Analytical Comparisons Between 2D Heat and Mass Capillary Porous Media Problem with and without Momentum Transfer

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Abstract. *Multidimensional drying problem in capillary porous media is analytically solved for the associated temperature, moisture and pressure content distributions. Luikov's model with linear transport coefficients and two-dimensional plate geometry is adopted for the description of the simultaneous heat, mass and pressure transfer phenomena. The generalized integral transform technique (G.I.T.T.) is applied to the problem and the automatically global error-controlled solution of the coupled partial differential equations is used to achieve the solutions. The convergence behavior of the proposed eigenfunction expansions is illustrated.*

Keywords *Integral Transform Method, Simultaneous Heat and Mass Transfer, Porous Media, Luikov Problem*

1. Introduction

The system of equations proposed by Luikov (1975) is by far the most frequently adopted in the study of drying phenomena in capillary porous media with various applications in the engineering and applied sciences. The integral transform method (Cotta, 1993, 1997) has been successfully utilized in the hybrid numerical-analytical solution of such problems, for both the linear (Duarte, 1995, 1998, Ribeiro et al, 1993) and non-linear versions (Ribeiro et al., 1995, Duarte, 1998), offering the attractive feature of automatic global error control in the final results. Both applications were previously considered (Duarte, 1995, 1998, Ribeiro et al, 1993, 1995) and the interest in studying multidimensional situations are still increasing, as demonstrated by the finite element method numerical solution in Ferguson and Lewis (1993). Therefore, the present contribution bring the integral transform methodology to be applicable in multidimensional drying problems, such as the one formulated in Lewis et al. (1996) and Thomas et al. (1980), and demonstrates another attractive feature of this class of hybrid method, i.e., the just very mild increase in computational effort for increased number of dimensions in the problem (independent variables). Essentially, it is reconfirmed that the overall computational cost in implementing the one-dimensional simulation is exactly comparable to that of solving the two-dimensional problem here proposed.

2. Analysis

We consider the heat, mass and pressure balance equations written in dimensionless form, for a symmetric plate geometry as depicted in Ferguson and Lewis (1993), Fig. (1) subjected to uniform prescribed boundary temperatures, moisture and pressure contents, and evaluated from uniform initial distributions (Duarte et al. 1995, 1998). The transport coefficients are assumed constant and the problem formulation according to Luikov's theory (Luikov, 1975) is given by Ferguson and Lewis (1993):

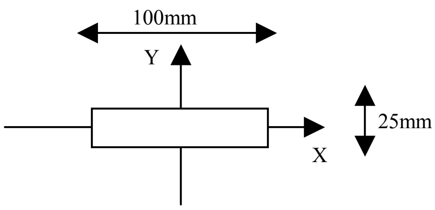

Figure 1 Geometry and coordinate system for contact drying for a moisture porous sheet.

$$\frac{\partial \Theta_1(X,Y,\tau)}{\partial \tau} = K_{11}\,\nabla^2\Theta_1(X,Y,\tau) + K_{12}\,\nabla^2\Theta_2(X,Y,\tau)K_{12} + K_{13}\,\nabla^2\Theta_3(X,Y,\tau)\,;$$

0<X<1, 0<Y<1; τ >0 (1)

$$\frac{\partial \Theta_2(X,Y,\tau)}{\partial \tau} = K_{21}\,\nabla^2\Theta_1(X,Y,\tau) + K_{22}\,\nabla^2\Theta_2(X,Y,\tau) + K_{23}\,\nabla^2\Theta_3(X,Y,\tau)\,;$$

0<X<1,0<Y<1; τ >0 (2)

$$\frac{\partial \Theta_3(X,Y,\tau)}{\partial \tau} = K_{31}\,\nabla^2\Theta_1(X,Y,\tau) + K_{32}\,\nabla^2\Theta_2(X,Y,\tau) + K_{33}\,\nabla^2\Theta_3(X,Y,\tau)\,;$$

