GAIN COMPUTATION STRATEGY FOR AN ATTITUDE CONTROL SYSTEM

Ramos, F. O., Leite Filho, W. C., Moreira, F. J. O.
Space Systems Division, Institute of Aeronautics and Space (CTA-IAE-ASE)
Praça Marechal Eduardo Gomes, 50, São José dos Campos, 12228-904, SP, Brazil
fausto@iae.cta.br, waldemar@iae.cta.br, fernando@iae.cta.br

Abstract. *The computation strategy adopted to define the controller gains of the attitude control system of VLS (acronym for Brazilian launcher vehicle) is stated here. The strategy relies on four main steps: LQ design, gain computation, stability analysis and performance indexes evaluation. Launcher's aerodynamic and structural parameters are used as input to the process. Two models – simplified and detailed ones – are considered appropriately to those steps. The investigation of the preliminary design is emphasised to specific events, which occur during the flight (lift-off and engine(s) burnout). Furthermore, the gain values are reviewed regarding restrictions of passive sensor fault tolerance. Finally, wind disturbances are inserted during simulation so that system's behaviour with reference to stability and saturation of the actuator signal is observed. The design is achieved when stability and performance requirements are satisfied, according an iterative procedure.*

Keywords. gain computation, attitude control, fault tolerance, controller design, launcher.

1. Introduction

Definition of the design methodology in an aerospace application is dependent on the knowledge of system architecture and mission objectives. Therefore, methodology is constrained basically by (i) plant, (ii) control physical layer (e.g. sensors and actuators) and (iii) control algorithm. A detailed explanation of these aspects to VLS launcher is given by Leite Filho (1999), oriented by the Methodology of Computer Simulation (Leite Filho and Carrijo, 1996)

The second part of this article gives an overview of the launcher, including the models. The third part describes the steps required by the Gain Computation Strategy (GCS). The implementation of GCS is commented in part four, where some of the constraints related to each step are discussed. A design session example is presented in part five.

2. Launcher overview

2.1. Characteristics and mission

The main characteristics of the VLS launcher are shown below:

- mass: 50000 kg, height: 19 m;
- circular orbit insertion capability: 100-350 kg, 250-1000 km;
- 4 stages (solid propellant), first one is a set of 4 boosters;
- sensors: inertial module for Euler angles, rate gyros for angular velocity, and accelerometers;
- control devices: movable nozzles, bi-propellant and cold gas on-off thrusters.

More details can be found in Leite Filho (1999).

2.2. Control system (movable nozzle)

The control system is composed of three independent controllers for pitch, yaw and roll manoeuvring; one of them is given on Fig. (1) (pitch plane, detailed model). Pitch and yaw controllers are of proportional-integral type (PI) with angular rate feedback. Gain scheduling is adopted, where gain values are defined *a priori* by a linear-quadratic (LQ) design. Two models – simplified and detailed ones – are built from launcher's database including aerodynamic, structural, trajectory, propulsion and other data. The simplified model is used for LQ design. Stability and performance evaluation is based on the detailed model.

2.3. Model building

A database (Institute of Aeronautics and Space, 1992) provides the many parameters used to build 3rd order (or detailed) plant transfer functions for each manoeuvring plane. These functions are obtained from the launcher's linear model (Greensite, 1970), taking account of simplifying assumptions, according Kienitz and Moreira (1993).

