MODELING SURFACE FLAW TRANSITION TO A THROUGH CRACK

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Abstract. *Part-through cracks (such as surface, corner or internal flaws) are one of the most common cracks in structural components. Traditionally, part-through cracks are assumed to have an elliptical shape (as indicated by fractographic observations), however their transition to a through crack is normally not addressed in fatigue life predictions. In general it is assumed that the surface crack is immediately transformed into a through crack as its depth reaches the specimen thickness. However, this approach creates a large increase in the stress intensity value, often causing the crack growth prediction to be excessively conservative. Experimental results reveal that elliptical surface flaws do not immediately transform into a through-crack. Instead, the crack essentially retains its elliptical shape as it grows into a through-crack. Johnson proposed a model to describe the surface flaw transition to a through-crack, however his original approach uses overly simplified SIF expressions for surface cracks, without considering the specimen width effect. In addition, it does not guarantee continuity of the SIF expressions between the transitioning period and the through-crack growth regime. This discontinuity problem is particularly deceiving when retardation models are considered in the calculation, as false overload events might be generated, leading to non-conservative predictions. In this work, an improved model for describing surface crack transition to a through crack is introduced, based on Newman-Raju's SIF equations for 2D cracks, obtained from Finite Element analyses.*

Keywords: fatigue, crack propagation, surface crack, transition to through crack.

1. Introduction

Fatigue cracks commonly nucleate as part-through surface or corner cracks, which propagate in 2D. The main characteristic of these cracks is a non-homologous fatigue propagation: in general, the crack front tends to change form from cycle to cycle, because ΔK varies from point to point along the crack front.

Some analytical expressions are available for the stress intensity factor of 2D cracks, such as surface, corner or internal cracks under combined tension and bending. If the cracks have ellipsoidal fronts, and if they are built in a plate of width **w** or **2w** and thickness **t**, the stress intensity range ΔK is a function of the stress range $\Delta\sigma$, the ratios **a/c**, **a/t** and **c/w**, and an angle θ (Anderson, 1995)

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi a} \cdot f_\theta(a/c,\ a/t,\ c/w) \tag{1}$$

where **a** and **c** are the ellipsis semi-axes, f_θ is a crack shape function and θ is defined in Fig. (1).

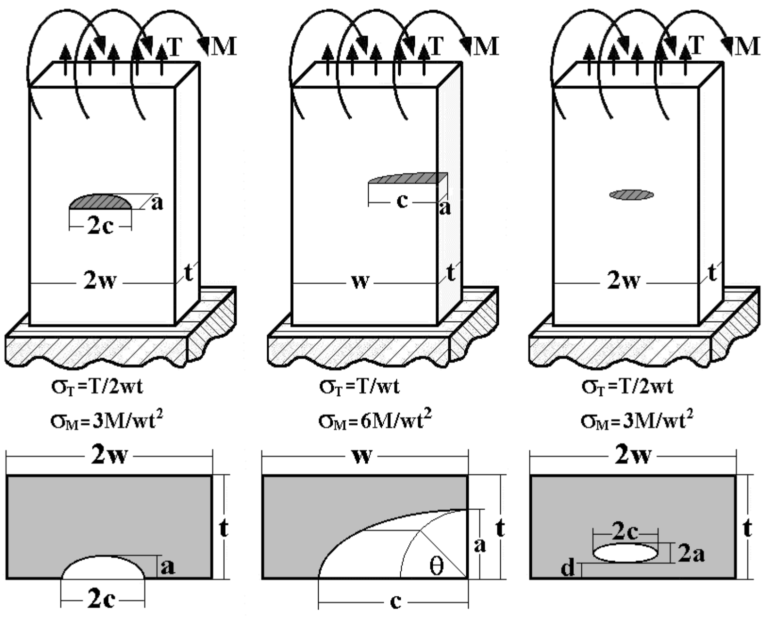

Figure 1. Surface semi-elliptical, corner quart-elliptical, and internal elliptical cracks.

