LOADING RESPONSE OF TRANSMISSIONS SETS USING EPICYCLIC GEAR TRAINS

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Abstract. *Epicyclic gear trains are important part of automotive transmissions. They are present in automatic transmissions, the focus of this work, and in differentials of most vehicles. The dynamic behaviour of a single epicyclic gear is well known, and this work presents improvements on a model of a complete transmission. Initially the Wilson gearbox, which is modelled later, is presented. The torque flow across the transmission is modelled using the model previously built. Finally, after the modelling, the results of a simulation using the original model and the new improved model are presented.*

Keywords *planetary gear trains, modelling, simulation, loading*

1. Introduction

A lot of effort has been put on dynamic modeling of epicyclic gear trains, EGTs, also known as planetary gear trains, PGTs. However, all those efforts have been used to describe just one set, as made by Kahraman (1994). Some improvement has yet to be done in the study of transmission sets using more than one epicyclic gear train. This kind of mechanism is used in some types of automatic transmission of automobiles. It is usually associated with torque converters. There are several construction forms using two to four EGTs and up to two clutches, as said by Amaral (2000). Morais et al.(2002a) and Morais et al. (2002b) obtained some results about dynamic simulation of the Wilson gearbox recently.

The main objective of this work is to present the influence of the loading on the dynamic model of the Wilson gearbox. This transmission was chosen because it is one of the most common examples of epicyclic gear trains. The motion equations are determined using the Lagrangian equations. The following part of this work involves the simulation of the transmission with a control set of inertias. After this step some sets of loading are simulated and the results obtained are presented.

2. The wilson gearbox

Major Wilson invented the Wilson Gearbox, Amaral (2000) . It is used mainly in off-road vehicles, such as tractors and tanks. It consists of 4 planetary gear trains and one clutch. Figure 1 shows the Wilson gearbox and components used to set the gear ratios.

A given gear ratio is set with the components as marked in Figure 4. It is necessary to set the 1st gear to brake the ring gear of EGT 1. The 2nd is set when the ring gear of EGT 2 is braked. The solar gear of EGT 3 is the element that has to be braked to set the 3rd gear. The coupling of the clutch gives the 4th gear and the reverse is set when the ring gear of EGT R is braked. The braking of all the gears, except the case of 4th gear, must reach the full stop situation, as an essential condition for the gear ratio to be set.

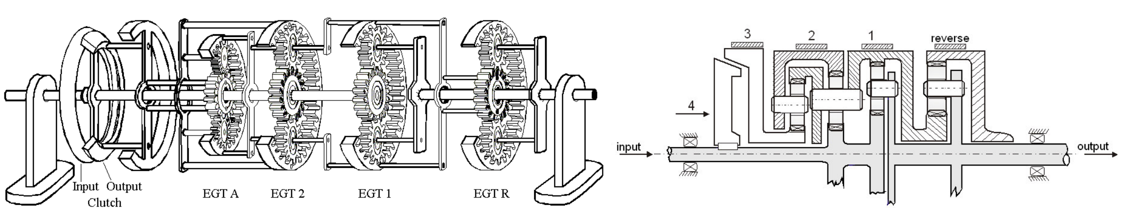

Figure 1. The components of the Wilson Gearbox and the components used to set the gears and sketch of the Wilson Gearbox

3. Modeling the Wilson gearbox

A model of the Wilson gearbox was presented in recent past; see Morais et al. (2002a) and Morais et al. (2002b). This model is used in this work, and some improvement has been done to better characterize the simulation results. In this work the main concern is to model the torque flow in the components of the transmission. That means to evaluate how much of the torque is used on each gear of each EGT.