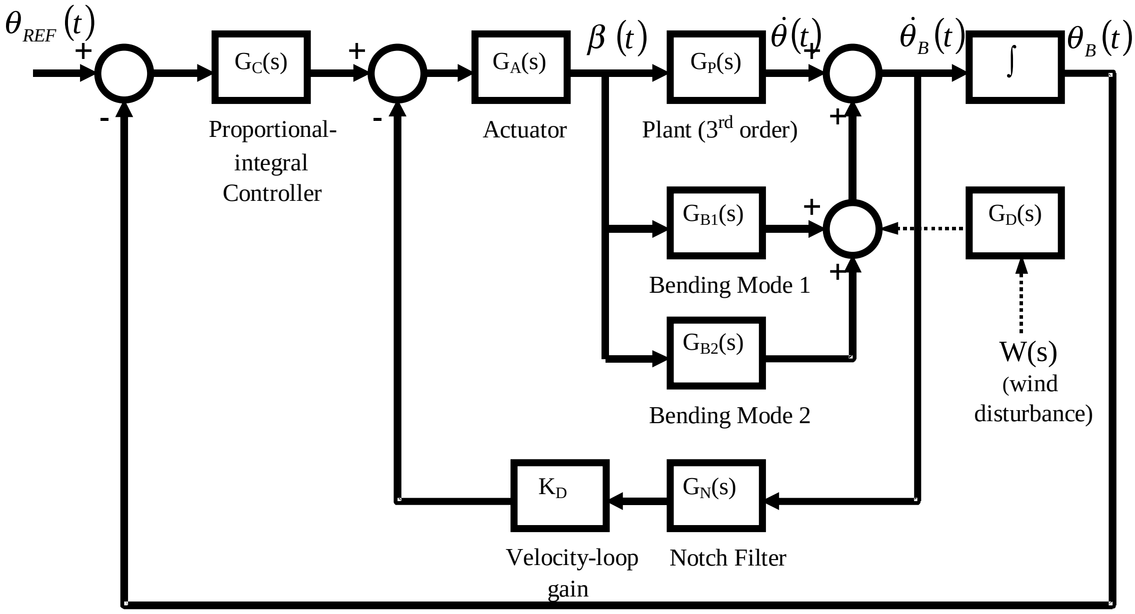

Figure 1. Launcher's detailed control system for pitch plane.

For example, the 3rd order pitch plane transfer function in s-domain is

$$G_P(s) = \frac{\dot{\Theta}(s)}{\mathrm{B}_Z(s)} = -\frac{M_{\beta z}\, s^2 + \left(M_{\beta z}\, Z_\alpha - M_\alpha\, Z_{\beta z}\right) s / U}{s^3 + \left(M_q + Z_\alpha / U\right) s^2 + \left(M_q Z_\alpha / U - M_\alpha\right) s + M_\alpha g / U} \tag{1}$$

where $\dot{\Theta}(s)$ is the pitch rate referred to the inertial frame, $\mathrm{B}_Z(s)$ is the actuator's output and U is the velocity vector referred to the x-axis of the body frame (see Fig. 2). The coefficients $M_{\beta z}$, M_α and M_q (angular momentum) and $Z_{\beta z}$ and Z_α (force) vary with flight time, but may be considered constant - for design purposes - if an interval small enough is considered.

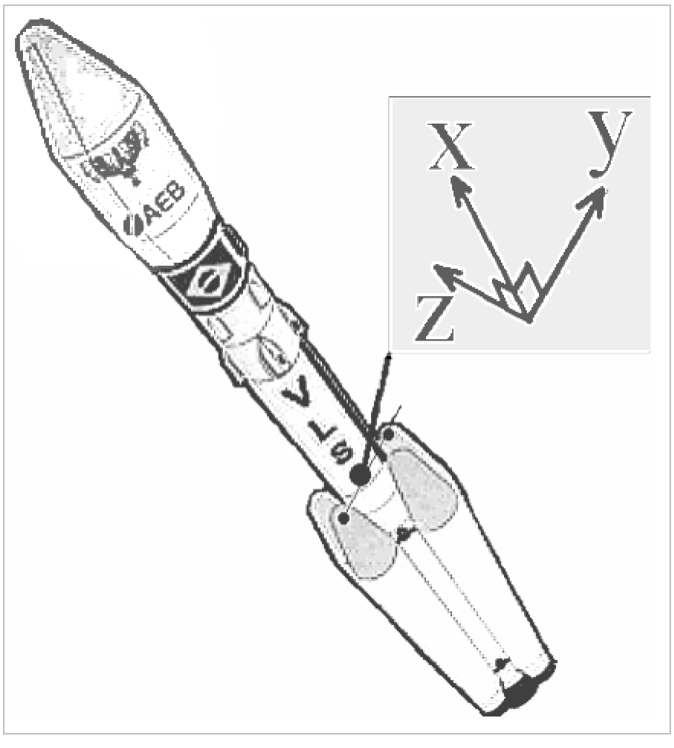

Figure 2. Launcher drawing and body axes (y-axis aligned with boosters, as shown).