The 2D ellipsoidal crack propagation problem is a reasonable approximation for many actual surface, corner, or internal cracks. Fractographic observations indicate that the successive fronts of those cracks tend to achieve an elliptical form, see Fig. (2), and to stay approximately elliptic during their fatigue propagation, even when the initial crack shape is far from an ellipsis (Castro et al., 1998). Therefore, it can be quite reasonable to assume in the modeling that fatigue propagation just changes the shape of the 2D cracks (given by the ratio **a/c** between the ellipsis semi-axes, which quantifies how elongated the cracks are), but preserves their basic ellipsoidal geometry. The idea is then to maintain the fundamental hypothesis of the ellipsoidal geometry preservation, accounting for the coupled growth in the depth (**a**) and surface width (**2c** for surface and **c** for corner cracks) directions.

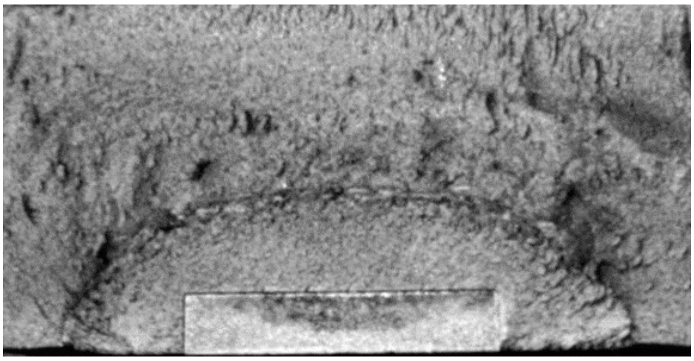

Figure 2. Surface fatigue crack that started from a sharp rectangular notch, but quickly changed its shape to grow with an approximately semi-elliptical front (Castro et al., 1998).

However, the state-of-the-art in the geometric considerations of part-through cracks (surface flaws) is not as well characterized as that for through-the-thickness cracks, because of its three-dimensional nature. Traditionally, the surface flaw transition to a through crack, as depicted in Fig. (3), is not addressed. In general it is assumed that, under tension, the surface crack is immediately transformed into a through crack as its depth reaches the specimen thickness **t** (at the back surface). This approximation is based on the “catch up effect” (Grandt et al., 1984), where the crack front at the back surface experiences a much higher SIF (and therefore faster crack growth) than at the front surface, straightening its profile. However, this approach creates a discontinuity in the calculated stress intensity values, often causing the crack growth prediction to be excessively conservative.

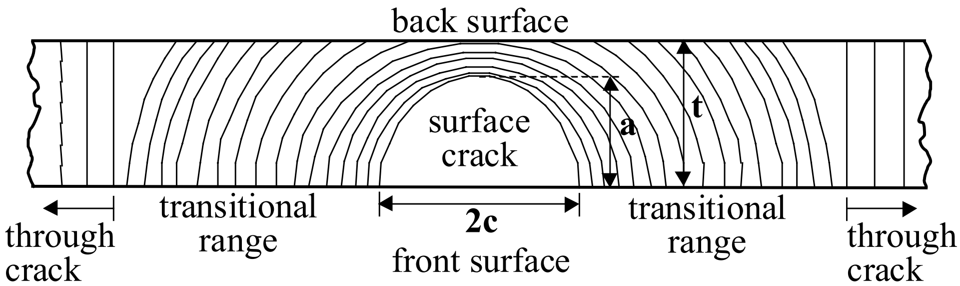

Figure 3. Plate cross-section showing a typical surface crack growth behavior during the transition.

Experimental results reveal that the elliptical surface flaw does not immediately transform into a through-crack. Fawaz (1997) studied the fatigue crack fronts on the fracture surface starting at a hole in sheet specimens loaded under combined tension and bending. He found out that the crack essentially retains its elliptical shape as it grows into a through-crack, see Fig. (4).

