MODELLING THE DYNAMICAL BEHAVIOUR OF SMART BEAMS WITH ER/MR FLUIDS USING FEM.

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Abstract. *Aeronautical structures are more susceptible to vibrations that can become aggravated by aeroelastic phenomena. Therefore, methodologies for suppressing undesired structural responses are important for aeronautics. One of the new technologies involve the concept of smart structures which are becoming common. One of the promissing material for use in smart structures are the fluids which presents rheological variations.This smart fluids in the presence of electrical or magnetical field change their rheological properties very fast allowing applications in control.The modelling the behaviour of this fluids has been tried for many years. Because of the viscoelastic nature of this fluid the modelage using Finite Element Method is not trivial. Therefore it will be employed one model developed for a sandwich beam with viscoelastic core. This work presents a sandwich beam with a smart fluid core modeled using a Golla-Hughes-McTavish method and the results compared with analitical models and experimental results available in literature. Future application of this model is intended by coupling with an unsteady aerodynamics model, and subsenquent design of control law.*

Keywords: *Smart Structures, Smart Fluids, Finite Elements, GHM Method, Structural Dynamics.*

1. Introduction

Aeronautical industry are recognized as a huge technology consumer and it has required novel approaches and advances in many knowledgement areas. One challenge for aircraft designers has been obtained lighter structures with a low vibration susceptibility. The level of this vibration can be aggravated by aeroelastic instabilities which is the coupling among inertia, aerodynamic and structural effects (Bisplinghoff *et al.*, 1996). Aeroelastic phenomena like flutter, buffeting, divergence and aeroelastic response can lead to a drastic reduction in accuracy and precision of aircraft and may also result in severe structural damage. To keep the dynamical responses in acceptable levels many strategies have been carried out, such as: new structural materials, passive and active control techniques and advanced design methods.

In order to suppress undesired vibration many studies have been done using the so-called *smart structures.* This approach integrates active elements and controllers to the structure and it has been mainly related to advances in smart materials and computational progress. Smart materials research for smart structures have increased since the 80's. Shape memory alloys, piezoelectric, electrostrictive, magnetostrictive materials, smart fluids and optic fiber have furnished reasonable results in structural control, which have motivated aeronautical applications.

Novel materials suitable for smart structures are under development in a number of research centres. Among them, fluids that can change their rheological properties start to grow in importance for active control applications. Such materials usually present change in their properties due to action of an external field, such as: electrical or magnetical. It has been observed that the changes occur very fast, allowing applications in control. There are two main classes of smart fluids, one exploiting the electro-rheological (ER) effect, and the other exploiting the magneto-rheological (MR) effect. It is possible to identify three ways to use the smart fluids, flow mode, shear mode and squeeze flow mode (Sims et al., 1999). Although the large use of flow mode particularly into semi-active suspensions the shear mode is more indicated to use in smart strucutures because its integrating to the structure

According to Gamota and Filisko (1991) this kind of fluids can be modeled and understood at least qualitatively in terms of three regions: a pre-yield, a yieldd and a post yield region. Each region presents diferent behaviour like viscoelastic in the pre-yield region, plastic in the post yield region and viscoelastic-plastic in the transition through yield. They also purpose a model consisting of a Zener element (Voigt element in series with an elastic element) in series with a Coulomb frictional element and a viscous element. Other models have been purposed like Stanway et al (1987) and Kamath and Wereley (1997) who created a model composed of parameters which weighting the influence of the viscoelastic and the plastic regions according to shear strain velocity. Considering that smart fluids exhibit linear shear behaviour at small strain levels, similar to many viscoelastic damping materials, it is believed that modeling approaches developed for viscoelastically damped structures are potentially applicable to smart fluids adaptive structures as well (Yalcintas and Coulter, 1998). Without any applied field the smart fluids behave like a Newtonian fluids while with a applied field they appeared like a Bingham plastic fluids (Yanju et al., 2001). Gamota and Filisko (1991) investigating the dynamic properties of ERFs found that ERFs had various stress responses such as viscoelasticity, linear elasticity, non-linear elasticity, plasticity and all of these responses were affected by the electric field strength, strain frequency and stress amplitude. The model developed by them includes the viscoelastic and plastic elements resulting in a model involving three degrees of freedom. In this work it will be explored the use of the smart fluids in a shear configuration. For that, a sandwich beam with a smart fluid core is modelled. First efforts involving this modellage was presented by Yalcintas , Coulter and coworkers (Yalcintas and Coulter, 1998, Yalcintas and Dai, 1999, Yalcintas, Coulter and Don, 1995) who based your approach in RKU (Ross- Kerwin-Ungar) model. Because of the viscoelastic nature of the smart fluids it seems clear that the modellage follow the theory developed for this kind of material. The use of finite elements as a solution for dynamical equations of motion have been increased along the years. Because of the viscoelastic dependence on temperature and frequency some theories have been developed like ADF (Anelastic Displacement Field), GHM (Golla-Hughes-McTavish), and others. In this study the GHM model which was developed in the mid 80's will be used.