The first step in this torque flow modeling is to introduce 2 relationships between the gears of a given EGT. Equations (1) and (2) show these values.

$$\tau_{y,k} + r_k \tau_{x,k} = 0 \tag{1}$$

$$\tau_{z,k} + (1 - r_k)\tau_{x,k} = 0 \tag{2}$$

The value of r_k is given by Eq. (3), if k=1, 3, 5, or 7, and Eq. (4), if k=2, 4, 6, or 8.

$$r_k = \frac{Z_A}{Z_P} \tag{3}$$

$$r_k = -\frac{Z_P}{Z_S} \tag{4}$$

If k is equal to 1 or 2, then the components of EGT A are taken in account. EGT 2 leads to r_3 and r_4. EGT 1 is associated to r_5 and r_6. Finally, r_7 and r_8 are linked to EGT R.

A torque flow on an EGT representation is necessary to model it. This model take the torque balance on each gear, and on each gearing, and set the values of each torque applied, Amaral (2000). Figure 3 shows the complete torque flow diagram for the Wilson gearbox. In this figure, the green lines show the input torques of the transmission. These torques are the resistive torques that set each gear ratio (τ_A, τ_B, τ_C, τ_D) and the input torque of the transmission (τ_{in}). The red lines show the connected components. In this case the balance of torque must take in account all components linked. The blue line shows the output torque of the transmission (τ_{out}). The black lines represent the torques that are applied on each gear and gearing of the Wilson gearbox.

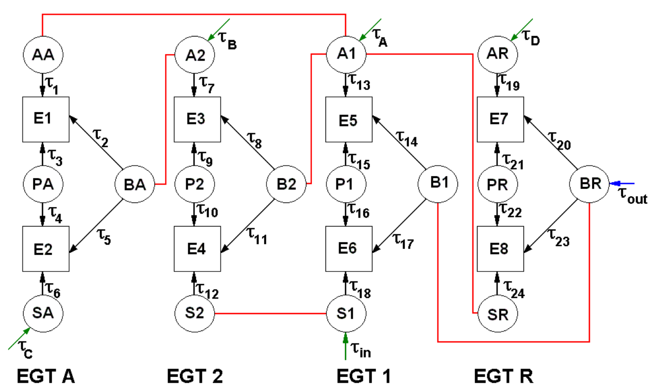

Figure 2. Torque flow diagram of the Wilson Gearbox

Figure (2) shows the output torque as an input torque. This occurs because the impact of the loading torque is not as simply as is on a conventional gearbox. The torque flow diagram show that a torque applied on any gear may affect all the system. This result of this modeling is a set of equations that give how the loading torque is seen by the input axis. Equation (5) gives the torque balance of the planet carriers of the EGTs 1 and R. This procedure is used to all gears, but the number of equations is smaller than the number of components because of the links among them.

$$\tau_{14} + \tau_{17} + \tau_{20} + \tau_{23} = \tau_{out} \tag{5}$$

Each gear has a resistive torque that characterizes it. In the first gear the resistive torque, τ_A, is applied on the annular gear of EGT 1. The torque balance of this component is presented in Equation (6). This equation has two parts: the first, that considers the resistive torque, is used to describe the first gear. The other is used to describe all other gear ratios.

$$\tau_1 + \tau_8 + \tau_{11} + \tau_{13} + \tau_{24} = \begin{cases} \tau_A \\ 0 \end{cases} \tag{6}$$

The other resistive torques are applied to the other gears. τ_B is used to describe the second gear, τ_C sets the third and fourth gears and τ_D is used in order to obtain the reverse speed.

This small procedure described here leads to a system of 26 equations. Once that this system is solved all the torque in transmission are known. The most important is to know how the output torque is seen by the input axis of the transmission. Equation (7) shows the output torque as seen by the input axis when the first gear is set.

$$\tau_{eng,1} = -\left(\frac{1}{1-r_6}\right)\left\{\frac{(1-r_5)}{r_5}\tau_A - \tau_{load}\right\} \tag{7}$$

This equation shows that the loading torque in the input axis is dependent not only of the loading, but also is dependent of the resistive torque. Equation (8) gives the expression obtained for the second gear.

$$\tau_{eng,2} = \left(\frac{1}{1-r_6}\right)\tau_{load} + \left\{\frac{1}{r_3 r_4} + \frac{(1-r_5)(1-r_3)}{(1-r_6)r_3 r_5} + \frac{(1-r_5)(1-r_4)}{(1-r_6)r_4}\right\}\tau_{load} \tag{8}$$