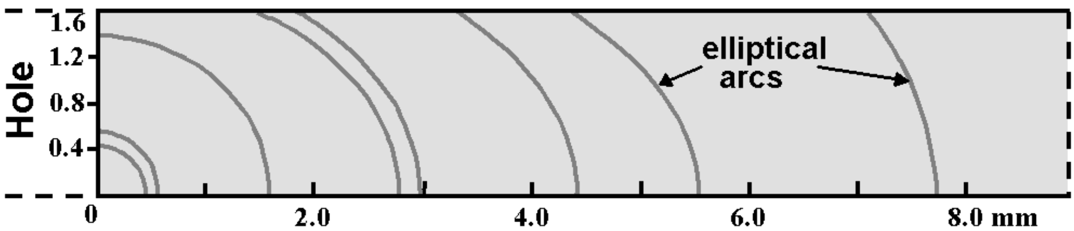

Figure 4. Measured fatigue crack fronts starting at a hole on an Al 2024-T3 Alclad plate with thickness 1.6mm, under combined tension and bending with stress ratio R = 0 (Fawaz, 1997).

However, no closed-form K-solution is available for through-cracks with oblique fronts. Some approximations have been suggested based on the boundary element method (Murakami, 1993), however they do not account for the specimen width effect. In this work, improved models for the transition from surface to through cracks are proposed. These models are based on Johnson's original idea, described next.

2. Johnson's model for the transition from surface to through cracks

Johnson (1979) introduced an idea to describe the surface flaw transition to a through crack. According to him, after the crack depth reaches the specimen thickness **t**, the crack is assumed to keep its elliptical shape, represented by

$$\left(\frac{c'}{c}\right)^2 + \left(\frac{t}{a'}\right)^2 = 1 \tag{2}$$

where **a'** is the imaginary crack depth (for **a** > **t**) and **c'** is the back face surface crack length, see Fig. (5).

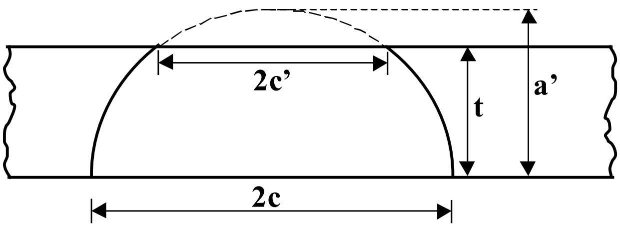

Figure 5. Model of the surface crack transition to a through crack.

As the crack is allowed to grow in the **a'** and **c** directions, **c'** can be calculated using Eq. (2). The flaw is assumed to be a through-crack when **c' = 0.9c**, after which the appropriate 1D stress intensity factor expression is used.

Johnson used an adaptation of the surface (2D) stress intensity factor (SIF) expressions described by Hall et al. (1974) to model the surface crack transition. The SIF used for the surface crack width direction is

$$K_I(c) = \sigma\sqrt{\pi c} \cdot \frac{1.1}{\sqrt{Q}} \cdot \frac{a}{c} \tag{3}$$

where **σ** is the applied nominal stress, **1.1** stands for an estimate of the front surface effect, and **Q** is the crack shape parameter

$$Q\left(\frac{a}{c}\right) = \begin{cases} 1 + 1.464(a/c)^{1.65}, & a \le c \\ 1 + 1.464(c/a)^{1.65}, & a > c \end{cases} \tag{4}$$

The SIF used for the depth direction is calculated by

$$K_I(a) = \sigma\sqrt{\pi a} \cdot \frac{M(a/c,\ a/t)}{\sqrt{Q}} \tag{5}$$

where **M(a/c**, **a/t)** is the back face magnification factor, defined in (Hall, 1974). During the transitioning period, Eqs. (3-5) are used assuming **a = t**, and Eq. (5) is multiplied by **1.1** to account for the back face becoming a free surface

$$K_I(a) = \sigma\sqrt{\pi a} \cdot \frac{M(a/c,\ a/t = 1)}{\sqrt{Q}} \cdot 1.1 \tag{6}$$

The modified expression above is used to calculate crack growth in the imaginary **a'** direction, until **c' = 0.9c**.

However, Johnson's approach may be inaccurate, since it uses overly simplified SIF expressions for surface cracks, without considering the effect of the specimen width **2w** and the variation of the front surface effect (assumed equal to **1.1**). In addition, it does not guarantee continuity of the width SIF expression $K_I(c)$ between the transitioning period and the 1D crack growth regimen, see Fig. (6). This problem is particularly deceiving when retardation models are considered in the calculation. If the considered SIF equation calculates a **smaller** value for the through crack than for the transitional range (e.g. for **c/t** = **0.5** or **0.2** in Fig. (6)), the retardation model may consider the larger SIF as an overload, retarding the subsequent 1D growth. Therefore, when considering load interaction models, Johnson's approach may predict crack growth retardation, even under constant amplitude loading, resulting in **non-conservative** predictions.

