# BOUNDARY ELEMENT ANALYSIS OF PANELS REINFORCED BY ADHESIVES PLATES

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**Abstract.** In this work, the boundary element method is used for the analysis of shear stresses in the adhesive layer of reinforced aeronautical panels with bonded composites plates. Plates are modeled by formulations of plane elasticity; the adhesive is modeled considering its shear modulus as constant, and computing the difference of displacements between the plates. Due to the adhesive, body forces appear in the plates that produce domain integrals in the formulation. These domain integrals are transformed into boundary integrals by using the dual reciprocity boundary element method. Numerical examples are modelled. Results were compared with analytical results.

***Keywords***. *Reinforcement by adhesion, boundary elements, adhesive patches.*

## 1. Introduction.

Cracked structures that work either with fatigue loads or in an aggressive ambient medium, are very frequent in several types of industries, especially in aeronautics, naval and petrochemistry. When the crack length is close to a critical length, the life of the structure or mechanical component is over. The change of this component for a new one is usually an expensive solution even when it is possible to carry out it in the required time. An alternative solution, cheaper and faster, is the repair of the damaged structure.

The repair and/or recovery of cracked structures can be carried out by many processes as for example, the structural reinforcement with the fixation of welded or mechanically fastened patches.

The most important role of the patch in a cracked structure is to carry out load deviation, so that the patch absorbs an appreciable amount of efforts that were initially supported by the structure, diminishing the stress intensity factor and inhibiting the crack growth, Salgado (1998).

Structural adhesive reinforcements have been investigated since 1970, and used in the aeronautical industry since 1980 (Wang et all, 1997). At the moment they are considered efficient solutions to repair cracked fuselages of airplanes. They present many advantages if compared to the welded repair, where many metallurgic problems exist.

Patches that are mechanically fixed by fasteners or by rivets has a shorter life due to the effects of stress concentration caused by these elements. Riveted patches are generally made either of aluminium or of the same material of the repaired structure. Contrarily, the use of adhesives to fix patches allows to use a diversity of materials that cannot be usually perforated, as for example composites. These materials present high stiffness and specific mechanical resistance, besides being chemically stable. Another important advantage of adhesive patches is that the area of load transference is bigger than in riveted patches.

From the presented advantages, we can conclude that, in a near future, repair techniques and union by adhesion will compete very well in some industrial fields with the traditional industrial processes of union, especially in the repair and production of aeronautical and naval structures.

At the moment, there is little diffusion of computational approaches for the design of adhesive joints. This is one of the main reason for the lack of use of adhesives in patch repairs.

The analysis of cracked structures can be carried out by methodologies developed by Fracture Mechanics. In this application it is necessary the use of numerical methods, because closed analytical solutions exist only for cases of very simple geometry. Nowadays, the two most used numerical methods are the finite element method and the boundary element method. A basic difference between these methods is that while in the finite element method the discretization and numerical integration in the whole domain is required, the boundary element methods only needs to discretize and integrate the contour of the analyzed structure or component (Brebbia et all, 1989; and Kane, 1993).

In this work, structures reinforced by adhesives patches are analyzed by the use of the boundary element method for two-dimensional analysis. Interaction forces between the panel and the patch are considered as body forces. Since these forces act in the domain, it is necessary to use the dual reciprocity boundary element method (DRM) in order to transform domain integrals into boundary integrals.

In order to model the structure and the adhesive patch, boundary element formulations for isotropic plates are used; the adhesive layer was mo deled considering a linear elastic behavior.

## 2. Formulation of Boundary Elements.

The displacement and traction fundamental solution for plane elasticity are given by;

$$u^*_{ij} = \frac{1}{8pm(1-n)}\left[(3-4n)\ln\frac{1}{r}d_{ij} + r_{,i}r_{,j}\right] \tag{1}$$

$$t^*_{ij} = -\frac{1}{4pr(1-n)}\left\{\frac{\partial r}{\partial n}\left[(1-2n)d_{ij} + 2r_{,i}r_{,k}\right] - (1-2n)\left(n_i r_{,j} - n_j r_{,i}\right)\right\} \tag{2}$$

References as Dominguez (1993) and Kane (1993) have a complete demonstration of these solutions. A recent application of the boundary element method for adhesive joint analysis is found in Wen et all (2002).