Equation (9) gives the expression obtained for the third and fourth gears.

$$\tau_{eng,3/4} = \left\{\frac{(1-r_2)+r_2(1-r_1)}{r_3 r_4} + \frac{\gamma(1-r_1)}{r_5(1-r_6)}\right\}\tau_{C,D} - \frac{1}{r_3 r_4(1-r_6)}\tau_{load} \tag{9}$$

In this case the value of γ is given by Eq. (10).

$$\gamma = r_1 r_2\left(\left[\frac{(1-r_4)}{r_3 r_4}\right] - \left[\frac{(1-r_3)}{r_3}\right]\right)\left[(1-r_2)+r_2(1-r_1)\right] \tag{10}$$

Equation (11) gives the expression obtained for the reverse.

$$\tau_{eng,R} = \frac{1}{1-r_6}\left[\tau_{load} - \left\{\frac{1-r_7}{r_7} + \frac{1-r_8}{r_7 r_8} + \frac{1-r_5}{r_5 r_7 r_8}\right\}\tau_D\right] \tag{11}$$

4. Simulation

The simulation is based on the model established previously by Morais et al. (2002a) and Morais et al. (2002b). In those cases the simulation had no loading. This fact does not lead to a realistic set of results, besides this set is very important in knowing the influence of the component inertias on the dynamic behavior of the transmission.

The modeled torques are the engine torque and the brake torque. The engine torque was modeled as a polynomial curve as shown in Figure (3). The brake torque was given by Equation (12), and its graphical form is shown in Fig. (3). M represents the maximum torque applied to the gears, and, T, the time needed to reach this torque. The values used for T and M were 1 s and 50 Nm, respectively. The brake torque model is hypothetical and is available in the literature, as showes by Doughty (1988).

$$T = \begin{cases} \frac{1}{2}M\left(1-\cos\left(\frac{\pi t}{T}\right)\right) & \text{if } t \le T \\ M & \text{if } t > T \end{cases} \tag{12}$$

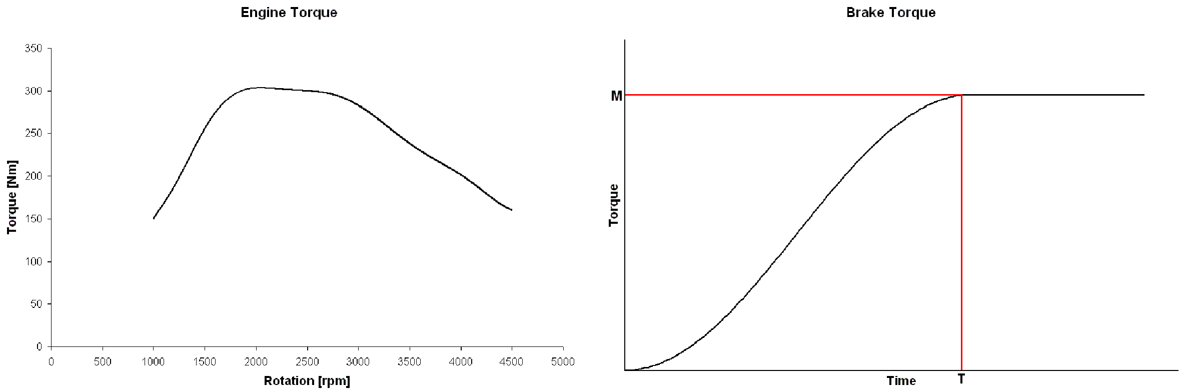

Figure 3. The engine torque curve and the schematic brake torque curve used in the simulation.

The data shown in Table (1), were used to describe the EGTs. The loading used to be applied on the simulation is given by Eq. (13).

$$\tau_{load} = Amp \cdot sin(\omega t) \tag{13}$$

Table 1. Numerical properties of the Wilson gearbox.