The aim of this work is to present a finite element model which includes the damping effects of rheological fluids and reproduces the dynamical behaviour viewing further use control. Future application of this model is intended by coupling with an unsteady aerodynamics model, and subsenquent design of control law. The results has been compared with analitical models and experimental results available in literature.

2. Physical Principles and Mathematical Model

A sandwich beam with a ER/MR fluid core is considered. In this model, the following assumptions are considered (1) the beam geometry is constant along the length of the beam, (2) the core material is isotropic and of much greater thickness than the face sheets. (3) the shear strain is constant through the core and negligible in the face sheets, (4) the longitudinal displacements of the face sheets are uniform through the thickness of the face sheets, (5) the transverse displacement does not vary through the thickness of core material and is small relative to the beam length.

The starting point in modelling the dynamics of the smart beam is to apply the principle of Hamilton, which states that,

$$\delta\int_{t_1}^{t_2}(T-V)dt+\int_{t_1}^{t_2}\delta W dt=0 \tag{1}$$

where T, V and δW are respectively the kinetic, potential (electro-mechanical coupling) energies and the virtual work of non-conservative loads.

The kinetic energy T for a beam can be expressed as:

$$T=\frac{1}{2}\int_V\{(\rho_1 A_1+\rho_2 A_2+\rho_3 A_3)\dot{w}^2+\rho_1 A_1\dot{u}_1^2+\rho_3 A_3\dot{u}_3^2\}dx \tag{2}$$

where the subscript indicates the layer, ρ is mass density, w is the transverse displacement along the y axis, and u is the translation of the neutral axis along the x-axis direction.

The potential energy formulation includes the bending in the faces, the shear in the ER material, and the extensional energy.

$$V_b=\frac{1}{2}(E_1 I_1+E_3 I_3)\int_0^L\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{3}$$

$$V_a=\frac{1}{2}E_1 A_1\int_0^L\left(\frac{\partial u_1}{\partial x}\right)^2 dx+\frac{1}{2}E_3 A_3\int_0^L\left(\frac{\partial u_3}{\partial x}\right)^2 dx \tag{4}$$

$$V_s=\frac{1}{2}G^* A_2\int_0^L\gamma^2 dx \tag{5}$$

where E, G^*, γ, I and A are respectively the Young's modulus of the face sheets, the shear modulus of the ER material, the shear strain of the viscoelastic shear layer, the moment of inertia and the area of the cross section.

Kinematics.

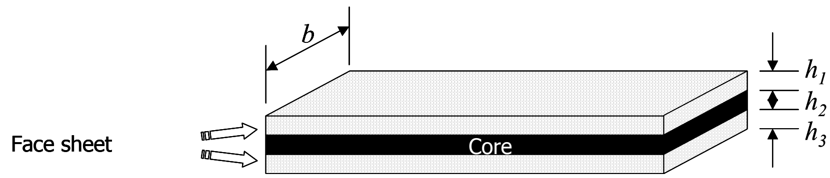

Figure 1. Sandwich beam.

Axial displacements are assumed to be linear through thickness, whereas transverse one w are supposed constant. The geometry and deformation of the beam with ER fluid is show in a Figure 2. The relation for the shear strain is obtained from the figure and is based on the axial and transverse displacements.

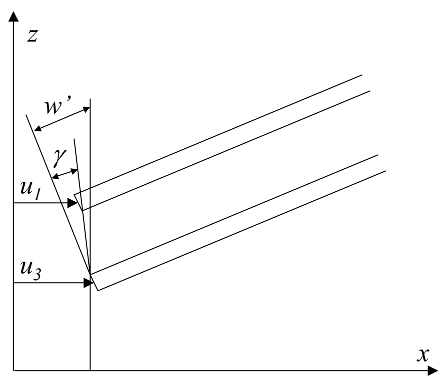

Figure 2. Geometry and deformation of the sandwich beam.

$$\gamma = \frac{1}{h_2}\left[u_1 - u_3 + \left(\frac{h_1 + 2h_2 + h_3}{2}\right)\frac{\partial w}{\partial x}\right] \tag{6}$$

where h is the thickness of the layer.