With the application Betti’s theorem, and the identity of Somigliana in the fundamental solution, an integral equation can be determined as:

$$\int_\Gamma t_i\, u^*_{ik}\, d\Gamma + \int_\Omega b_i\, u^*_{ik}\, d\Gamma = \int_\Gamma t^*_{ik}\, u_i\, d\Gamma + c_{ij}\, u_i \tag{3}$$

Disregarding body forces and dicretizing the boundary, equation (3) can be rewritten as:

$$c_{ij}u_i + \sum_{j=1}^{ne}\int_\Gamma t^*_{ik}u_i\, d\Gamma_e = \sum_{j=1}^{ne}\int_\Gamma t_i\, u^*_{ik}\, d\Gamma_e \tag{4}$$

Since Eq. (4) is applied for each one of the *ne* boundary element nodes, a linear system is obtained that can be written in a matrix form as:

$$\mathbf{Hu} = \mathbf{Gt} \tag{5}$$

This system of equations is manipulated algebraically to obtain a unique solution.

The solution of the integral Eq. (3) considers forces in the domain (body forces) in their formulation. The DRM is a method developed by Brebbia and Nardini in 1982 (Alburquerque, 2001; Nardini et all, 1982; and Wen, 2000) that allows the transformation of domain integrals into boundary integrals.

With the application of the DRM, the body forces can be written as:

$$b_k = \sum_{j=1}^{N+L} f^j a^j_k \tag{6}$$

Where $\alpha_k$ are coefficients to be determined and $f^j$ is an approximation function.

The DRM uses particular solutions as proposed by Nardine and Brebbia (1982). Equation (3) can be written in a discretized form, applying DRM to transform domain integrals into boundary integral, as:

$$\mathbf{c}^i\mathbf{u}^i = \int_\Gamma \mathbf{u}^*\mathbf{t}\, d\Gamma - \int_\Gamma \mathbf{t}^*\mathbf{u}\, d\Gamma + \sum_{j=1}^{N+L}\left(\mathbf{c}^i\hat{\mathbf{u}}^{ij} + \int_{\mathbf{G}} \mathbf{t}^*\hat{\mathbf{u}}^j\, d\Gamma - \int_\Gamma \mathbf{u}^*\hat{\mathbf{t}}^j\, d\Gamma\right) \tag{7}$$

That can be written in a matrix form as:

$$c^i u^i + \sum_{k=1}^{N}\overline{H}_{ik}u_k - \sum_{k=1}^{N}G_{ik}t_k = \sum_{j=1}^{N+L}\left(c^i\hat{u}_k^{\ ij} + \sum_{k=1}^{N}\overline{H}_{ik}\hat{u}^j_k - \sum_{k=1}^{N}G_{ik}\hat{t}^j_k\right)\cdot a^j \tag{8}$$

This final equation doesn't have any domain integral, since they were transformed into boundary integrals. If Eq. (8) is applied for all *N* boundary nodes, a matrix equation given by:

$$\mathbf{H}\cdot\mathbf{u} - \mathbf{G}\cdot\mathbf{t} = \left(\mathbf{H}\cdot\hat{\mathbf{U}} - \mathbf{G}\cdot\hat{\mathbf{T}}\right)\cdot\mathbf{F}^{-1}\cdot\mathbf{b} \tag{9}$$

is obtained.

**3. Numerical implementation of DRM.**

The approximation function used in this work is given by;

$$f = 1 - r \tag{10}$$

This approximation function is well known from many other previous works see for example Salgado et all (1998).