	EGT 1			EGT 2			EGT A			EGT R		
	A	P	S	A	P	S	A	P	S	A	P	S
Moment of Inercia (kgm²)	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
Number of Teeth	156	50	56	120	40	40	120	43	44	156	46	64

The simulation was done from first to fourth gears. The results presented here are that obtained with the loading applied. These results of the simulation where no loading is applied is also presented, it is showed as parameter of comparison.

The amplitude, A, of the loading torque is 500 Nm, for 1st, 3rd, and 4th gear, and 100 Nm for the 2nd gear. The frequency, ω, of all loadings is 100 rad/s.

Figure (4) shows variation of the speed for the first gear over time, Curve A represents input axis rotational speed, Curve B shows the values for the output axis speed and Curve B represents the speed of the braked element. The second half of each figure shows the gear ratio behavior over time, the instantaneous gear ratio is represented by the curve that is not constant and the constant line shows the expected value for that gear. Figure (5) show the results obtained when the loading is applied to the first gear.

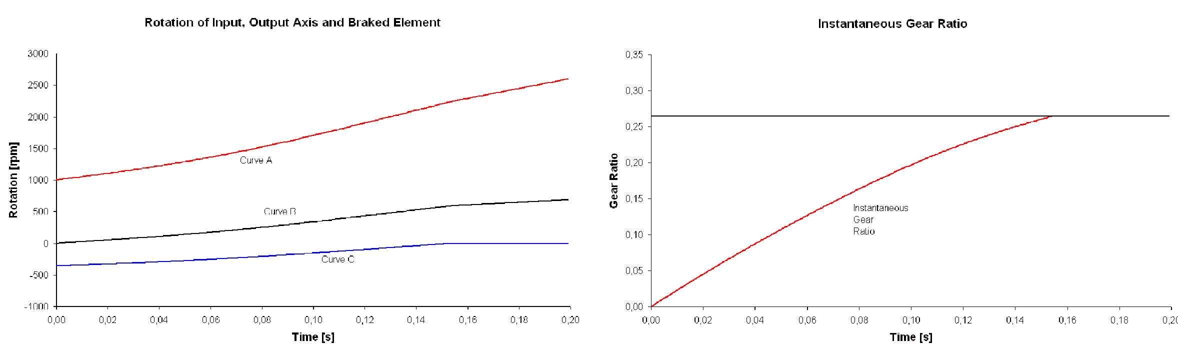

Figure 4. Variation of the speed of the main components for the first gear in the simulation.

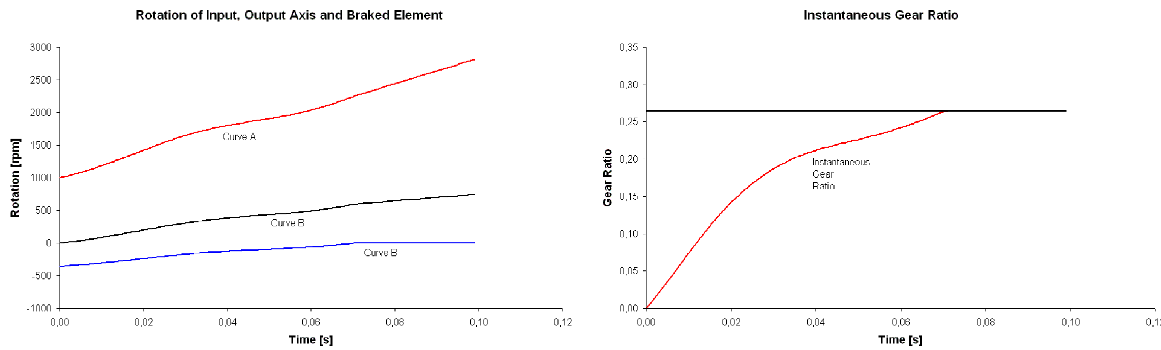

Figure 5. Variation of the speed of the main components for the first gear of the Wilson gearbox when the loading is applied.