2.1.Finite Element Discretization

The FE method is essentially a process through which a continuum with infinite degrees of freedom can be approximated by an assemblage of sub-regions (or elements) each with a specified but now finite number of unknowns. Further, each such element interconnects with others in way familiar to engineers dealing with discrete structural or electrical assemblies (Zienkiewicz, 1971). The assumed mechanical degrees of freedom $\{u_1\ u_3\ w\ \theta\}^T$ are respectively the axial and vertical displacements, and the rotation around y-axis.

For the k^{th} element the corresponding generalized displacement vector $\{q_k\}$ can be assembled, which allows the determination of the discrete form of the displacements and rotation at the node i and j, that is:

$$\mathbf{q}_k = \begin{bmatrix} u_{1i} & u_{3i} & w_i & \theta_i & u_{1j} & u_{3j} & w_j & \theta_j \end{bmatrix}^T \tag{9}$$

This discretization procedure is achieved by using the shape functions $\mathbf{N}_{u_1}$, $\mathbf{N}_{u_3}$, $\mathbf{N}_w$, and $\mathbf{B}_w$, which relates the continuum displacements to discrete ones

$$\begin{aligned} u_{1i} &= \mathbf{N}_{u_1}\mathbf{q}_i \\ u_{3i} &= \mathbf{N}_{u_3}\mathbf{q}_i \\ w_i &= \mathbf{N}_w\mathbf{q}_i \\ \theta_i &= \frac{dw_i}{dx} = \frac{d\mathbf{N}_w}{dx} = \mathbf{B}_w\mathbf{q}_i \end{aligned} \tag{10}$$

Taking in acount the Lagrange linear shape functions for the axial displacement and a cubic Hermitian functions for the transverse displacement,

$$\mathbf{N}_{u_1} = \begin{bmatrix} 1 - x/L & 0 & 0 & 0 & x/L & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

$$\mathbf{N}_{u_3} = \begin{bmatrix} 0 & 1 - x/L & 0 & 0 & 0 & x/L & 0 & 0 \end{bmatrix} \tag{12}$$

$$\mathbf{N}_{w} = \left[0 \quad 0 \quad 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \quad x\left(1 - \frac{x}{L}\right)^2 \quad 0 \quad 0 \quad \frac{x^2}{L^2}\left(3 - \frac{2x}{L}\right) \quad \frac{x^2}{L}\left(\frac{x}{L} - 1\right) \right] \tag{13}$$

Rewriting (2), (3), (4), and (5) into variational formulation and taking in account the generalized coordinate show in (10) and a relation for γ in (6):

$$\begin{aligned} \delta T = {} & \rho_1 A_1 \int_0^L \delta \ddot{\mathbf{q}}^T \mathbf{N}_{u1}{}^T \mathbf{N}_{u1} \ddot{\mathbf{q}} dx + \rho_3 A_3 \int_0^L \delta \ddot{\mathbf{q}}^T \mathbf{N}_{u3}{}^T \mathbf{N}_{u3} \ddot{\mathbf{q}} dx \rho_3 A_3 + \ldots \\ & + \left(\rho_1 A_1 + \rho_2 A_2 + \rho_3 A_3\right) \int_0^L \delta \ddot{\mathbf{q}}^T \mathbf{N}_w{}^T \mathbf{N}_w \ddot{\mathbf{q}} dx \end{aligned} \tag{14}$$

$$\delta V_b = \left(E_1 I_1 + E_3 I_3\right) \int_0^L \delta \mathbf{q}^T \mathbf{B}'_w{}^T \mathbf{B}'_w \mathbf{q} dx \tag{15}$$

$$\delta V_a = E_1 A_1 \int_0^L \delta \mathbf{q}^T \mathbf{B}'_{u1}{}^T \mathbf{B}'_{u1} \mathbf{q} dx + E_3 A_3 \int_0^L \delta \mathbf{q}^T \mathbf{B}'_{u3}{}^T \mathbf{B}'_{u3} \mathbf{q} dx \tag{16}$$

$$\delta V_s = G^* A_2 \int_0^L \delta \mathbf{q}^T \mathbf{B}'_\gamma{}^T \mathbf{B}'_\gamma \mathbf{q} dx \tag{17}$$

The kinetic energy in a variational form can be write as,

$$\delta T = \delta \ddot{\mathbf{q}}^T \left(\mathbf{M}_b + \mathbf{M}_a\right) \ddot{\mathbf{q}} \tag{18}$$

where **M** is the mass matrix and the subscripts *a* and *b* are related to a axial and transverse moviment respectively.