Table 1. Properties of material used by Elast_patch software

| Material | E [GPa] | G [GPa] | n |
|---|---|---|---|
| Aluminum 7075 | 72 | 28 | 0.32 |
| Steel | 200 | 75.8 | 0.33 |
| Titanium | 117 | 44.8 | 0.31 |
| Epoxy adhesive[1] | 1.1 | 0.44 | 0.30 |
| Boro-Epoxi[2] | 190 | 7 | 0.17 |
| Epoxy adhesive[3] | 2.8 | - | 0.40 |

[1] Structural Adhesive 3M AF-163-2k
[2] Composite used in aeronautical patches.
[3] Epoxy adhesive Ciba Geigy AY103

In the interface of the program it is possible to choose the material of the panel and of the patch, as well as the properties of the adhesive; Tab. (1) shows materials and their properties that are stored in the database of the program.

In numerical results, it is important to analyze the number of boundary elements used in the discretization, and also the number of internal nodes. Figure (1) presents the error of the numerical solution computed by DRM compared to the analytical solution, in this case radial displacements of a revolving disk with radius equal to 3 m, density equal to 8000 kg/m$^3$, and speed equal to 20 rad/seg. Firstly the solution was determined with 5 elements. In order to improve results, it is necessary to increase the number of elements and/or internal nodes. With 20 internal nodes, the answer has an error around 2%. The introduction of more internal nodes improves the accuracy of results. However it strongly increases the computer time.

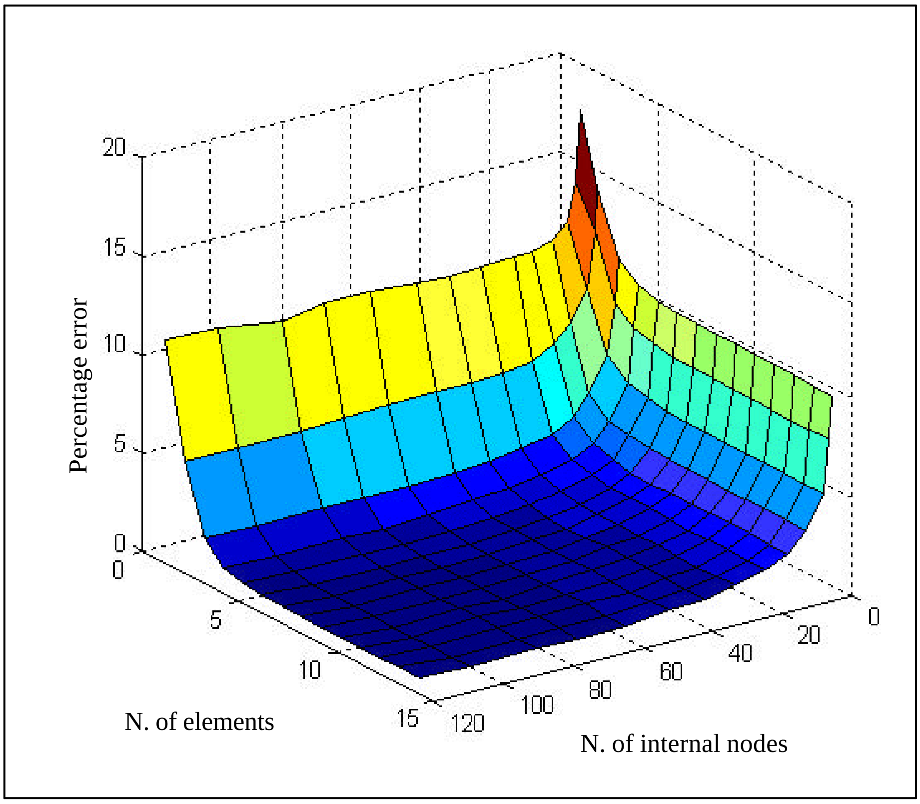



Figure 1. Percentage error in function of the elements number and the internal nodes number for revolving disk.

