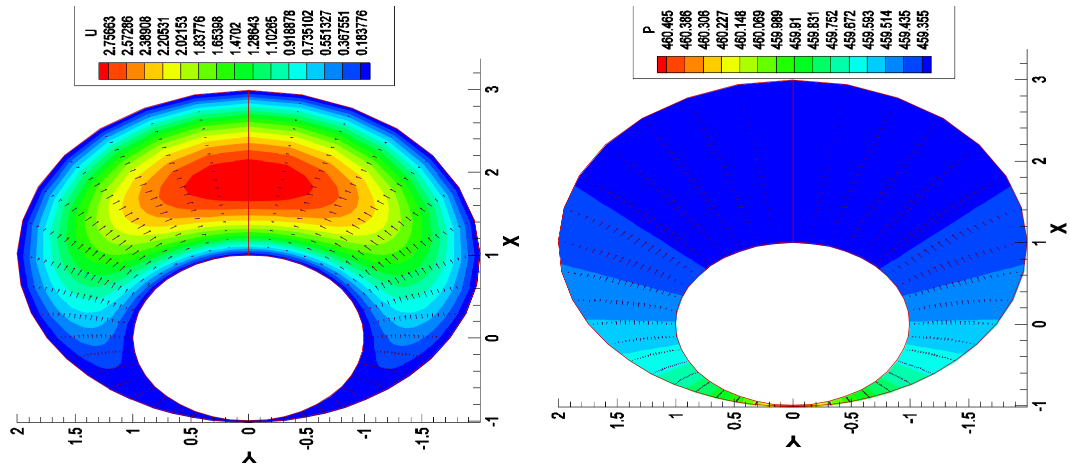

Figure 8: Velocity field at $z / L = 0.25$. Amplitude of the sinousoidal variation of the eccentricity is $A / R_0 = 0.49$.

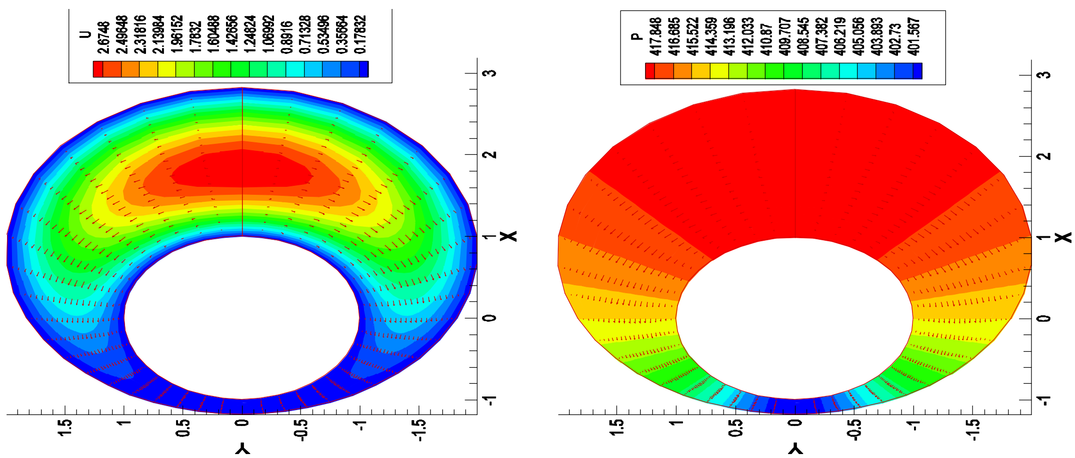

Figure 9: Velocity field at $z / L = 0.35$. Amplitude of the sinousoidal variation of the eccentricity is $A / R_0 = 0.49$.

The friction factor as a function of the amplitude of the sinousoidal variation of the eccentricity is presented in Fig.11. It decays as the amplitude of the variation rises.