Transfer functions with lower degree are preferable for design purposes; thus, a simplified model is also defined. If one considers that velocity U is very high and terms containing M_q can be neglected, then Eq. (1) simplifies to

$$G_{PS}(s) = -\frac{M_{\beta z}\, s}{s^2 - M_\alpha} \tag{2}$$

Since M_α may assume positive values during the flight, the plant is expected to be unstable.

3. GCS strategy

Gain computation strategy (GCS) relies on four main steps, namely: LQ design, gain computation, stability analysis and performance indexes evaluation; each step is outlined below.

3.1. LQ design

Using the simplified model given by Eq. (2), associated with Fig. (3), the LQ design is conducted by:

(i) Choosing a particular set of aerodynamic coefficients, needed by Eq. (1). This set is extracted from the database, for a time instant during flight when the aerodynamic load is maximal.

(ii) Calculating the closed loop t. f., regarding the simplified model, and rewriting it to an equivalent 3rd order transfer function given by Eq.3, where K_P , K_D and K_I represent proportional, derivative and integral gains, and $\Theta(s)$ and $\Theta_{REF}(s)$ are actual and reference pitch angles referred to the inertial frame.

$$G_{cl}(s) = \frac{\Theta(s)}{\Theta_{REF}(s)} = -\frac{(K_P\, s + K_I)\, M_{\beta z}}{s^3 - K_D\, M_{\beta z}\, s^2 - (M_\alpha + M_{\beta z}\, K_P)s - K_I\, M_{\beta z}} = \frac{K\,(s + \mu\, p_0)}{(s + p_0)(s^2 + 2\,\zeta\,\omega_d\, s + \omega_d^2)} \tag{3}$$

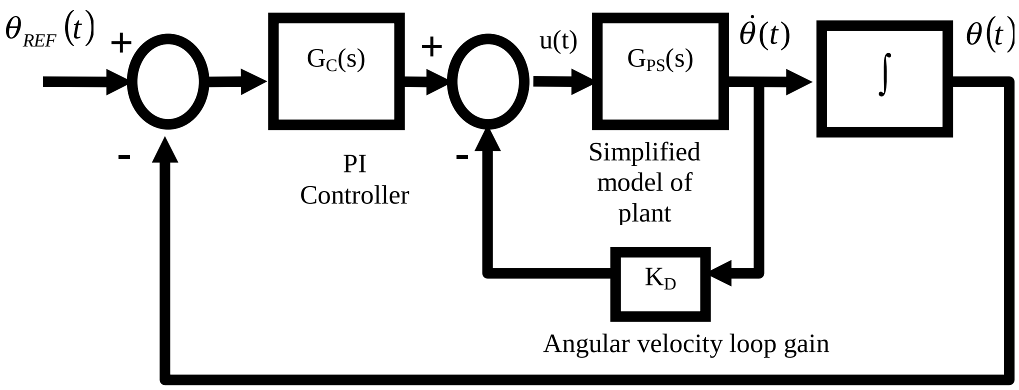

Figure 3. Simplified control system for pitch plane.

(iii) The fixed set of parameters ζ , ω_d and p_o is calculated, by applying the cost function

$$J = \int_0^\infty \left[\mathbf{z}^T(t)\mathbf{Q}\mathbf{z}(t) + \beta(t)^2 R \right] dt \; , \; \mathbf{z}(t) = \begin{bmatrix} \dot{\theta}(t) \\ \theta(t) \\ \int \left[\theta_{ref}(t) - \theta(t) \right] dt \end{bmatrix} \tag{4}$$

to the system

$$\begin{bmatrix} \ddot{\theta}(t) \\ \dot{\theta}(t) \\ [\theta_{ref}(t) - \theta(t)] \end{bmatrix} = \begin{bmatrix} 0 & M_\alpha & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}(t) \\ \theta(t) \\ \int [\theta_{ref}(t) - \theta(t)] dt \end{bmatrix} + \begin{bmatrix} -M_{\beta z} \\ 0 \\ 0 \end{bmatrix} \beta(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \theta_{ref}(t) \tag{5}$$

subject to the control law shown in Fig. (3):

$$\beta(t) = -[K_D \quad K_P \quad K_I] \begin{bmatrix} \dot{\theta}(t) \\ \theta(t) \\ \int [\theta_{ref}(t) - \theta(t)] dt \end{bmatrix} + K_P \, \theta_{ref}(t) \tag{6}$$

The functions $\theta(t)$ and $\dot{\theta}(t)$ are pitch angle and rate of the vehicle (inertial frame). The function $\beta(t)$ is related with the thrust vector deflection angle of the movable nozzle, and is associated with signal u(t) shown in Fig. (3) (actuator's dynamics is assumed instantaneous, for design purposes).