0<X<1,0<Y<1; τ >0 (3)

with initial conditions

$$\Theta_1(X,Y,0) = \Theta_2(X,Y,0) = \Theta_3(X,Y,0) = 0\,; \quad 0<X<1,\ 0<Y<1 \qquad (4,5,6)$$

and boundary conditions

$$\frac{\partial \Theta_1(0,Y,\tau)}{\partial X} = 0\,; \quad \frac{\partial \Theta_1(X,0,\tau)}{\partial Y} = 0\,; \quad \tau>0 \qquad (7,8)$$

$$\frac{\partial \Theta_2(0,Y,\tau)}{\partial X} = 0; \quad \frac{\partial \Theta_2(X,0,\tau)}{\partial Y} = 0\,; \quad \tau>0 \qquad (9)$$

$$\frac{\partial \Theta_3(0,Y,\tau)}{\partial X} = 0; \quad \frac{\partial \Theta_3(X,0,\tau)}{\partial Y}\,; \quad \tau>0 \qquad (10)$$

$$\Theta_1(1,Y,\tau) = \Theta_2(1,Y,\tau) = \Theta_3(1,Y,\tau) = 1\,; \quad \tau>0 \qquad (11,12,13)$$

$$\Theta_1(X,1,\tau) = \Theta_2(X,1,\tau) = \Theta_3(X,1,\tau) = 1\,; \quad \tau>0 \qquad (14,15,16)$$

where the K´s represent:

$$K_{11} = k_q + \varepsilon\lambda\delta^{'}\,; \quad K_{12} = \varepsilon\lambda k_m\,; \quad K_{13} = \varepsilon\lambda k_p \qquad (17,18,19)$$

$$K_{21} = \delta k_m\,; \quad K_{22} = k_m\,; \quad K_{23} = k_p \qquad (20,21,22)$$

$$K_{31} = -\varepsilon\lambda k_m\,; \quad K_{32} = -\varepsilon k_m\,; \quad K_{33} = k_p(1-\varepsilon) \qquad (23,24,25)$$

and $\boldsymbol{\theta_1}$ is the dimensionless temperature distribution, $\boldsymbol{\theta_2}$ is the dimensionless moisture content distribution, $\boldsymbol{\theta_3}$ is the dimensionless pressure distribution.

Without loss of generality, using the formalisms of the *integral transform* (Ribeiro, Cotta, 1993, Cotta and Mikhailov, 1993) method the solution for the system of eqs. (1-3) is now proposed in terms of auxiliary problems, expressed by three pairs of easily available decoupled eigenfunction expansions of Sturm-Liouville problems, for the temperature, moisture and pressure potentials (k = 1,2,3):

$$\frac{d^2\Psi_{ki}(X)}{dX^2} + \mu_{ki}^2\Psi_{ki}(X) = 0, \qquad X \in \mathbf{V} \tag{26}$$

$$\frac{d\Psi_{ki}(0)}{dX} = 0, \quad \Psi_{ki}(1) = 0 \qquad X \in \mathbf{S} \tag{27,28}$$

$$\frac{d^2\Gamma_{kj}(Y)}{dY^2} + \lambda_{kj}^2\Gamma_{kj}(Y) = 0, \qquad Y \in \mathbf{V} \tag{29}$$

$$\frac{d\Gamma_{kj}(0)}{dY} = 0, \quad \Gamma_{kj}(1) = 0 \qquad Y \in \mathbf{S} \tag{30,31}$$

These auxiliary problems permit the definition of the integral transform pairs that are necessary for the solution of the homogeneous problem:

Inverse,

$$\Theta_{kh}(X,Y,\tau) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{N_{ki}^{1/2}M_{kj}^{1/2}}\Psi_{ki}(X)\Gamma_{kj}(Y)\overline{\Theta}_{kij}(\tau) \tag{32}$$

Transform,

$$\overline{\Theta}_{kij}(\tau) = \int_0^1\int_0^1\frac{\Psi_{ki}(X)\Gamma_{kj}(Y)}{N_{ki}^{1/2}M_{kj}^{1/2}}\Theta_{kh}(X,Y,\tau)dXdY \tag{33}$$

The normalizations integrals are,

$$N_{ki} = \int_0^1 \Psi_{ki}^2(X)dX \tag{34}$$

$$M_{kj} = \int_0^1 \Gamma_{kj}^2(Y)dY \tag{35}$$

The problem now is to find numerically the eigenvalues (μ_{ki}, and λ_{kj}), eigenfunctions (Ψ_{ki}, and Γ_{kj}) and norms (N_{ki}, and M_{kj}).