$$\mathbf{M}_a = \rho_1 A_1 \int_0^L \mathbf{N}_{u1}{}^T \mathbf{N}_{u1} dx + \rho_3 A_3 \int_0^L \mathbf{N}_{u3}{}^T \mathbf{N}_{u3} dx \tag{19}$$

$$\mathbf{M}_b = \left(\rho_1 A_1 + \rho_2 A_2 + \rho_3 A_3\right) \int_0^L \mathbf{N}_w{}^T \mathbf{N}_w dx \tag{20}$$

Similarly the potencial energy can be expressed in a variational form like,

$$\delta V = \delta \mathbf{q}^T \left(\mathbf{K}_b + \mathbf{K}_a + \mathbf{K}_s\right) \mathbf{q} \tag{21}$$

where **K** is the stiffness matrix and the subscript *s* is related to shear energy.

$$\mathbf{K}_b = \left(E_1 I_1 + E_3 I_3\right)\int_0^L \mathbf{B}_w'^{\,T}\mathbf{B}_w' dx \tag{22}$$

$$\mathbf{K}_a = E_1 A_1 \int_0^L \mathbf{B}_{u1}'^{\,T}\mathbf{B}_{u1}' dx + E_3 A_3 \int_0^L \mathbf{B}_{u3}'^{\,T}\mathbf{B}_{u3}' dx \tag{23}$$

$$\mathbf{K}_s = G^* A_2 \int_0^L \mathbf{B}_\gamma'^{\,T}\mathbf{B}_\gamma' \, dx \tag{24}$$

2.2. The GHM Model of Material Properties.

In the theory of linear viscoelasticity for one dimensional structures the constitutive relation stress-strain could be represented by:

$$\sigma(t) = G(t)\varepsilon(0) + \int_0^t G(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau \tag{25}$$

where *G(t)* is the relaxation function of the viscoelastic material (the stress response to a unit step input). This stress relaxation represents energy loss from the material, hence damping (Balamurugan and Narayan, 2002).

The Golla-Hughes McTavish (GHM) method (McTavish and Hughes, 1993 and Golla and Hughes, 1985) represents the material modulus as a series of mini oscilator terms (Figure 3) or internal variables. requires the representation of material modulus function as a series of (damped) mini-oscillator terms or internal variables. This method was developed for direct incorporation into the finite element method. The material complex modulus can be written in the Laplace domain in the form.

$$s\widetilde{G}(s) = G^{\infty}\left[1 + \sum_k \alpha_k \frac{s^2 + 2\hat{\zeta}_k\hat{\omega}_k s}{s^2 + 2\hat{\zeta}_k\hat{\omega}_k s + \hat{\omega}_k^2}\right] \tag{26}$$

where the factor G^{∞} corresponds to the equilibrium value of the modulus – the final value of the relaxation function *G(t)*. Each mini-oscillator term is a second order rational function involving three positive constants($\alpha_k, \hat{\zeta}_k, \hat{\omega}_k$). These constants govern the shape of the modulus function over the complex *s-plane*. Depending on the nature of the material modulus function and the range of s over which it is to be modeled., any number of mini-oscillator terms may used in the GHM expression.

Consider the elementary mass-spring system with an applied force. With an elastic spring the motion of the system is described by the second-order equation of motion.

$$m\ddot{q}(t) + kq(t) = f(t) \tag{27}$$

Now, we allow the spring to be viscoelastic, described by relaxation function *k(t)*

$$m\ddot{q}(t) + \int_0^t k(t-\tau)\dot{q}(\tau)d\tau = f(t) \tag{28}$$

Initial conditions have been assumed to be zero for convenience. The material modulus function of *k(t)* is now modeled by a single term GHM expression thus:

$$s^2 m\widetilde{q}(s) + k\left[1 + \alpha\frac{s^2 + 2\hat{\zeta}\hat{\omega}s}{s^2 + 2\hat{\zeta}\hat{\omega}s + \hat{\omega}^2}\right]\widetilde{q}(s) = \widetilde{f}(s) \tag{29}$$

Now, an auxiliary coordinate z is introduced such that,

$$\tilde{z}(s)\frac{\hat{\omega}^2}{s^2+2\hat{\zeta}\hat{\omega}s+\hat{\omega}^2}\tilde{q}(s) \tag{30}$$

Using this new dissipation coordinate, the Laplace transformed equation of motion may be written as two coupled second-order equations:

$$\begin{aligned} &s^2 m\tilde{q}+(k+\alpha k)\tilde{q}-\alpha k\tilde{z}=\tilde{f} \\ &s^2\tilde{z}+2\hat{\zeta}\hat{\omega}s\tilde{z}-\alpha k\tilde{z}-\hat{\omega}\tilde{q}+\hat{\omega}^2\tilde{z}=0 \end{aligned} \tag{31}$$

2.3. GHM Viscoelastic Finite Element Matrices.

Multiplying the second equation of the above pair by $\alpha k/\hat{\omega}^2$, the resulting system of equations has a symmetric matrix second order time-domain realization.