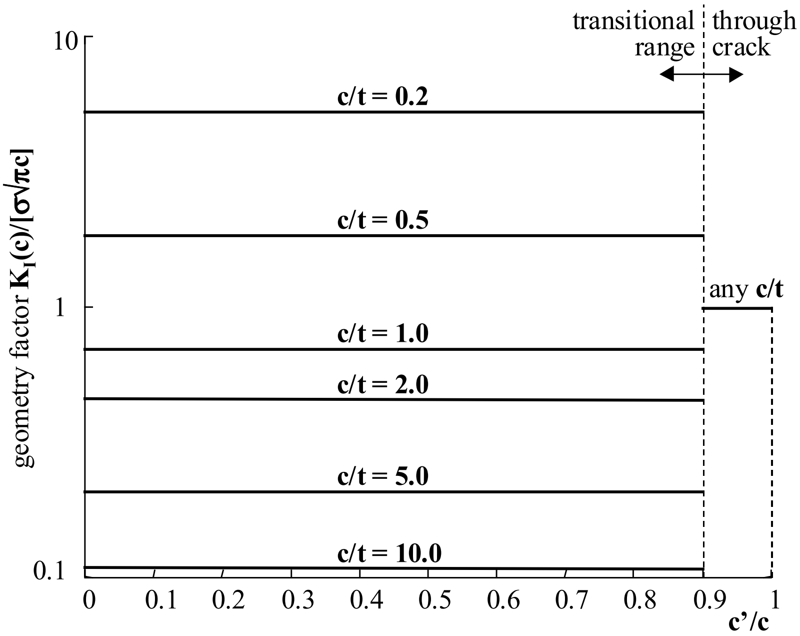

Figure 6. Width geometry factor modeled by Johnson (1979), showing discontinuity at **c'/c = 0.9**.

In the next section, an improved model for describing surface crack transition to a through crack is introduced, based on Newman-Raju's (1984) SIF equations for 2D cracks (to better model the width and front surface effects) and using modifications to Johnson's original idea (to guarantee continuity).

3. Improved models for the transition from surface to through cracks

In this section, Newman-Raju's equations (1984) are adapted to model the transition from surface (semi-elliptical) or corner (quarter elliptical) cracks to through-the-thickness (1D) cracks. In addition to the effects of the back face magnification and crack shape parameter, Newman-Raju's expressions also model the specimen width and front surface effects.

To model the transition, the fundamental hypothesis of the ellipsoidal geometry preservation is assumed. From Eq. (2), it is found that Johnson's criterion **c' = 0.9c** for the end of the transitioning period is equivalent to

$$\mathbf{a'} = \frac{\mathbf{t}}{\sqrt{1-(\mathbf{c'}/\mathbf{c})^2}} = \frac{\mathbf{t}}{\sqrt{1-0.9^2}} \cong \mathbf{2.3t} \tag{7}$$

Therefore, the transitioning period starts when the crack depth equals the specimen width (**a' = a = t**), ending when the imaginary crack depth **a'** reaches **2.3t**. Using this result, continuity of the width SIF expressions for the transitioning period, $\mathbf{K_I(c)}$, and for the 1D crack growth regimen, $\mathbf{K_{I,1D}}$, can be achieved replacing all occurrences of **c/t** by a parameter

$$\mathbf{r'} \equiv \alpha\left(\frac{\mathbf{c}}{\alpha\mathbf{t}}\right)^{\frac{2.3-\mathbf{a'}/\mathbf{t}}{1.3}} \tag{8}$$

where α is the value of **c/t** that satisfies $\mathbf{K_I(c)} = \mathbf{K_{I,1D}}$. Note that **r' = c/t** in the beginning of the transition (where **a' = t**) and **r' = a** when the crack becomes through-the-thickness (**a' = 2.3t**). The application of this approach to the transition from surface (semi-elliptical) cracks to through-cracks is presented next.