The next step is to find the ordinary differential equation transform system. Using the *transform concept* in eqs. (1-25) and the auxiliary problems (26-31,34,35), after truncation to a sufficient order (i =1 **...** I, and j = 1 **...** J) for the desired convergence, we obtain,

$$\frac{dY(\tau)}{d\tau} + A_{2N,2N}Y(\tau) = 0_{2N,1} \tag{36}$$

where,

$$Y(\tau) = \{\overline{\Theta}_{11}(\tau)...\overline{\Theta}_{1N}(\tau) \quad \overline{\Theta}_{21}(\tau)...\overline{\Theta}_{2N}(\tau)\}^T \tag{37}$$

The initial transform conditions are similarly obtained applying the integral transform concept to the initial conditions on the homogeneous problem, resulting:

$$Y(0) = \overline{F}(\tau) \tag{38}$$

Now, this initial value problem can be solved through matrix eigenvalue analysis or scientific libraries. Initial value problem solvers with local error control schemes are employed for solving the truncated version of the transformed initial value problem. An adaptive procedure is utilized to automatically reduce, along the integration path, the truncation orders required for a certain user-prescribed accuracy yielding, as a by-product, a global error estimator.

At this point, it is possible to write the complete solution to the original problem. Using inversion formulae, temperature and moisture potentials can now be numerically obtained as:

$$\Theta_k(X,Y,\tau) = \Theta_{ks}(X,Y) + \sum_{i=1}^{I}\sum_{j=1}^{J} \frac{\Psi_{ki}(X)\Gamma_{kj}(Y)}{M_{ki}^{1/2} N_{kj}^{1/2}} \overline{\Theta}_{kij}(\tau) \; ; \tau>0 \tag{39}$$

where Θ_{ks} are the steady-state solutions, Ψ_{ki} and Γ_{kj} are the normalized eigenfunctions, and $\overline{\Theta}_{kij}$ represent the transformed potentials, obtained from numerical solution of the resulting ordinary differential system, after the completion of the integral transformation process.

3. Results and Discussion

The Luikov problem as proposed above is now solved using the integral transform technique and the problem without pressure potential was exactly solved by substitution of two heat transfer problems (Duarte, 1998). The numerical results make it possible an inspection of the overall convergence behavior for the proposed eigenfunctions expansions. The governing parameters, according to the data in Lewis (1996), and Cunha et all. (2002), assume the following values: ρ_0 = 1170,0 $Kg.m^{-3}$, c_q = 1.400,0 $J.Kg^{-1}.^oK^{-1}$, c_m = 0,03 $Kg.Kg^{-1}.^oM^{-1}$, c_p = 0,05 $Kg.Kg^{-1}.Pa$, ε = 0,3, λ = 2,3.10^6 $J.Kg^{-1}$, Δ = 0,67 $^oM.\,^oK^{-1}$, k_q = 576,0 $J.h^{-1}.m^{-1}.^oK^{-1}$, k_m = 3,0.10^{-6} $Kg.h^{-1}.m^{-1}.^oM^{-1}$, K_p = 1,5.10^{-6} $Kg.h^{-1}.m^{-1}.Pa^{-1}$, and the truncation orders, *N*, were taken less or equal to 12, for temperature, moisture and pressure. The computer program was implemented on *Mathemtica®* software (Wolfram, 1996), on a Pentium 700 *MHz* microcomputer with 256 Mb of memory RAM, and a typical run took less than *5* minutes of CPU time.