3.2. Gain computation

The fixed set of parameters obtained from the last section, a time-variant set of aerodynamic coefficients, and the following equations provide the scheduled gain tables to be used during the launcher flight:

$$\begin{aligned} K_P &= -\frac{\left(M_\alpha + 2\zeta\omega_d \, p_0 + \omega_d^{\,2}\right)}{M_{\beta z}} \\ K_I &= -\frac{p_0 \omega_d^{\,2}}{M_{\beta z}} \\ K_D &= -\frac{\left(p_0 + 2\zeta \, \omega_d\right)}{M_{\beta z}} \end{aligned} \tag{7}$$

Open loop transfer functions for each time instant of flight are built from Fig. (1) and the gains given by Eq. (7). The table of gains K_P , K_D and K_I are inspected regarding passive sensor fault tolerance. This inspection is based on Ramos and Leite Filho (2001); see also Ramos (2002).

3.3. Stability analysis

The detailed control system depicted on Fig. (1) is verified according the following features:

(i) location of poles and zeros for every instant of flight time;

(ii) relative stability by Nichols chart, using a set of Nichols plots since lift-off to 3rd stage separation (when the attitude controller is discarded);

The investigation of the preliminary design is emphasised to certain events occurring during the flight time (e.g., lift-off and engine(s) burnout). Wind disturbances are also inserted into the simulation, together with different thrust curves associated with the engines.

3.4. Performance indexes evaluation

The detailed control system, which comprises the detailed model, is simulated with reference input $\theta_{ref}(t)$ equals to ramp and step functions. The parameters of **plant, bending modes, proportional-integral controller** and **velocity loop gain** are time-variant. Additionally, wind disturbances and small sensor faults are inserted during simulation so that system's behaviour with reference to stability and saturation of the actuator signal is observed. Finally, performance indexes are extracted from the simulation runs for each scenario considered. When necessary, parameters **Q** and R are adjusted appropriately until those performance indexes are inside an admissible range.

4. Implementation of GCS

Next sections present some comments about the procedures required by GCS in order to define a suitable gain table used by the control algorithm of the launcher.

4.1. LQ design and gain computation

One of the inputs of the LQ design is a particular data set extracted from the launcher's database (Institute of Aeronautics and Space, 1992), regarding the maximum aerodynamic load, given by:

$$Q_\alpha = P_{din}\,\alpha \tag{8}$$

where Q_α is the aerodynamic load, P_{din} is the dynamic pressure and α is the angle of attack. The maximum value of Q_α indicates the time instant where the design parameters are chosen, as suggested in (Moreira, 1995).

For the LQ design is necessary to define the weighting factors given by matrices **Q** and R, acting on **z** vector and β respectively (see Eq. 4). In order to increase the importance of the pitch angle θ and actuation signal β, a suitable choice could be:

$$\mathbf{Q} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, R = 0.4 \tag{9}$$

A single set of gain values K_P, K_D and K_I are so obtained, by means of the MATLAB ® function **lqr**(A,B,Q,R), where, according Eq. (5):

$$\mathrm{A} = \begin{bmatrix} 0 & M_\alpha & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad , \quad \mathrm{B} = \begin{bmatrix} -M_{\beta z} \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Note that M_α and $M_{\beta z}$ are chosen when Q_α is maximum. Now, substituting the gain values and aerodynamic coefficients into Eq. (3), a fixed set of parameters ζ, ω_d and p_o is found. Finally, the gain table for the entire flight is calculated, according the above mentioned set of parameters (fixed) and aerodynamic coefficients (variant), for every instant of flight, by using Eq. (7).