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \alpha\frac{1}{\hat{\omega}^2}\mathbf{K} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{q}} \\ \ddot{\hat{\mathbf{z}}} \end{bmatrix}+\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \alpha\frac{2\hat{\zeta}}{\hat{\omega}}\mathbf{K} \end{bmatrix}\begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\hat{\mathbf{z}}} \end{bmatrix}+\begin{bmatrix} \mathbf{K}(1+\alpha) & -\alpha\mathbf{K} \\ -\alpha\mathbf{K} & \alpha\mathbf{K} \end{bmatrix}\begin{bmatrix} \mathbf{q} \\ \hat{\mathbf{z}} \end{bmatrix}=\begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{32}$$

This mechanical system has the mechanical representation shown in Figure(3).

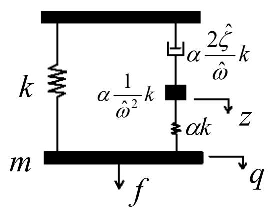

Figure 3. Mini-oscillator

Since the elastic element stiffness matrix **K** is usually positive semi definite (one or more eigenvalues representing rigid body motion) the mass matrix in this formulation will not usually be positive definite. To remedy this situation , spectral decomposition of the elastic stiffness matrix **K** is used:

$$\mathbf{K}=G^{\infty}\overline{\mathbf{K}}=G^{\infty}\overline{\mathbf{R}}\overline{\mathbf{\Lambda}}\overline{\mathbf{R}}^T \tag{33}$$

Here, $\overline{\mathbf{\Lambda}}$ is a diagonal matrix of the nonzero (necessarily positive) eigenvalues κ_p of the modulus factored out stiffness matrix **K**. The corresponding orthonormalized eigenvectors $\mathbf{r}_p$ form the columns of the matrix R,

$$\overline{\mathbf{\Lambda}}=diag\{\kappa_p\},\quad \overline{\mathbf{R}}=row\{r_p\},\quad \overline{\mathbf{R}}^T\overline{\mathbf{R}}=\mathbf{1} \tag{34}$$

such that

$$\overline{\mathbf{K}}\mathbf{r}_p=\mathbf{r}_p\kappa_p,\quad \kappa_p>0 \tag{35}$$

To achieve our objective of fewer dissipation coordinates and a positive definite viscoelastic mass matrix, we first factor the equilibrium modulus G back into the diagonal eigenvalue matrix $\overline{\mathbf{\Lambda}}$, ie, $\mathbf{\Lambda}=G^{\infty}\overline{\mathbf{\Lambda}}$ a , then let:

$$\mathbf{z}=\mathbf{R}^{\mathbf{T}}\hat{\mathbf{z}}\quad and\quad \mathbf{R}=\overline{\mathbf{R}}\mathbf{\Lambda} \tag{36}$$

Substitution thies equation in equation ,

$$\mathbf{M}_v\begin{bmatrix} \ddot{\mathbf{q}} \\ \ddot{\hat{\mathbf{z}}} \end{bmatrix}+\mathbf{D}_v\begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\hat{\mathbf{z}}} \end{bmatrix}+\mathbf{K}_v\begin{bmatrix} \mathbf{q} \\ \hat{\mathbf{z}} \end{bmatrix}=\begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{37}$$

where the viscoelastic matrices are

$$\mathbf{M}_v = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \alpha \dfrac{1}{\hat{\omega}^2} \mathbf{\Lambda} \end{bmatrix} \quad \mathbf{D}_v = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \alpha \dfrac{2\hat{\zeta}}{\hat{\omega}} \mathbf{\Lambda} \end{bmatrix} \quad \mathbf{K}_v = \begin{bmatrix} \mathbf{K}(1+\alpha) & -\alpha\mathbf{R} \\ -\alpha\mathbf{R} & \alpha\mathbf{\Lambda} \end{bmatrix} \tag{38}$$

These element matrices have the symmetry and definiteness properties desired for a standard second order structural dynamics model:

$$\mathbf{M}_v^T = \mathbf{M}_v > 0, \quad \mathbf{D}_v^T = \mathbf{D}_v, \quad \mathbf{K}_v^T = \mathbf{K}_v \tag{39}$$

3. GHM Model Validation.

In this section a validation procedure has been implemented in order to validate the GHM model for a smart damped sandwich beam. Comparisons with experimental and numerical results of other researchers are provided to ensure proper performance. The first validation procedure has been carried out in terms of verifying the present GHM model in order to validate the primary model. For this purpose a free rod in a longitudinal vibration and a cantilever rod both admensionalized have been considered according Golla and Hughes (1985). The viscoelastic properties of the rod modeled with 4 mini-oscillator and the non dimensionalized parameters are presented in Table 1 and the results obtained for a longitudinal vibration in Table 2 and for transverse vibration in Table 3.