3.1. Transition of surface semi-elliptical cracks

Newman and Raju (1984) modeled the SIF on the width and depth directions of surface cracks, $\mathbf{K_I(c)}$ and $\mathbf{K_I(a)}$, as

$$\mathbf{K_I(c)} = \sigma\sqrt{\pi \mathbf{c}} \cdot \mathbf{F_{s,w}} \cdot \frac{\mathbf{M_s}}{\sqrt{\mathbf{Q}}} \cdot \frac{\mathbf{a}}{\mathbf{c}} \cdot \mathbf{F_{s,c}} \tag{9}$$

$$\mathbf{K_I(a)} = \sigma\sqrt{\pi \mathbf{a}} \cdot \mathbf{F_{s,w}} \cdot \frac{\mathbf{M_s}}{\sqrt{\mathbf{Q}}} \cdot \mathbf{F_{s,a}} \tag{10}$$

where $\mathbf{Q}$ is the crack shape parameter defined in Eq. (4), $\mathbf{F_{s,w}}$ is the specimen width effect, $\mathbf{M_s}$ is the back face magnification factor, and $\mathbf{F_{s,c}}$ and $\mathbf{F_{s,a}}$ are respectively the front surface effects on the width and depth directions, given by

$$F_{s,w}\left(\frac{c}{w},\frac{a}{t}\right)=\sqrt{\sec\left(\frac{\pi c}{2w}\sqrt{\frac{a}{t}}\right)} \tag{11}$$

$$M_s\left(\frac{a}{c},\frac{a}{t}\right)=\begin{cases}1.13-0.09\frac{a}{c}+\left(-0.54+\frac{0.89}{0.2+a/c}\right)\left(\frac{a}{t}\right)^2+\left(0.5-\frac{1}{0.65+a/c}+14\left(1-\frac{a}{c}\right)^{24}\right)\left(\frac{a}{t}\right)^4, & a\le c\\ \frac{c}{a}+0.04\left(\frac{c}{a}\right)^2+\left(\frac{c}{a}\right)^{4.5}\left(\frac{a}{t}\right)^2\left[0.2-0.11\left(\frac{a}{t}\right)^2\right], & a>c\end{cases} \tag{12}$$

$$F_{s,c}\left(\frac{a}{c},\frac{a}{t}\right)=\begin{cases}1.1+0.35(a/t)^2, & a\le c\\ 1.1+0.35(c/a)(a/t)^2, & a>c\end{cases} \tag{13}$$

$$F_{s,a}=1.0 \tag{14}$$

For the 1D crack growth regimen, Tada et al. (1973) modeled the SIF of a center-cracked plate as

$$K_{I,1D}=\sigma\sqrt{\pi c}\cdot\sqrt{\sec\left(\frac{\pi c}{2w}\right)}\cdot\left[1-0.025\left(\frac{c}{w}\right)^2+0.06\left(\frac{c}{w}\right)^4\right] \tag{15}$$

where $\mathbf{2c}$ and $\mathbf{2w}$ are the through-crack and plate widths. The polynomial term under brackets improves the precision of the above equation from 2.6% to less than 0.2%. Therefore, comparing Eqs. (9) and (15), and noting that the only term in Eq. (9) dependent on $\mathbf{c/w}$ is $\mathbf{F_{s,w}}$, a modification for Eq. (11) is proposed

$$F_{s,w}\left(\frac{c}{w},\frac{a}{t}\right)=\sqrt{\sec\left(\frac{\pi c}{2w}\sqrt{\frac{a}{t}}\right)}\cdot\left[1-0.025\left(\frac{c}{w}\sqrt{\frac{a}{t}}\right)^2+0.06\left(\frac{c}{w}\sqrt{\frac{a}{t}}\right)^4\right] \tag{16}$$

Using the above equation, Newman-Raju's SIF solutions are improved to better model the influence of the plate width.