The gain table is inspected regarding their absolute gain values, so that factors as noise or small faults due to the sensors do not be excessively amplified by the controller and applied to the actuator (movable nozzle). If that happens, the actuator may saturate and system's stability is not guaranteed. For more details about this influence see (Ramos and Leite Filho, 2001).

4.2. Stability analysis

As said before, two methods are used in order to evaluate the stability; both of them consider the detailed model of the system:

- Map of poles and zeros. Some components of the launcher structure are not considered during the LQ design (e.g., bending modes, which effect should be attenuated appropriately by the notch filter). Therefore, it is necessary to verify when the system may become unstable during the flight due to those components.
- Nichols chart. Required to evaluate the gain and phase margins as indicators of the system's robustness caused by (i) variations of the coefficients (e.g., M_α) and (ii) shift in time between the real trajectory and the anticipated one used for the design.

4.3. Performance indexes evaluation

The resultant performance characteristics of the launcher depends on the LQ design, passive sensor fault tolerance and stability analysis. Some comments about each of the characteristics are included below:

- Rise time (t_R): must obey the equation $t_{min} < t_R < t_{max}$. The minimum time t_{min} comes from the structural integrity of the launcher.
- Overshoot: must be limited so that the bending modes are not excessively excited. Another factor to be considered is the maximum deflection of the nozzle, which should not exceed ± 3 degrees; if that maximum is reached, the actuator is no longer considered linear in the design, and stability is not assured. External disturbances (wind gusts) and sensor faults may also be taken into account when defining the damping.

- Attitude errors. The integrator must be avoided during lift-off and burnout of the second and third stages. However, errors due to this requisite can be minimised by choosing adequate values of proportional and derivative gains.

4.4. CAD tool for GCS

For the VLS launcher, GCS is implemented on MATLAB ® scripts. Additional tasks are also included (which were not cited in this work):

- calculation of the aerodynamic coefficients for LQ design;
- generation of data files as input of digital and hybrid (hardware-in-the-loop) simulations.

GCS is oriented by the Methodology of Computer Simulation (Leite Filho and Carrijo, 1996) which comprises four procedures: (i) design simulation (GCS is included here), (ii) validation simulation, (iii) statistical simulation and (iv) hardware in the loop simulation. These procedures can also be found in Malyshev *et al.* (1996).

A large number of conventional m scripts were modified and integrated inside the MATLAB's ® Graphical User Interface (GUI), making GCS easier to execute and better traceable. The result was a tool named "PACA", acronym for Design and Analysis of Attitude Controller (Ramos, 2003). Automatic reports are generated for every design. Project files can be defined for each scenario, so that it can be loaded later with most of data already defined. The next section presents the data obtained for a typical design session of the VLS launcher in PACA environment.

5. Example of a design session

The aerodynamic coefficients M_α (Fig. 4) and $M_{\beta z}$ are calculated based on files containing aerodynamic characteristics of the launcher, trajectory, thrust, mass and inertia, structure, and other data. The input to the LQ design are the weighting factors **Q** and R, and the coefficients, chosen in a particular time instant of the flight.

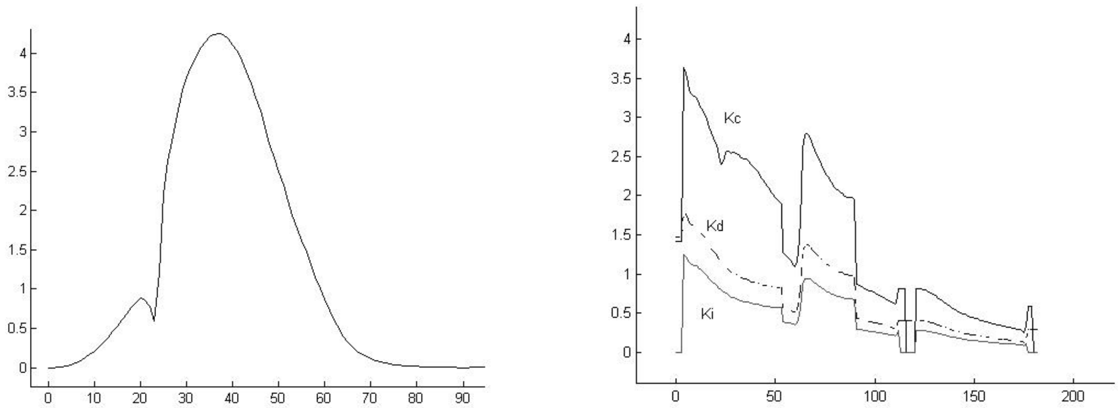

Figure 4. Graph of the coefficient M_α (left), and gain values after design, versus time of flight (seconds).

