MIMO Modal Parameter Identification Under Stochastic Noise Influence

Roberto Pedro Amaro Baldeón
Departamento de Projeto Mecanico - UNICAMP - Caixa Postal 6122 CEP: 13.083-970 - Campinas - SP
amarobrp@fem.unicamp.br

Paulo Roberto Gardel Kurka
Departamento de Projeto Mecanico - UNICAMP - Caixa Postal 6122 CEP: 13.083-970 - Campinas - SP
kurka@fem.unicamp.br

Abstract: The vector autoregressive moving average with exogenous inputs (VARMAX) Model is used to identify Multiple Input- Multiple Output (MIMO) dynamical systems. This class of technique permits to model the presence of stochastic noise in data that permits better modal parameter identification. In order to estimate the parameters of the VARMAX model the Spliid's fast algorithm is used. The advantage of this algorithm is its high computational efficiency, which permits to deal with a great quantity of data. When there is a great quantity of data and the regulariy conditions are satisfied the results obtained are very similar to to the results obtained by the maximum likelihood technique. On the other hand, to determine the modal parameters the companion matrix is built with the autoregressive part of the VARMAX model. The performance of this method here discussed is presented by means of simulations using a three degrees of freedom mass-damping-stiffness vibrating system.

Key Words: VARMAX, MIMO, Modal Analysis, Companion Matrix

1. Introduction

Modal parameter identification can be done in the time domain or in the frequency domain. Frequency domain are widely used by experimentalist despite it gives satisfactory results only for cases where there are neither specific problems such as combination of significant noise and damping [1].

On the other hand, time series analysis is being used in modal analysis satisfactory when there is significant noise (Smail et all, 1999, Prevosto and Olagnon, 1991, Amauri, 1996, Saito and Yokota, 1996, Bazan, 1993). In Multiple Input – Multiple Output (MIMO) systems, it is common to find that the output Y_t is the result of a sum of controlled and stochastic inputs, (Fassois 2000, 2000, Petsounis, 2001, Spliid,1983). In this case a model that permits to model this stochastic term is needed. The vector autoregressive moving average with exogenous inputs (VARMAX) Model permits to model the stochastic noise presence in data due to the presence of the MA term which is the different with the VARX model. This model is very important in the time domain for modal analysis, it is used to identify MIMO dynamical systems (Fassois, 2000).

To estimate the parameters of the VARMAX model the Spliid's fast algorithm is introduced. This algorithm has two important characteristics: 1. High computational efficiency, which permits to deal with a great quantity of data. 2. It has an interesting property: when there is a great quantity of data and the regularity conditions are satisfied the results obtained by this one are very similar to the results obtained by the maximum likelihood technique (Spliid, 1983, Shumway and Stoffer, 2000). To determine the modal parameters of the mechanical system the companion matrix is built with the autoregressive part of the VARMAX model (Larbi And Lardies, 2000, Huang, 2001, Maiaet all 1987). The performance of this method here discussed is presented by means of simulations using three degrees of freedom mass-damping-stiffness vibrating system influenced by stochastic noise.

2. Multivariate autoregressive moving average with exogenous variables model (VARMAX)

Fassois (2000, 2000, Petsounis, 2001) shows that the VARMAX model can represent a mechanical system with stochastic and controlled inputs; the VARMAX model is represented by:

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=0}^{r-1} \beta_i X_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

Where X_t is $(m \times 1)$ vector of the controlled input, with $t = 1,...,T$

Y_t is $(k \times 1)$ response vector, with $t = 1,...,T$

ε_t is $(k \times 1)$ non observable white noise vector, independent of X_t, having $t = 1,...,T$, with zero mean and a covariance matrix Σ described as:

$$E\{\varepsilon\varepsilon'\} = \Sigma, \ \varepsilon \ (k \times T)$$

It can also be said that:

ϕ_i is a $(k \times k)$ matrix

β_i is a $(k \times m)$ matrix

θ_i is a $(k \times k)$ Matrix

p is the order of the autoregressive part.

q is the order of the moving average part.

r is the order of the variable exogenous part.

The model order for the case of acceleration measurements is: $pk = 2n$ for the autoregressive part, $r = p$ for the exogenous variable.

2.1 A Fast Algorithm for Parameter Estimation of the parameters of the VARMAX Model

A fast parameter estimation algorithm developed by Spliid (1983) is presented next.

Let Y be a matrix $T \times k$ of measured data:

$$Y = \begin{bmatrix} Y_{11} & \cdots & Y_{1T} \\ \vdots & \ddots & \\ Y_{T1} & \cdots & Y_{Tk} \end{bmatrix} = \{Y_{ij}\}, \ i = 1,..,T \quad j = 1,...,k$$

A residual matrix $\mathrm{E} = \{\varepsilon_{ij}\}, i = 1,..,T \quad j = 1,....,k$ and a regression matrix $X = \{X_{ij}\}, i = 1,..,T \quad j = 1,....,m$ are likely defined.