The convergence behavior below, Tab. (1), illustrates of the three expansions (different *N*'s) for temperature (**Θ_1**), moisture (**Θ_2**) and pressure potential (**Θ_3**), obtained at the plate centerlines (*Y = 0.0*) and different *X* positions. Since the heat, mass and pressure transfer processes have, for this problem, markedly different time constants, the values of dimensionless time considered in each case, are different. The convergence characteristics are, in both potentials, quite evident, with full convergence to four digits to moisture and pressure distribution and three digits to temperature distribution being achieved at *N* as low as *12*. Such results open up broad perspectives for extension of this approach into even more involved coupled parabolic problems.

Figures (2-5) show drying process, and the temperature, moisture and pressure distributions are obtained with the converged values, and the drying process for temperature and moisture exactly solved. As expected, the material with and without pressure variation show a low thermal inertia spending about *0.1* dimensionless time to achieve the thermal equilibrium in the most deep layer. The moisture potential present a different behavior in each case, as expected, the presence of pressure in the system carriage to deep inside a considerable quantity of humidity mass, allied to thermo-gradient effect and are needed more than *1500* dimensionless times steps to porous media to meet the moisture equilibrium. This effect does not happen with the no pressure model, and the humidity achieving the equilibrium state in no more than 55 dimensionless times steps, but in comparison with the temperature potential it characterize a very right mass inertia. The pressure, in that case, present a very right inertia, achieving the equilibrium state with *1400* dimensionless time steps. The different time equilibrium make the difference in O.D.E convergence number, for moisture, because the negative pressure work over the material in the same time. Such process create some difficulty to the numerical convergence, and of course, to the real drying process, as it take place after *300* dimensionless times steps, as can be seen in Fig (4). The drying process can be observed, when the pressure potential reached a half value of the prescribed boundary and the temperature is established over the material (τ > *300*).

Table 1 . Convergence behavior of temperature, Θ_1 moisture, Θ_2 and pressure, Θ_3 expansions.

Θ_1(X,0.0,0.15)					Θ_1(X,0.0,0.03)			
X/N	6	9	12		X/N	6	9	12
0.0	0.3308	0.3315	0.3309		0.0	0.6482	0.6489	0.6484
0.2	0.3311	0.3315	0.3311		0.2	0.6491	0.6494	0.6490
0.4	0.3315	0.3312	0.3318		0.4	0.6548	0.6553	0.6552
0.6	0.3486	0.3489	0.3489		0.6	0.6895	0.6900	0.6899
0.8	0.5109	0.5104	0.5104		0.8	0.8027	0.8020	0.8021
1.0	1.000	1.000	1.000		1.0	1.000	1.000	1.000

$\Theta_2(X,0.0,160)$					$\Theta_2(X,0.0,650)$			
X/N	6	9	12		X/N	6	9	12
0.0	3.271	3.273	3.273		0.0	1.968	1.968	1.968
0.2	3.278	3.276	3.276		0.2	1.959	1.959	1.959
0.4	3.320	3.320	3.320		0.4	1.910	1.910	1.910
0.6	3.522	3.528	3.522		0.6	1.758	1.758	1.758
0.8	3.368	3.367	3.367		0.8	1.448	1.448	1.448
1.0	1.000	1.000	1.000		1.0	1.000	1.000	1.000
$\Theta_3(X,0.0,200)$					$\Theta_3(X,0.0,450)$			
X/N	6	9	12		X/N	6	9	12
0.0	0.2181	0.2181	0.2181		0.0	0.6339	0.6339	0.6339
0.2	0.2177	0.2177	0.2177		0.2	0.6345	0.6348	0.6348
0.4	0.2161	0.2161	0.2161		0.4	0.6435	0.6435	0.6435
0.6	0.2300	0.2299	0.2299		0.6	0.6858	0.6859	0.6859
0.8	0.4168	0.4168	0.4168		0.8	0.8038	0.8038	0.8038
1.0	1.000	1.000	1.000		1.0	1.000	1.000	1.000