The transition from 2D to 1D crack growth is then modeled using Eqs. (9-10), (12-13), and (16), considering also $\mathbf{F_{s,a} = 1.1}$ (to account for the back face becoming a free surface) and $\mathbf{a = t}$, resulting in

$$K'_I(c)=\sigma\sqrt{\pi c}\cdot F'_{s,w}\cdot\frac{M'_s}{\sqrt{Q'}}\cdot\frac{t}{c}\cdot F'_{s,c} \tag{17}$$

$$K'_I(a)=\sigma\sqrt{\pi t}\cdot F'_{s,w}\cdot\frac{M'_s}{\sqrt{Q'}}\cdot F'_{s,a} \tag{18}$$

where the prime (') symbol denotes the expressions for the transition from part-through to through cracks ($\mathbf{t < a' < 2.3t}$), and thus

$$F'_{s,w}\left(\frac{c}{w}\right)=\sqrt{\sec\left(\frac{\pi c}{2w}\right)}\cdot\left[1-0.025\left(\frac{c}{w}\right)^2+0.06\left(\frac{c}{w}\right)^4\right] \tag{19}$$

$$M'_s\left(\frac{c}{t}\right)=\begin{cases}1.09-0.09\frac{t}{c}+\frac{0.89}{0.2+t/c}-\frac{1}{0.65+t/c}+14\left(1-\frac{t}{c}\right)^{24}, & c\ge t\\ \frac{c}{t}+0.04\left(\frac{c}{t}\right)^2+0.09\left(\frac{c}{t}\right)^{4.5}, & c<t\end{cases} \tag{20}$$

$$Q'\left(\frac{c}{t}\right)=\begin{cases}1+1.464(t/c)^{1.65}, & c\ge t\\ 1+1.464(c/t)^{1.65}, & c<t\end{cases} \tag{21}$$

$$F'_{s,c}\left(\frac{c}{t}\right)=\begin{cases}1.45, & c\ge t\\ 1.1+0.35(c/t), & c<t\end{cases} \tag{22}$$

$$F'_{s,a}=1.1 \tag{23}$$

To guarantee continuity of the width SIF expression, it is required that Eqs. (15) and (17) be equivalent at the end of the transitioning period (i.e. $\mathbf{a' = 2.3t}$), resulting in

$$K'_I(c) \equiv K_{I,1D}\big|_{a'=2.3t} \quad \Rightarrow \quad \frac{M'_s(c/t)}{\sqrt{Q'(c/t)}}\cdot\frac{t}{c}\cdot F'_{s,c}(c/t) = 1 \tag{24}$$

But Eq. (24) is a function of **c/t** only, having a unique solution for **c/t = 1.23**. Therefore, if the ratio **c/t** is replaced in Eqs. (17-23) by a function **r'(c/t, a'/t)** that tends to **1.23** as **a'** tends to **2.3t**, then continuity of the SIF is guaranteed. So, from Eq. (8), **r'** is expressed as

$$r'(\frac{c}{t},\frac{a'}{t}) = 1.23\cdot\left(\frac{c}{1.23t}\right)^{\frac{2.3-a'/t}{1.3}} \tag{25}$$

and the SIF during the transitioning period is then modeled replacing **c/t** by **r'** in Eqs. (17-23). When the imaginary crack depth **a'** reaches **2.3t**, Eq. (15) is then used to model 1D crack growth. Figure (7) shows the ratio between the transition and the 1D stress intensity factors calculated using the proposed approach.

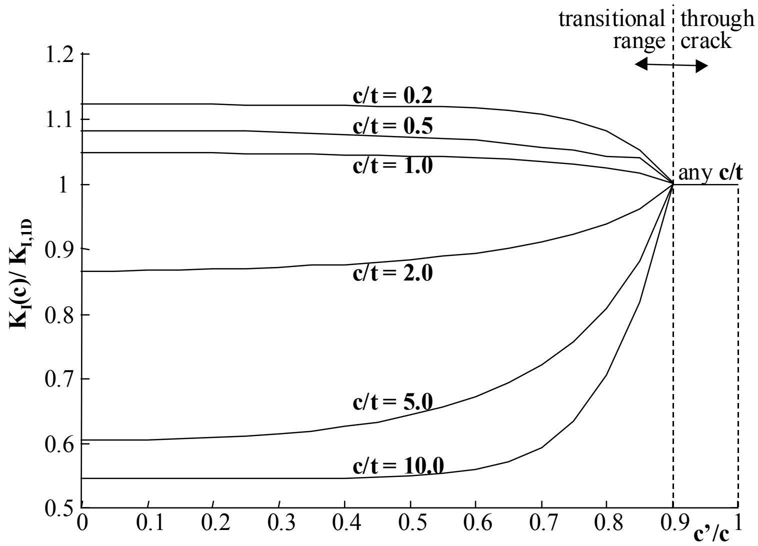

Figure 7. Ratio between the transition and 1D SIF using the proposed approach.