This numerical example gives us a clear idea of the magnitude of the error and the quantity of nodes and elements that are needed in order to have a reasonable accuracy.

#### 4. Modeling of the adhesive patch

Figure (2) shows a schematic elliptic pattern of patches that at the moment is one of the most accepted in aircraft repairs. Aplied loads can be uniaxials or in the plane.

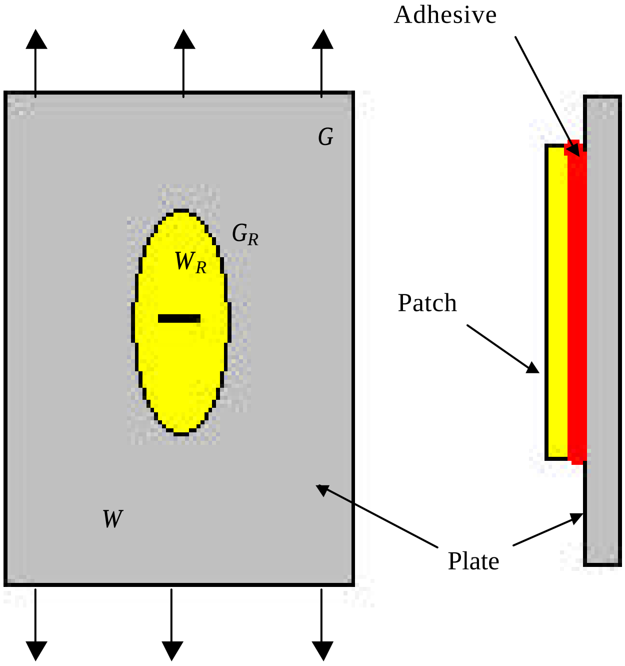



Figure 2 - Reinforced plate with adhesive patch.

The form of the elliptic patch becomes circular when the relationship between length and width of the ellipse is one.

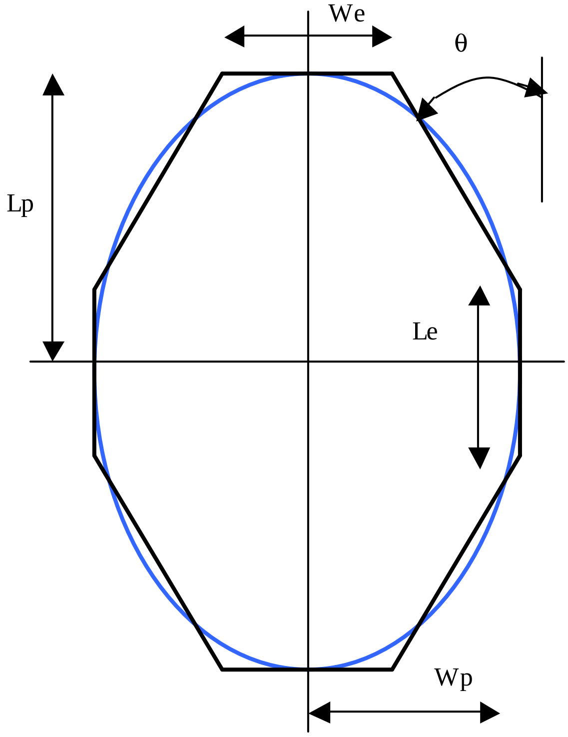



Figure 3 - Equivalent octagonal patch.