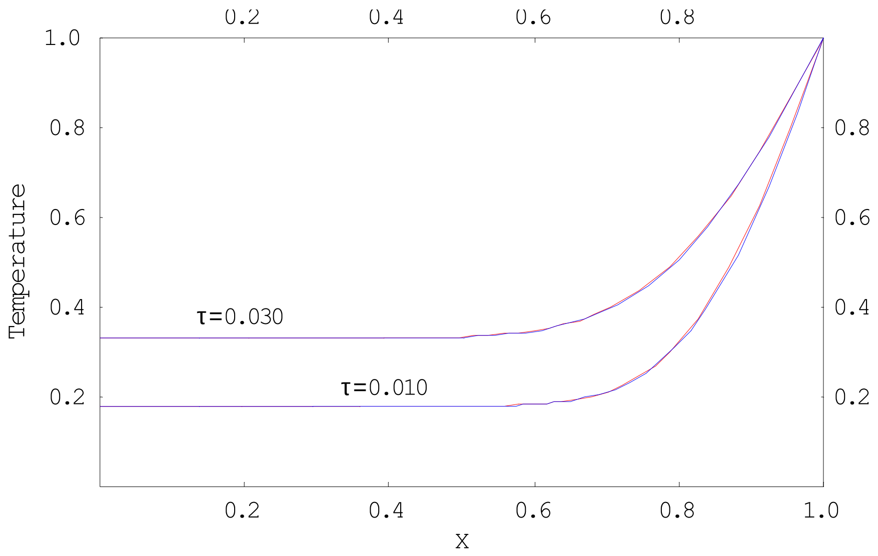

Figure 2. Evolution of temperature profiles during the process.