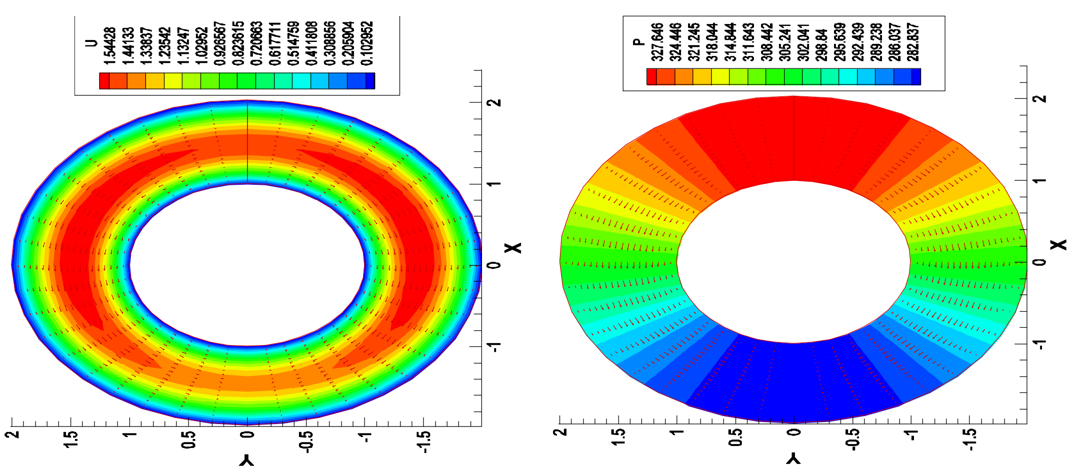

Figure 10: Velocity field at $z / L = 0.5$. Amplitude of the sinousoidal variation of the eccentricity is $A / R_0 = 0.49$.

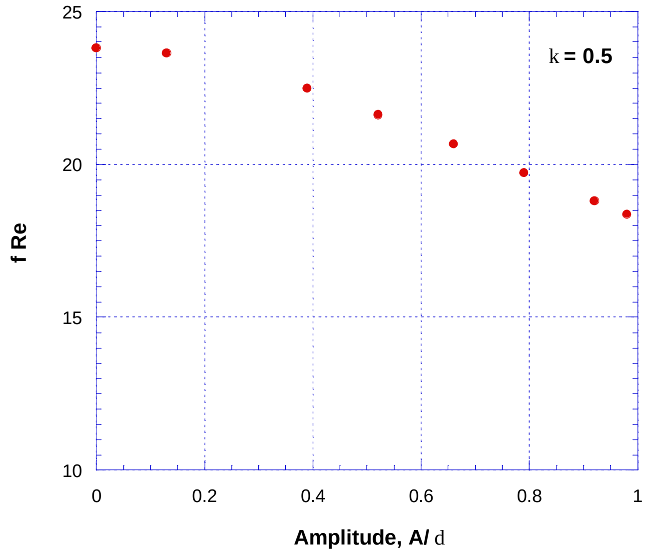

Figure 11: Friction factor as a function of the amplitude of the sinousoidal variation of the eccentricity along the main flow direction.

5. Final comments

Lubrication approximation was used to simplify the three-dimensional momentum equation that describes the flow in annular space with eccentricity varying along the axial direction. In the particular case of constant eccentricity, the proposed model is able to recover the results of more complex and computationally expensive well developed flow models available in the literature.

The results presented exemplified the effects of the variation of the eccentricity along the length of the annular space. It has to be consider in the analysis of the flow between the drilling pipe and the wellbore walls during drilling operations. Although the results were limited to the case of non rotating inner cylinder, the model does not make any restriction to the rotation of either walls. This model is currently being extended to include non Newtonian behavior of the flowing liquid and to study the displacement of one liquid by injecting another liquid. The goal is to develop a simulator for well flow analysis that can be used with small computational cost.

6. Acknowledgement

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