For easiness of production and cut, a method exists to calculate an octagonal patch equivalent to the modeling patch (Duong et all, 2002), see Fig. (3). Equations (11), (12), and (13) allow the calculation of this equivalent patch.

$$Le = 0.345 * 2Lp \quad (11)$$

$$We = 0.345 * 2Wp \qquad (12)$$

$$q = \tan^{-1}[\frac{2Wp - We}{2Lp - Le}] \qquad (13)$$

In order to design the patch, it is necessary to analyze shear stresses on the adhesive layer. A failure criteria that is reasonably accepted for adhesives is the modified von Mises failure criteria proposed by Gali, mentioned in Mortesen and Thomsem (2002). This criteria is defined by:

$$s_{vm} = Cs \times \sqrt{J_{2D}} + Cv \times J_1 \qquad (14)$$

where:

$$Cs = \frac{\sqrt{3} \times (1 + l)}{2l} \qquad (15)$$

$$Cv = \frac{l - 1}{2l} \qquad (16)$$

$$l = \frac{S_c}{S_t} \qquad (17)$$

$J_{2D}$ is the second invariant of the deviatoric stress, and $J_1$ is the first invariant of the stress state, $\lambda$ is the rate between compression strength and traction strength of the adhesive.

$$J_{2D} = \frac{1}{6}[(s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2] \qquad (18)$$

The adhesive that joins the two panel pieces and reinforces them is modeled as a linear stiffness (spring) that resists shear efforts. This model, proposed by Volkersen mentioned in Mortesen and Thomsem (2002), considers that all points of the adhesive layer are in a state of pure shear stress. As a consequence, the first invariant of the stress state is null (hydrostatic stress).

To avoid failure, then it is necessary that:

$$s_{vm} \leq S_{ult} \qquad (19)$$

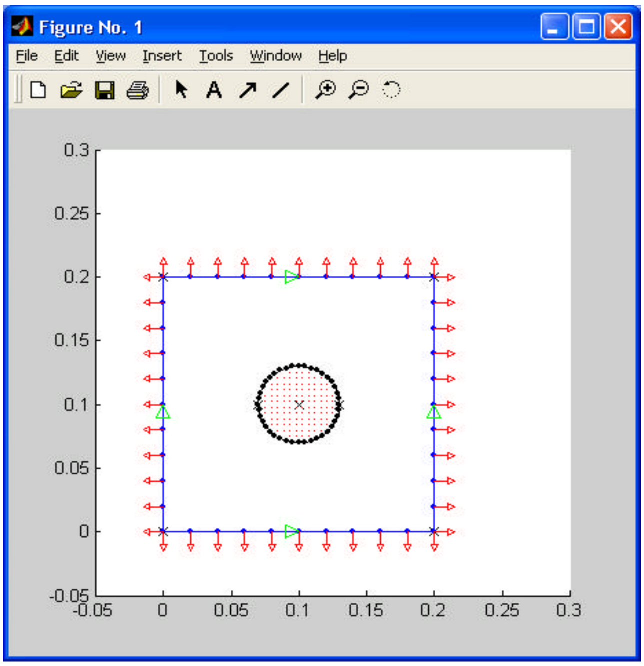



Figure 4. Circular Patch: Mesh and boundary elements generation.

The stress on the adhesive, computed by considering differences between panel and patch displacements deformed by external loads, is given by:

$$t_j(x') = \left[u_j(x') - u_j^R(x')\right] \times \frac{G_A}{h_A} \tag{20}$$

Where $x'$ represents a generic point of the deformed domain.

Undeformed points of the panel and patches are coincidents, displacements that occur due to external loads can be computed by:

$$u_j(x') - u_j^R(x') = \frac{h_A}{G_A} \cdot b_j(x') \tag{21}$$

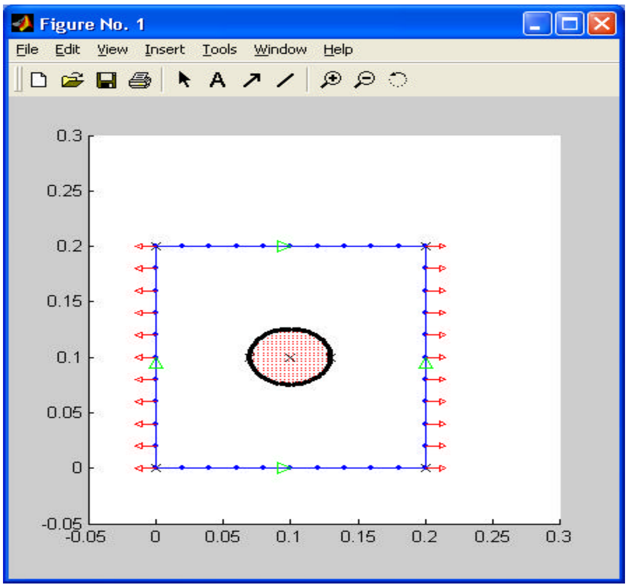
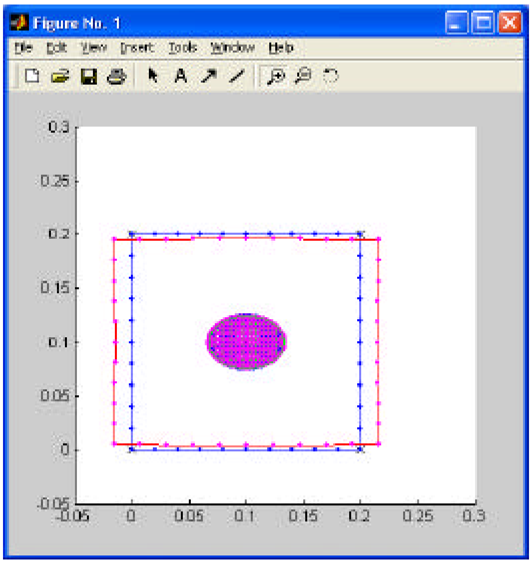



Figure 5. Patch elliptic, generation of elements, non deformed mesh, with solicitation in X direction.

**5. Numerical Examples.**

The example of the Fig. (4) is reported in the literature. In this examples a squared plate reinforced with a circular patch loaded in the X and Y directions with the same load magnitude ($9.10^9$ Pa) is modeled. This example was used in order to validate the model developed in this work. Results obtained by Lourenço (2000) and Salgado (1998) were used in order to check accuracy of the proposed method.



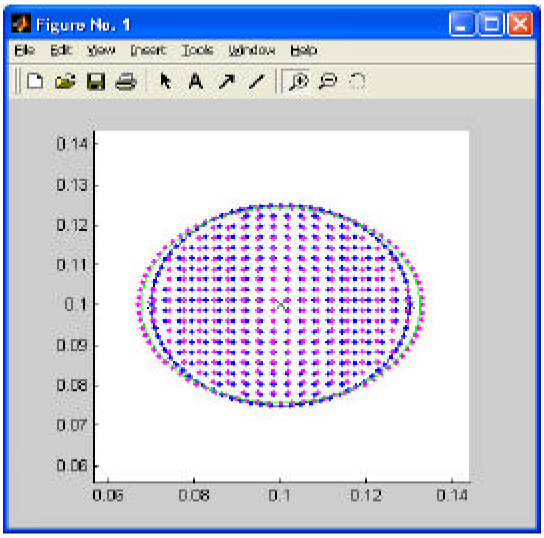




Figure 6. Patch elliptic, analysis result, deformed geometry.

The necessity of patches with more flexible geometries than the circular ones, leads to the development of an elliptic geometry. With this geometry you can guide the patch in such a way that the direction of the biggest stiffness coincides with the direction of the biggest load component that acts on the damaged plate. Figure (2) shows schematically how as a plate weakened by a crack can be reinforced with an elliptic patch, providing an area saving and good stiffness in the direction of the load that tries to extend the crack.

Additionally, the elliptic geometry allows a variation of the relationship width / length that provides great flexibility in the selection of patches to be used in according to the necessities of the problem.

In Fig.(5) a numerical example of an elliptic patch is presented. Figure (6) shows results of the example, where you can see the deformation of the panel and the deformation of the adhesive patch.

Points in blue (dark) in Fig. (6) represent the non deformed points of the panel that are coincident with the points of the adhesive patch. Points in magenta (clear) represent the deformed points of the panel, and the green line (clear) marks represent the boundary of the deformed patch.