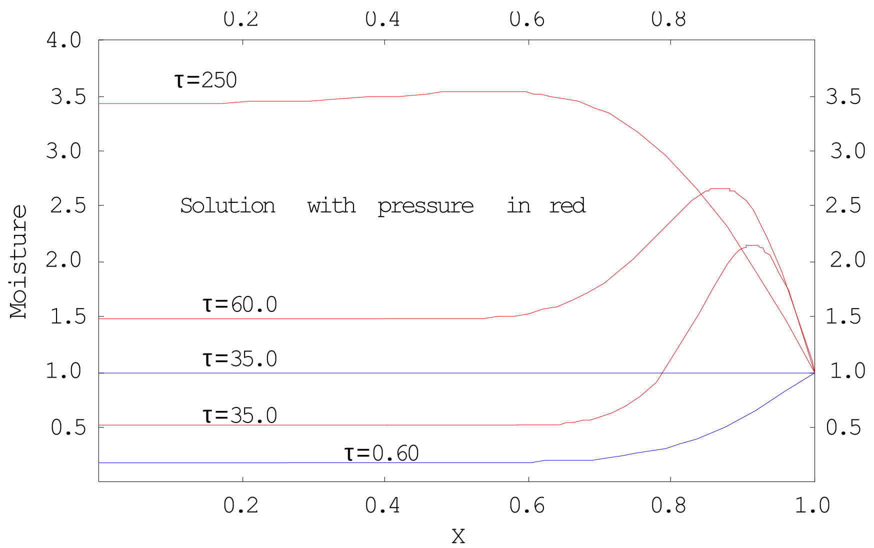

Figure 3. Evolution of moisture profiles during the process

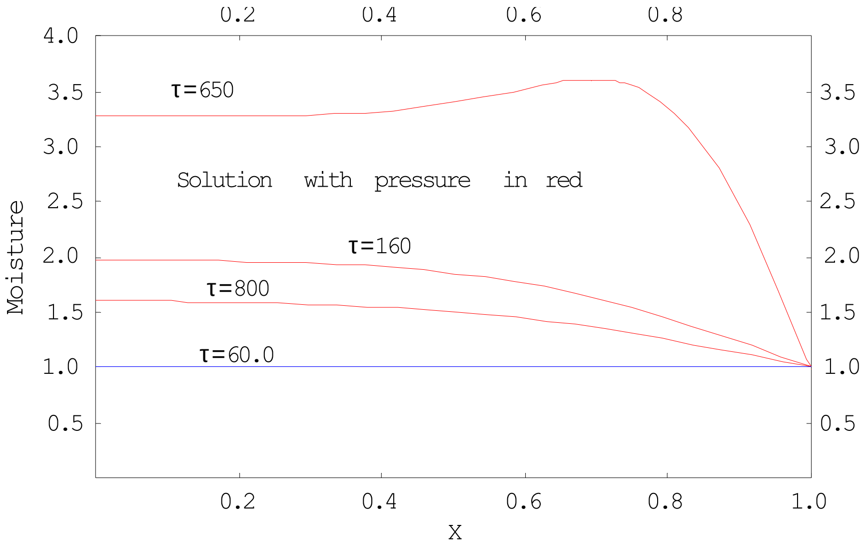

Figure 4. Evolution of moisture profiles during the process

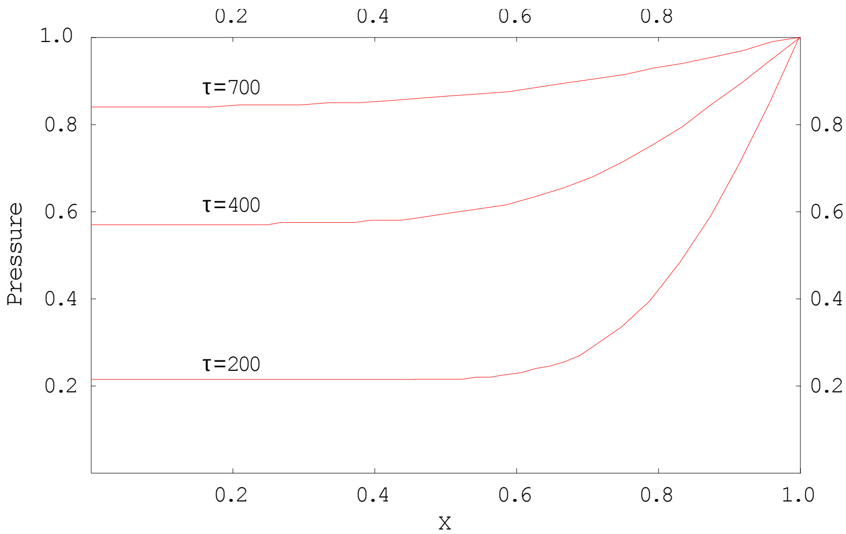

Figure 5. Evolution of pressure profiles during the process

4. Conclusion

In this paper the multidimensional drying problem in a capillary porous media was analytically solved for the associated temperature, moisture and pressure distributions, using Luikov´s model. The generalized integral transform technique (G.I.T.T.) was applied to the problem and an exact solution was achieved for the no pressure model. Convergence behavior of the adopted numerical methods and results of the temperature, moisture and pressure distributions showed very interesting aspects of drying process on such porous media.

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Symbols

$\mathbf{a_m}$	Moisture diffusion coefficient
$\mathbf{c_m}$	Specific moisture capacity
$\mathbf{c_p}$	Air capacity
$\mathbf{c_q}$	Heat capacity
$\mathbf{k_m}$	Coefficient of moisture conductivity
$\mathbf{k_p}$	Moisture filtration coefficient
$\mathbf{k_q}$	Thermal conductivity
θ_1	Dimensionless temperature distribution
θ_2	Dimensionless moisture distribution
θ_3	Dimensionless Pressure distribution
X	Dimensionless co-ordinate
Y	Dimensionless co-ordinate
τ	Dimensionless time.
δ	Thermo-gradient coefficient
λ	Latent heat
ε	Ratio of vapor diffusion coefficient to the coefficient of total moisture diffusion.