Table 1. Mini-oscillator non dimensionalized properties.

Mini oscillator	α	β	δ
1	3.0×10^{-2}	4.16×10^{-1}	3.16×10^{-2}
2	3.0×10^{-2}	4.16	3.16
3	3.0×10^{-2}	4.16×10^{1}	3.16×10^{2}
4	3.0×10^{-2}	4.16×10^{2}	3.16×10^{4}

Table 2. Non dimensionalized frequency and damping for elastic and viscoelastic FEM in a longitudinal vibration.

			Viscoelastic FEM			
Mode	Elastic PDE (Golla and Hughes, 1985)	Elastic FEM (Golla and Hughes, 1985)	Frequency (Golla and Hughes, 1985)	Damping (Golla and Hughes, 1985)	Frequency Present	Damping Present
1	0	0	0	0	0	0
2	3.14	3.22	3.33	8.55×10^{-3}	3.33	8.55×10^{-3}
3	6.28	6.93	7.19	1.01×10^{-2}	7.19	1.01×10^{-2}
4	9.42	11.26	11.73	1.02×10^{-2}	11.73	1.02×10^{-2}
5	12.57	13.85	14.46	9.71×10^{-3}	14.46	9.71×10^{-3}

Table 3. Non dimensionalized frequency and damping for elastic and viscoelastic FEM in a transverse vibration.

			Viscoelastic FEM			
Mode	Elastic PDE (Golla and Hughes, 1985)	Elastic FEM (Golla and Hughes, 1985)	Frequency (Golla and Hughes, 1985)	Damping (Golla and Hughes, 1985)	Frequency Present	Damping Present
1	3.52	3.52	3.63.	8.60×10^{-3}	3.63.	8.60×10^{-3}
2	2.20×10^{1}	2.21×10^{1}	2.31×10^{1}	8.27×10^{-3}	2.31×10^{1}	8.27×10^{-3}
3	6.17×10^{1}	6.22×10^{1}	6.53×10^{1}	8.42×10^{-3}	6.53×10^{1}	8.42×10^{-3}
4	1.21×10^{2}	1.23×10^{2}	1.30×10^{2}	7.46×10^{-3}	1.30×10^{2}	7.46×10^{-3}
5	2.00×10^{2}	2.28×10^{2}	2.42×10^{2}	3.94×10^{-3}	2.42×10^{2}	3.94×10^{-3}

The second validation performed use the experimental results acquired by Lam (1997). In this experiment a free-free sandwich beam with aluminum face sheets constrained the core which is admited as ISD 112. Faces sheets have dimensions of *0.381m* in length, *0.0381 m* in wide and *0.0032 m* in thick, physical properties are assumed as *70 Gpa* for stiffness and $2700\ Kg/m^3$ for mass density. The core is made from the viscoelastic ISD 112 10 mil ($0.381m \times 0.0381m \times 0.000254m$ and $G^{\infty} = 5 \times 10^4$) which has the parameter for the GHM model present in Table 4.

Table 4. Parameters for ISD 112 GHM model.

Mini oscillator	$\hat{\alpha}$	$\hat{\zeta}$	$\hat{\omega}$
1	9.6	73.4	1×10^4
2	99.1	1.1	5×10^4
3	26.2	3.28	0.5×10^4

The free-free beam is experimented through a impact hammer and a accelerometer. The measurement point is located at the center of the beam and the impact occur at the center too. Figure (4) presents a schema for this test.

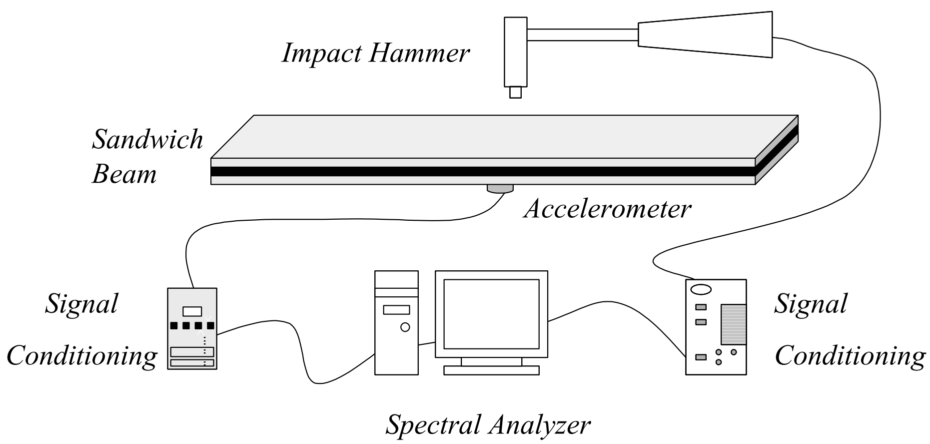

Figure 4. Set up for the experiment (Lam, 1997).