It can be seen in Fig. (4) that the first gear needs only a short period of time to reach the final gear ratio. In this case the engine had not reached its maximum speed before the final gear ratio was set. Another important factor is the output speed, which always increases, as expected in an automobile. Figure (5) shows that the loading influences the dynamic behavior of the transmission. The time needed to set the final gear ratio is smaller than that needed in the case where there is no loading. The patterns are almost the same; this shows that the loading has an influence that is not about the time of setting the gear ratio only.

The variation of the speed over time is shown in Fig. (6) for second gear. And the results obtained for the simulation with the loading are shown in Fig. (7).

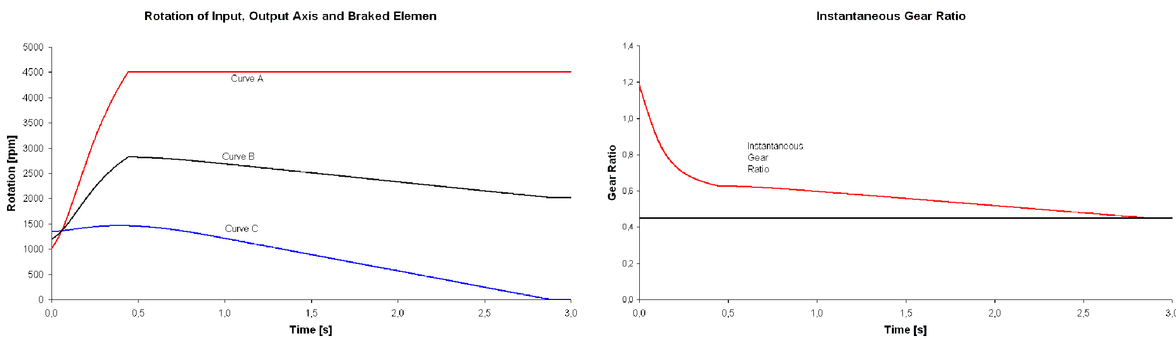

Figure 6. Variation of the speed of the main components for the second gear in the simulation.

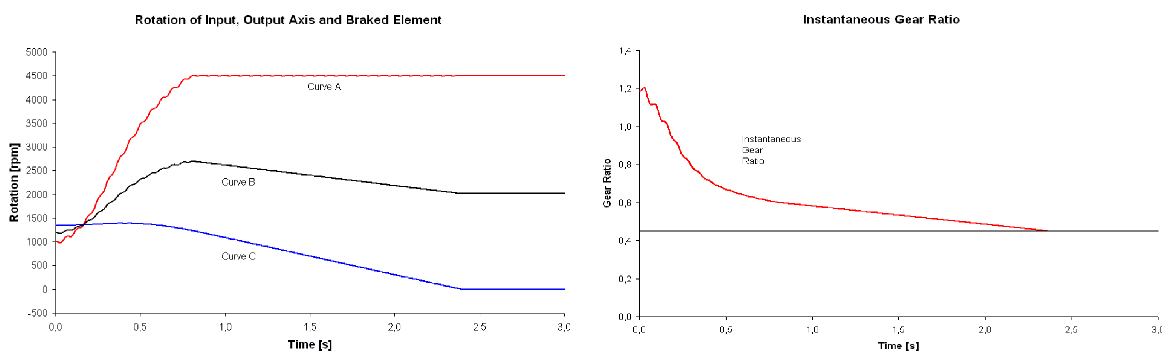

Figure 7. Variation of the speed of the main components for the second gear of the Wilson gearbox when the loading is applied.

As shown in Fig. (6), the second gear needs a time almost twenty times greater than first gear to reach its final gear ratio. In this case the engine had reached its maximum speed so soon that it was kept for a long time in the acceleration process. For second gear it was observed that the output speed increased while the input speed have not reached its maximum value.

At that moment the output speed began to decrease its value related to the speeds of the input and the braked element, following the braked element. It is obvious that the process of braking in the second gear must be controlled to avoid this effect. As seen in the first gear the time needed for set the final gear ratio is smaller with the loading applied. The rotation patterns are, again, almost equal to the simulation with no loading. Figure (7) shows the influence loading function.