Finally the lagged matrices are defined as:

$$\begin{aligned} LaY &= (LY, L^2Y, ..., L^PY) \\ LaA &= (L\varepsilon, L^2\varepsilon, ..., L^q\varepsilon) \\ LaX &= (X, LX, ..., L^{r-1}X) \\ U &= (-LaA, LaY, LaX) \end{aligned}$$

where $L^sY_t = \{Y_{t-s,i}\}$, $i = 1,...,k \ \ t = 1,...,T$.

The parameters are ordered in one matrix of dimension $(kq + kp + kr) \times k$:

$$\delta = \left(\theta_1', \theta_2', ..., \theta_q', \phi_1', \phi_2', ..., \phi_1', \beta_0', \beta_2', ..., \beta_{r-1}'\right)'$$

The algorithm is initiated without the moving average term but with an increased number of autoregressive terms, $s = p + q$. It is important to mention that the algorithm does not require initial values.

The following steps are thus defined in Spliid's algorithm:

Step 0: Build the matrix $W = \left(LY, L^2Y, ..., L^sY, LaX\right)$. An estimated white noise signal ε, for iteration 0, is obtained by:

$$\hat{\varepsilon}(0) = Y - W(W^T W)^{-1} W^T Y$$

where $\hat{\varepsilon}(0) = \varepsilon$.

The first assumed values for the parameters matrix is $\hat{\delta}(0) = 0,\ j = 0$ and the process can evolve to step 2, after this initial assumption.

Step 1: recursively compute the residuals for t = 1,2,......,*T*:

$$\hat{\varepsilon}_t(j) = Y_t - \sum_{i=1}^{p} \hat{\phi}_i(j) Y_{t-i} - \sum_{i=0}^{r-1} \hat{\beta}_i(j) X_{t-i} + \sum_{i=1}^{q} \hat{\theta}_i(j) \hat{\varepsilon}_{t-i}$$

Step 2: Build $\hat{A}(j)$, $\hat{U}(j)$ and compute the new estimates via linear regression, by solving the equation:

$$\hat{U}^T(j)\hat{U}(j)\hat{\delta}(j+1) = \hat{U}^T Y$$

Step 3: If $\hat{\delta}(j+1) \neq \hat{\delta}(j)$, increment *j* by 1 and repeat steps 1 to 3. Stop if

$$\hat{\delta}(j+1) = \hat{\delta}(j)$$

Spliid (1983) shows that these estimators are asymptotically normal distributed. Another important property is that in most cases these estimators are very close to the ML estimator, (Shumway 2000, Spliid 1983).

2.2 Model Order Choice

In order to choice the model order the Bayesian Criterion of Schwarz (BCS),(SAS Intitute, 2000), is used:

$$SBC = \log\left(\left|\hat{\Sigma}\right|\right) + \frac{r \log(T)}{T}$$

where: r is the number of estimated parameters.
T is the number of samples.
Σ is the covariance matrix.

2.3 Modal Parameter Estimation

The coefficients of the autoregressive part of the model contain the modal characteristics of the system (Fassois, 2000). To obtain its modal parameter, it is necessary to build the companion matrix M (Larbi And Lardies, 2000, Huang, 2001, Maiaet all 1987),

$$M = \begin{bmatrix} -\phi_1 & \cdots & -\phi_p \\ I_{(p-1)} & & 0_{(p-1)x1} \end{bmatrix}$$

The eigenvalues and eigenvectors of M are related to the modal parameters [5,12,13] in the following way:

$$\lambda_s = \frac{\ln(\lambda_M)}{dt} \qquad \psi_M = \begin{bmatrix} \psi_S \\ \cdots \\ \psi_2 \end{bmatrix}$$

where:

λ_M represents the eigenvalues of M

λ_s represents the eigenvalues of the mechanical system

ψ_M represents the eigenvectors of M

ψ_S represents the eigenvectors of the mechanical system, which is the upper half partition of ψ_M .

3. Application

A simulated system of two inputs and three outputs is analyzed under the influence of stochastic noise , the Noise/Signal ratio (SNR) is 0.3699.The results are presented in table 1 and table 2.

Table 1- Results for the Poles of the system

Theorical Poles	Estimated Poles
-0.0196 - 6.6533i	-0.0256 + 6.6517i
-0.0496 -10.5300i	-0.0597 +10.5303i
-0.1307 -16.2737i	-0.1908 +16.2836i

Table 2- Results for the Vibrating modes

Theorical Vibrating Modes	Estimated Vibrating Modes
1.0000 6.5732 + 0.0404i 4.2208 + 0.0250i	1.0000 7.1617 + 0.7703i 4.5648 + 0.5325i
1.0000 -0.0884 + 0.0008i -0.0992 + 0.0010i	1.0000 -0.1004 - 0.0070i -0.1097 + 0.0017i
1.0000 -15.4828 - 0.2628i 23.8749 + 0.4254i	1.0000 -16.0515 - 5.8693i 24.4180 +10.0270i

It can be seen in table 1 and table 2 that this technique provides good estimation results in both poles and vibrating modes of the mechanical system.

4. Conclusion

This paper shows an implementation of the VARMAX identification model. The results obtained from numerical simulation show that such method are able to produce identification results with higher accuracy in the presence of stochastic noise. The fast algorithm gives goods results in the identification of the VARMAX's parameters.

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