Using this configuration, Lam (1997) has proceeded an experimental measurement of frequency response functions (impact by gravitacional acceleration). The present GHM FE model has been used to predict the frequency response function (FRF) in the same conditions as in the Lam (1997) experiment. Figure 5 shows both Lam (1997) and the present GHM FE model FRFs, which clearly indicates how good is the present FE model in assessing the dynamic characteristics of the smart beam. Such results, reinforces the reliability of the present model in predicting dynamic characteristic of a smart beam. Unfortunately the location of the measurement and the excitation point is a node therefore the even modes have been lost.

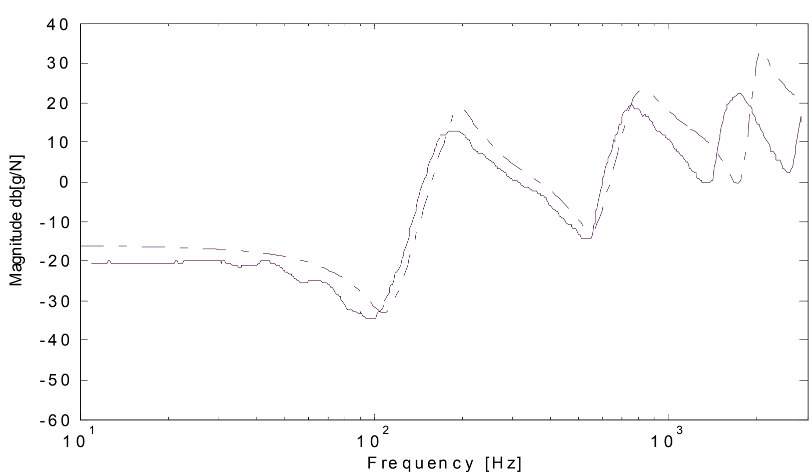

Figure 5. FRF produced Lam (1997) experiment (solid line) and calculated with the present FE model (dashed line).

4. Dynamical Characteristics of a Sandwich beam with smart fluids

After validating the GHM model FE with a variety of results, it encorouge to use the smart fluids as a core in a sandwich beam. For this purpose the dynamical characteristics of sandwich beam containing smart fluid has been analyzed. The experimental data has been performed by Yalcintas and Dai (1999) for a sandwich beam with a ER fluid as a core. The test beam has *381 mm* in length and *25.4 mm* in width and mass density of $2700\ Kg.m^{-3}$ and stiffness of *70 Gpa*. The elastic upper and lower plate material is aluminum at a thickness of *0.79 mm* and the ER material damping layer has *0.50 mm* in thickness and mass density of $1700\ Kg.m^{-3}$ The test has been done using a simple support configuration and a applied voltage of $3.5\ KV.mm^{-1}$. If one of this support is the reference, the actuation is applied at *115 mm* away from the reference support and the transverse vibration response is measured at *231 mm* away. The GHM model has the following parameters: $\alpha = 1$, $\zeta=5000$, $\omega = 4$, and $G^{\infty} = 0.4\ 10^6$.

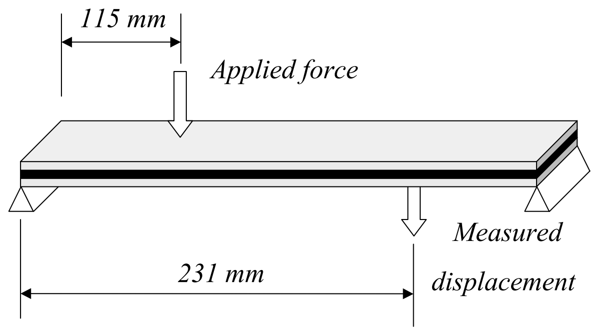

Figure 6. Experimental setup for ER sandwiched beam (Yalcintas and Dai, 1999).

The results have been obtained using 10 elements and 4 physical degrees and 3 dissipation degrees per node and they get good agreement especially if is considered that only one mini-oscillator were carried out. Representing better the behaviour of the ER/MR fluid is one of the major challenge in the development of this kind of fluid. Some procedures for curve fitting and hence better choices of the GHM parameters are available (Park et al., 1999).