Figure (8) shows the variation of speed over time for the third gear. Figure (9) shows the results of the simulation with the loading applied, as seen before.

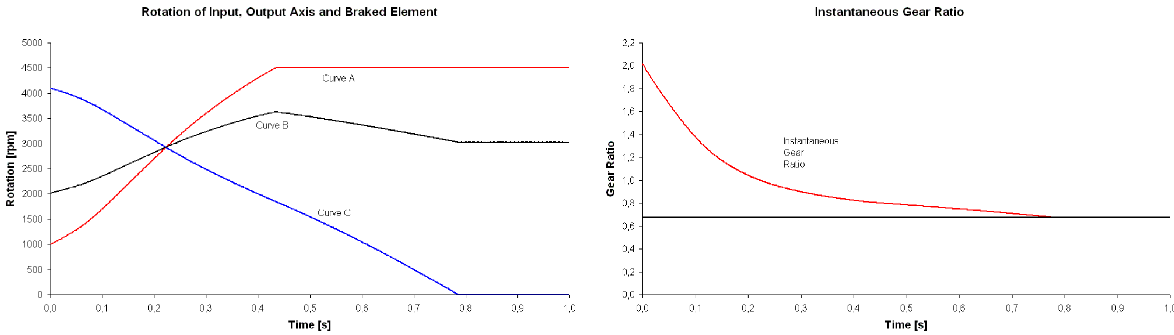

Figure 8. Variation of the speed of the main components for the third gear in the simulation.

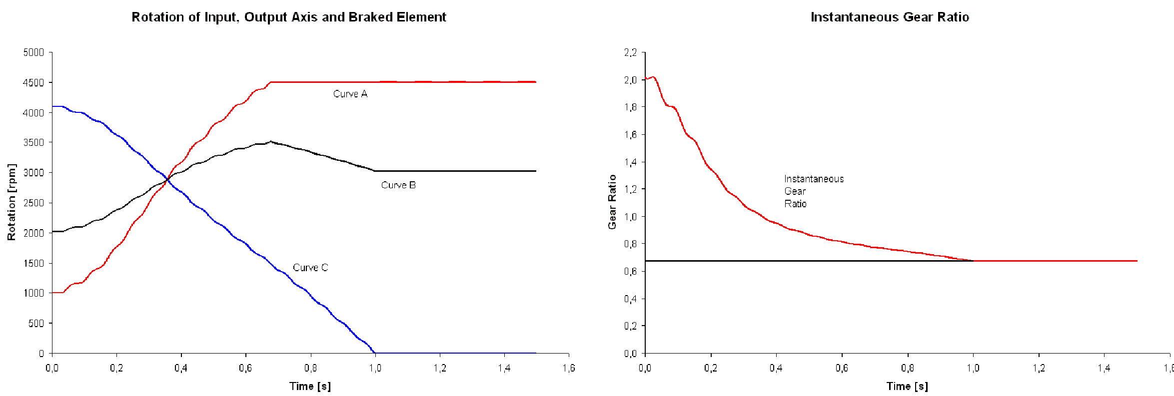

Figure 9. Variation of the speed of the main components for the third gear of the Wilson gearbox when the loading is applied.

All that was said about the second gear is also valid for the third, however in this case a slightly shorter period of time was required. In all gears, but the first, the output speed has a value larger than the input speed, which means that initially the gear ratio is bigger than 1. This occurs because of the method for evaluating the initial output speed. This was predictable since the output speed is expected to keep positive acceleration. The third gear shows a change of behavior. The time need to set the final gear ratio is greater when the loading in applied. This result is close to that results that were expected.

Figure (10) shows the variation of speed over time for the fourth gear. Figure (11) shows the results of the simulation with the loading applied, as seen before.