The gain table is built from the fixed set of parameters ζ , ω_d and p_o (obtained from the LQ design), and the aerodynamic coefficients. The table contains the gains K_P , K_D and K_I , seen on Fig. (4). Note that the integral action (K_I) is turned off during the launching, in order to avoid a collision of the launcher with the launching pad, as suggested by Kienitz and Moreira (1993).

The next step is the evaluation of the stability, specially for certain events of flight (e. g., engine burnout, shown by the map of poles and zeros and Nichols chart in Fig. 5).

Besides the plant being unstable during the flight, the effects of the bending modes must also be considered, when verifying the design. A situation as shown in Fig. (5) can be avoided, by correctly choosing the controller gains and the coefficients of the notch filter - $G_N(s)$ in Fig. (1). For controller gains, another time instant around the one proposed in section 4.1 (Eq. 8), together with a different choice of **Q** and R values, may be used to reduce the amplitude of oscillations generated due to the actuator's nonlinearities (limit cycle), by increasing the derivative gain K_D .

For notch filter, its tuning frequency and damping value must be careful chosen so that closed-loop poles due to the first bending mode are stable. The final values are obtained from hardware-in-the-loop simulation, which corresponds to a closer configuration to the real flight.

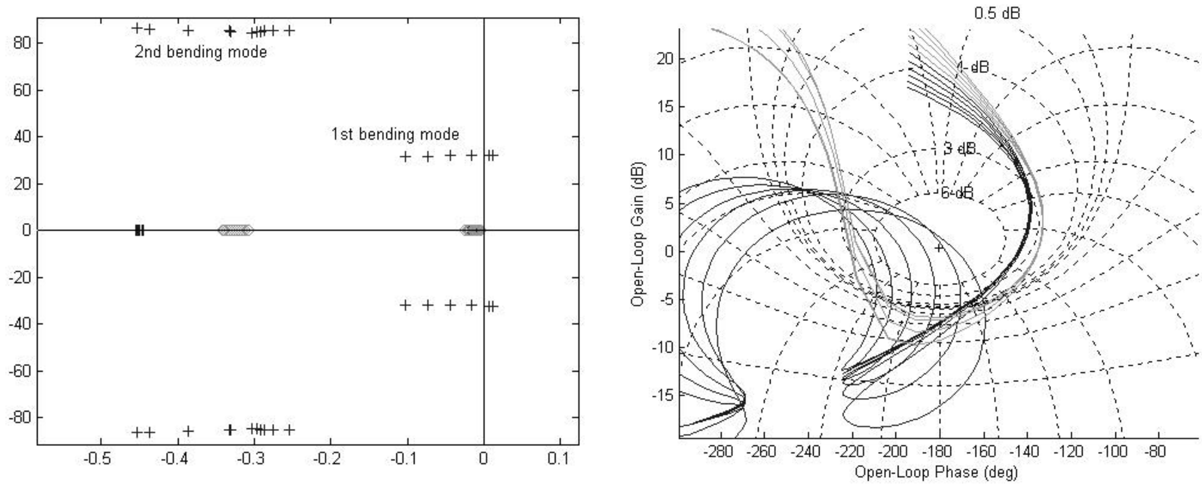

Figure 5. Map of closed-loop poles (+) and zeros (o) and Nichols chart of an unstable system, near 1st stage engines burnout (between 55 and 65 seconds of flight). Left graph: complex plane (imaginary versus real co-ordinates).

After the design is verified, the performance indexes are inspected from the step response of the control system. The detailed model (Fig. 1) is used for the simulation. A typical result is seen in Fig. (6); the oscillation is mainly due to the first bending mode of the launcher.