Two main improvements are achieved using this approach. First, the effect of **c/t** on $\mathbf{K_I(c)}$ is much better modeled by Newman-Raju's equations than the expressions used by Johnson. In fact, for extreme values of the **c/t** ratio such as **c/t = 0.2** and **c/t = 10**, Johnson's equations resulted approximately in a 400% error (comparing Figs. (6) and (7)), because it does not include in $\mathbf{K_I(c)}$ the back face magnification factor (it is included only in $\mathbf{K_I(a)}$).

Second, continuity in the $\mathbf{K_I(c)}$ function is guaranteed for the threshold from the transitional range to 1D growth through the use of Eq. (25). Note in Fig. (7) that the function **r'(c/t, a'/t)** only starts influencing significantly the calculated $\mathbf{K_I(c)}$ when **c'/c > 0.5**.

Figure (8) plots the normalized stress intensity factor (also known as geometry factor) in the width direction, calculated using the proposed approach. Note the smooth transition between the surface and through-crack growth regimes, in special under high **c/w** ratios.

The only remaining discontinuity in this model is due to the front surface effect on the depth direction, $\mathbf{F_{s,a}}$. However, since this discontinuity predicts a **larger** stress intensity factor as the crack enters the transition, the calculation problems when retardation effects are considered (discussed in the previous section) are not present.

In the next section, the proposed approach to model of the transition from part-through to through cracks is applied to corner (quarter-elliptical) cracks.

3.2. Transition of corner quarter-elliptical cracks

Newman and Raju (1984) modeled the SIF on the width and depth directions of corner cracks, $\mathbf{K_I(c)}$ and $\mathbf{K_I(a)}$, as

$$K_I(c) = \sigma\sqrt{\pi c}\cdot F_{q,w}\cdot\frac{M_q}{\sqrt{Q}}\cdot\frac{a}{c}\cdot F_{q,c} \tag{26}$$

$$K_I(a) = \sigma\sqrt{\pi a}\cdot F_{q,w}\cdot\frac{M_q}{\sqrt{Q}}\cdot F_{q,a} \tag{27}$$