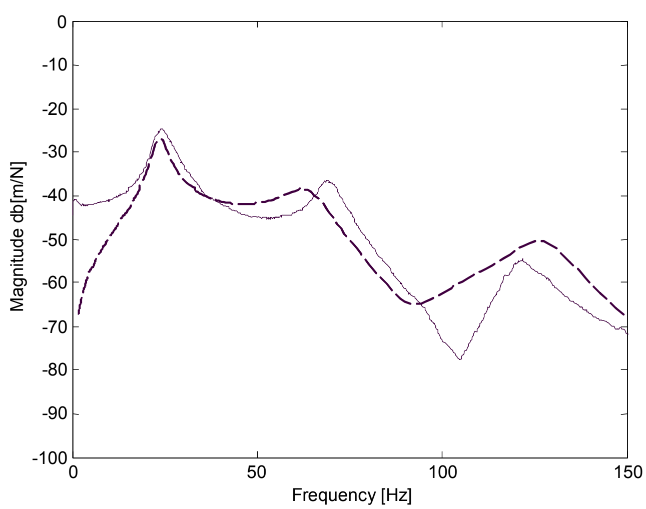

Figure 7. FRF produced in Yalcintas and Dai (1999) experiment for a voltage of $3.5\ KV.mm^{-1}$ (solid line) and calculated with the present FE model (dashed line).

5. Conclusions

A GHM finite element model including a ER/MR fluid core has been developed from the viscoelastic theories. The main steps in formulating the GHM FE model have been presented and the resulting motion equations has been compared to results found in the related literature, presenting good agreement. The validation has been carried out with a non dimensionalized model present in the first works. Experimental results extracted from the technical literature have been also used, proving that the GHM FE model is capable of adequate dynamic characteristic prediction. The ER/MR model has been also tested in terms of its capabilities in simulation the experimental data performed by the previous researchers. The results present a good agreement permiting the use of the GHM FE model in ER/MR sandwiched beam. One disadvantage of this method is that it increases the number of degrees of freedom because the dissipation modes. It could be solved by the use of the reduction methods which are widely available in literature. So it could be concluded that this model is applicable for control studies.

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6. References

Balamurugan, V., and Narayanan, S., 2002, "Finite Element Formulation and Active Vibration Control Study on Beams using Smart Constrained Layer Damping (SCLD) Treatment", Journal of Sound and Vibration, Vol. 249, No 2, pp. 227-250.

Bisplinghoff, R. L., Ashley, H. and Halfman, R., 1996, "Aeroelasticity", Dover, New York, 860 p.

Gamota, D. R., and Filisko, F.E., 1991, "Dynamic mechanical studies of electrorheological materials: Moderate frequencies", Journal of Rheology, Vol 35, No 3, pp. 399-425.

Golla, D.F., and Hughes, P.C., 1985, "Dynamics of Viscoelastic Structures – a time domain, finite element formulation", Journal of Applied Mechanics, Vol. 52, pp. 897-906.

Kamath , G.M., and Wereley, N.M.,1997, "A nonlinear viscoelastic-plastic model for electrorheological fluids", Smart Materials and Structures, Vol. 6, pp. 351-359.

Lam, M.J. (1997), "Hybrid Active/Passive Models with frequency dependent damping", PhD. Thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA, 146p.

McTavish, D.J., and Hughes, P.C., 1993, "Modeling of Linear Viscoelastic Space Structures", Journal of Vibration and Acoustics, Vol 115, pp. 103-110.

Park, C.H., Inman, D.J., and Lam, M.J., 1999, "Model Reduction of Viscoelastic Finite Element Models, "Journal of Sound and Vibration, Vol. 219, No 4, pp. 619-637.

Sims, N. D., Stanway, R., and Johnson, A.R., 1999, "Vibration Control Using Smart Fluids: A State of the Art Review", The Shock and Vibration Digest, Vol. 31, No. 3, pp. 195-203.

Yalcintas, M., and Dai, H., (1999), "Magnetorheological and electrorheological materials in adaptive structures and their performance comparison", Smart Materials and Structures, Vol. 8, pp.560-573.

Yalcintas, M., Coulter, J.P., and Don, D.L., 1995, Vol. 4, pp. 207-214.

Yalcintas, M.Y., and Coulter, J.P., 1998, "Electrorheological material based non-homogeneous adaptive beams", Smart Materials and Structures, Vol. 7, pp128-143.

Yanju, L., Hejun, D., and Dianfu, W., "ER fluid based on inorganic/polymer blend particles and its adaptive viscoelastic properties", Colloids and Surfaces, Vol. 189, pp. 203-210.