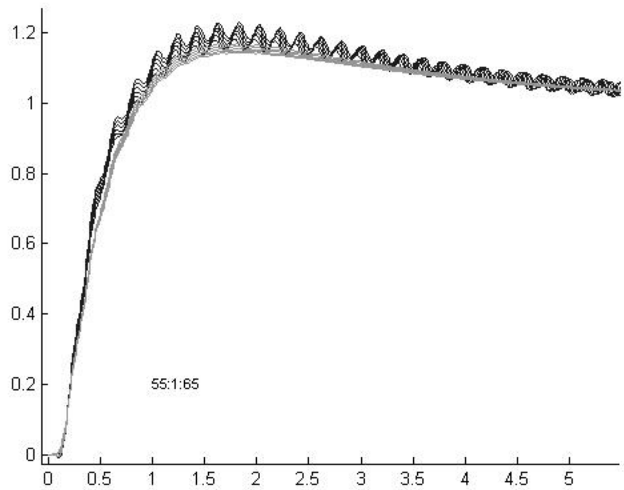

Figure 6. Unit step response of the control system, near 1st stage engines burnout (between 55 and 65 seconds of flight). Pitch angle $\theta(t)$ (degrees) versus time (seconds).

The choice of the linear-quadratic technique allows of controller low gain values, which is a desirable characteristic both for avoiding saturation of the actuator signal and also for reducing the impact due to sensor faults, as observed in Ramos (2002).

6. Conclusions

The computation strategy for building the gain tables needed by VLS launcher's controller was presented. Three main steps are considered: (i) a linear-quadratic design, using a data set extracted from a database, and two weighting factors; (ii) a stability analysis, based on tools of the classical control theory; (iii) evaluation of the performance indexes. The main aspects of the strategy are: (i) LQ design – the choice of the data set extracted from the database for a particular instant (worst case) during flight, (ii) stability analysis – obtaining gain and phase margins, and (iii) performance indexes – avoiding actuator saturation (maintaining the actuator typically as a linear device), and minimising attitude errors. Finally, some results of a design session for VLS launcher were presented, obtained by using a MATLAB's ® GUI based tool.

7. References

Greensite, A. L. (1970) Analysis and design of space vehicle flight control systems. NY: Spartan Books. Institute of Aeronautics and Space (1992).

Institute of Aeronautics and Space (1994). Banco de Dados do VLS. São José dos Campos (Brazil): Instituto de Aeronáutica e Espaço. N.590-000000/B3035.

Kienitz, K. H. ; Moreira, F. J. O. (1993) Design of VLS control algorithms for movable nozzle actuators. São José dos Campos (Brazil): Institute of Aeronautics and Space. N. 590-000000/B3033 and 590-000000/B3046.

Leite Filho, W. C. (1999) Control system of Brazilian launcher. Proc. 4th ESA International Conference on Spacecraft Guidance, Navigation and Control Systems, Noordwijk, 18-21 October 1999, pp. 401-405.

Leite Filho, W. C. ; Carrijo, D. S. (1996) Hardware-in-loop simulation of Brazilian launcher VLS. Proc. 3rd ESA International Conference on Spacecraft Guidance, Navigation and Control Systems, Noordwijk, 26-29 November 1996, pp. 355-358.

Malyshev, V. V. ; Krasilshikov, M. N. ; Bobronnikov, V. T. ; Dishel, V. D. ; Leite Filho, W. C. ; Ribeiro, T. S. (1996) Aerospace Vehicle Control – modern theory and applications. São José dos Campos (Brazil): FAPESP.

Moreira, F. J. O. (1995) Design of VLS control algorithms with movable nozzles actuators. São José dos Campos (Brazil): Institute of Aeronautics and Space. N.590-000000/B3046.

Ramos, F. O. ; Leite Filho, W. C. (2001) Transient effects related to a reconfigurable control design (satellite launcher). In: Proc. 15th IFAC Symposium on Automatic Control in Aerospace, Bologna/Forlì, pp. 79-84.

Ramos, F. O. (2002) Perturbation due to sensor fault and its implications on design with case study (VLS). MSc dissertation. Department of Electronic Engineering, Aeronautic Technologic Institute, São José dos Campos (Brazil).

Ramos, F. O. (2003) Design and Analysis of Attitude Controller (PACA). São José dos Campos (Brazil): Institute of Aeronautics and Space. N.590-000000/E1015.

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