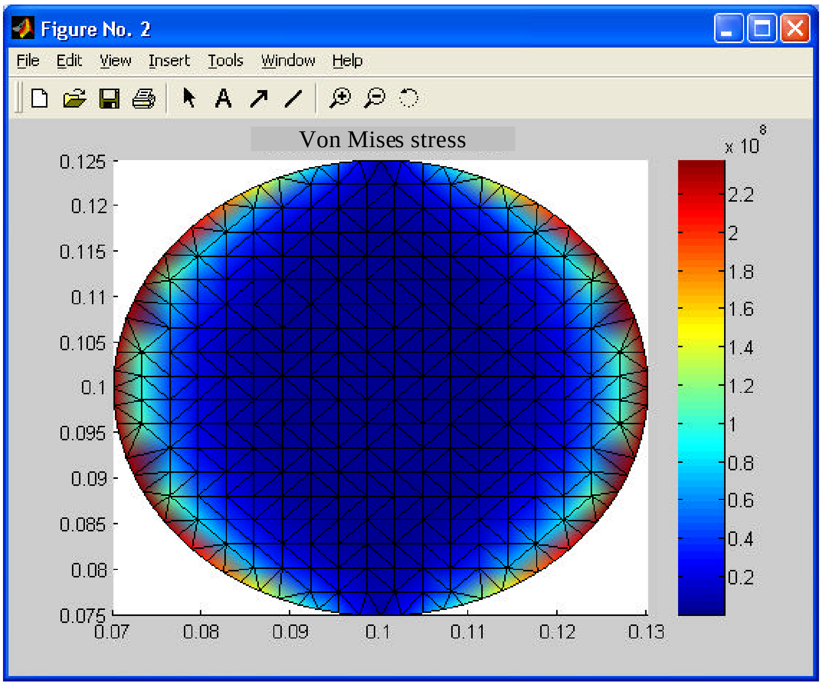



Figure 7. Analysis result, von Mises stress on the adhesive layer.

The results of the stress analysis with modified von Mises is shown in the Fig. (7). In the numerical example, an adhesive Ciba Geigy AY103 was used with an ultimate strength of 71 MPa, $l$ = 1.3, and elasticity module of 2800 MPa (Mortesen and Thomsem, 2002).

The comparison of the effective von Mises stresses with the strength of the adhesive allows to predict the failure, so that you can design the dimensions of the adhesive patch interactively to avoid the failure.

**6. Conclusion and Discussion.**

The method of dual reciprocity boundary elements is a method that allows efficient structural analysis, without the necessity of discretizing the domain, even when there are body forces. Thus, an the main advantage of the boundary element method that is the boundary only discretization is maintained.

The presented approach offers useful results in the analysis of adhesive patches of isotropic material. However the modern tendency of the use of composites materials in the repair of aeronautical structures makes necessary to extend the analysis to orthotropic and anisotropic patches.

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**Annex 1 – Nomenclature**

E - Young's modulus
$G_A$ - Shear modulus of adhesive
$h_A$ - Adhesive layer thickness
$\sigma_1, \sigma_2, \sigma_3$, - Principals components of stress
$\nu$ - Poisson's ratio
$\delta_{ij}$ - Kronecker Delta function
$u_{ij}$ - Displacement
$t_{ij}$ - Body forces
n - Normal to the boundary
$\hat{\mathbf{U}}, \hat{\mathbf{T}}$ - Particular solutions on the boundary
$c_{ij}$ - Coefficients of BEM
$b_k$ - Body forces of MDR
$a_k^j$ - Coefficients of MDR
$f^j$ - Approximation function of MDR.
**H, G** - Coefficients matrices of BEM
**b** - Shear load vector in a point of adhesive layer
N - Node number on the boundary
L - Node number in domain
Lp - Bigger side of elliptic patch.
Wp - Smaller side of elliptic patch.