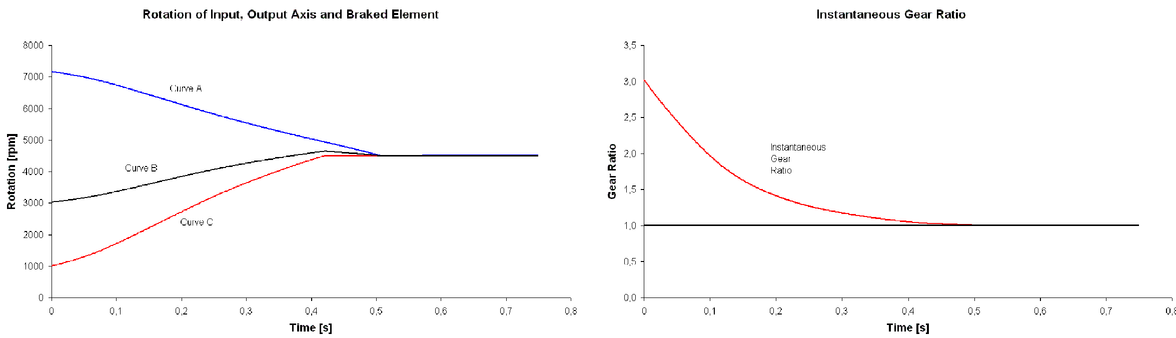

Figure 10. Variation of the speed of the main components for the fourth gear in the simulation.

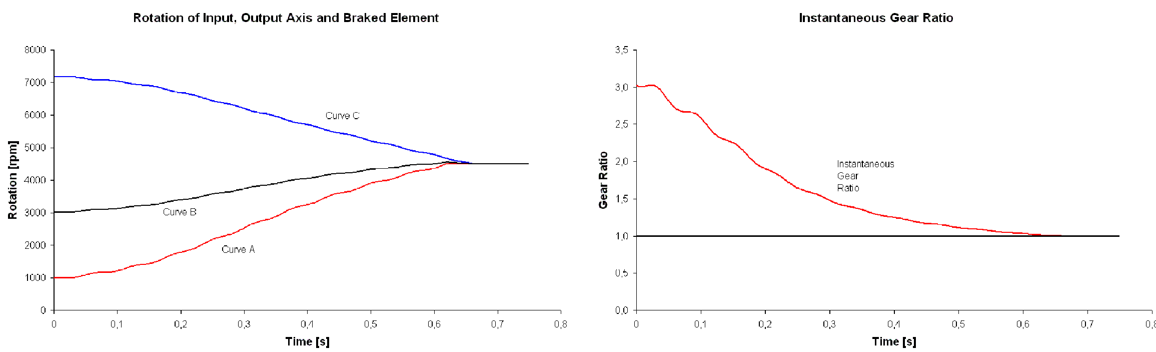

Figure 11. Variation of the speed of the main components for the fourth gear of the Wilson gearbox when the loading is applied.

In the fourth gear the same effects were present again, the increase in the output speed followed by a decreasing value. The engine reached its maximum value faster than the final gear ratio. In fourth gear, all elements converged to the same value of 4500 rpm. At this point the transmission starts to behave as a rigid body. This is not necessarily true, but gives a good idea of what actually happens in a transmission of this type. The time needed to set the final gear ratio is, again, greater when the loading applied. The time is about 20% greater in both cases, third and fourth gears.

In third and fourth gears the influence of the loading can be clearly noticed. This influence is the same of that seen in first and second gears, more clearly in the second gear. That means a change in the pattern of the rotation behaviors during the simulation.

5. Conclusions

The model built gave positive results since the final gear ratios where correct at the end of the simulations. Some of the results are the same as were expected, third and fourth gears. Unexpected results must be checked in a prototype in order to validate the model. Next steps in the research include considering actual loading in the output axle Another option is to set of gear that does not have inertias with the same value and check the influence of this condition. The plans for the future include building an experiment to confirm, or not, the model proposed here. The complete model and the experimental results will allow optimization of the inertias of the gears and an eventual control strategy.

6. Acknowledgement

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