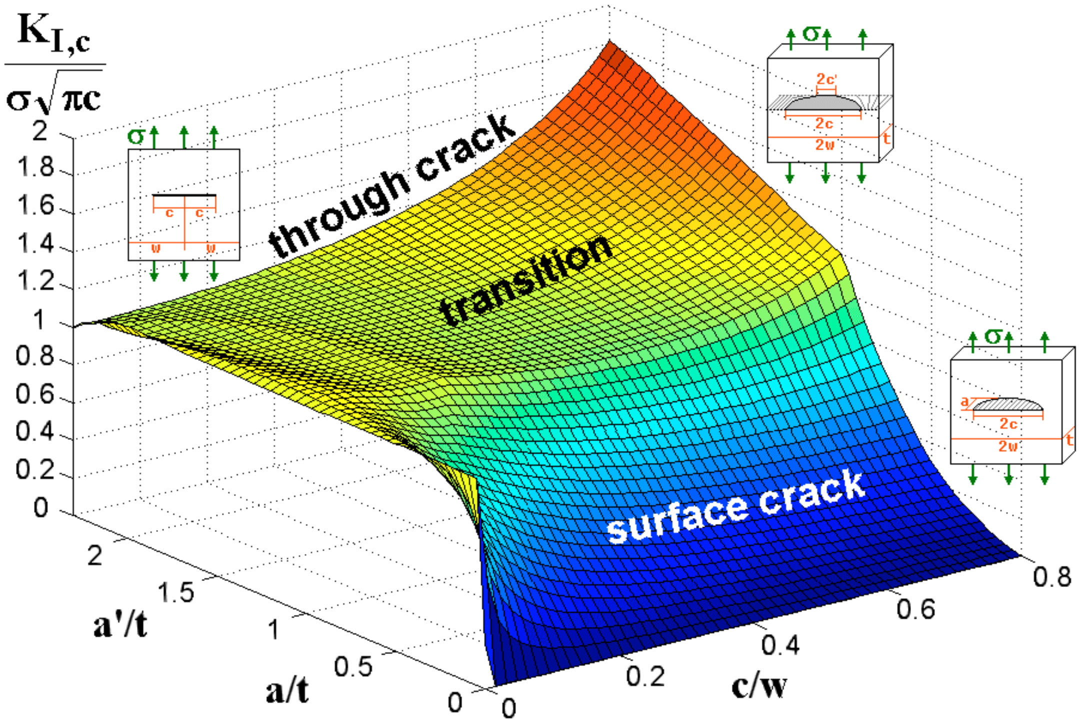

Figure 8. Geometry factor in the **c** direction for a surface crack on a rectangular plate (**w/t = 5**).

where **Q** is the crack shape parameter defined in Eq. (4), $\mathbf{F_{q,w}}$ is the specimen width effect (modeled by Newman and Raju using the same function for surface cracks), $\mathbf{M_q}$ is the back face magnification factor, and $\mathbf{F_{q,c}}$ and $\mathbf{F_{q,a}}$ are respectively the front surface effects on the width and depth directions, given by

$$F_{q,w}\left(\frac{c}{w},\frac{a}{t}\right)=\sqrt{\sec\left(\frac{\pi c}{2w}\sqrt{\frac{a}{t}}\right)} \tag{28}$$

$$M_q\left(\frac{a}{c},\frac{a}{t}\right)=\begin{cases}1.08-0.03\frac{a}{c}+\left(-0.44+\frac{1.06}{0.3+a/c}\right)\left(\frac{a}{t}\right)^2+\left(-0.5+0.25\frac{a}{c}+14.8\left(1-\frac{a}{c}\right)^{15}\right)\left(\frac{a}{t}\right)^4, a\le c\\ 1.08\frac{c}{a}-0.03\left(\frac{c}{a}\right)^2+\left(\frac{c}{a}\right)^{2.5}\left(\frac{a}{t}\right)^2\left[0.375-0.25\left(\frac{a}{t}\right)^2\right], a>c\end{cases} \tag{29}$$

$$F_{q,c}\left(\frac{a}{c},\frac{a}{t}\right)=\begin{cases}1.08+0.4(a/t)^2, a\le c\\ 1.08+0.4(c/a)^2(a/t)^2, a>c\end{cases} \tag{30}$$

$$F_{q,a}\left(\frac{a}{c},\frac{a}{t}\right)=\begin{cases}1.08+0.15(a/t)^2, a\le c\\ 1.08+0.15(c/a)^2(a/t)^2, a>c\end{cases} \tag{31}$$

For the 1D crack growth regime, Tada (1973) modeled the SIF of a single edge-cracked plate

$$K_{I,1D}=\sigma\sqrt{\pi c}\cdot\sec\left(\frac{\pi c}{2w}\right)\cdot\left(0.752+2.02\frac{c}{w}+0.37\left(1-\sin\frac{\pi c}{2w}\right)^3\right)\sqrt{\frac{2w}{\pi c}\tan\frac{\pi c}{2w}} \tag{32}$$

where **c** and **w** are the through-crack and plate widths. The above expression improves the precision of the SIF equation from **more than** 20% (due to the bending moment that must not be neglected) to less than 0.5%. Analogously to the surface crack modeling, a modification for Eq. (28) is proposed based on a comparison with Eq. (32)

$$F_{q,w}\left(\frac{c}{w},\frac{a}{t}\right)=\sec\left(\frac{\pi c}{2w}\sqrt{\frac{a}{t}}\right)\cdot\left[0.752+2.02\frac{c}{w}\sqrt{\frac{a}{t}}+0.37\left(1-\sin\left(\frac{\pi c}{2w}\sqrt{\frac{a}{t}}\right)\right)^3\right]\sqrt{\frac{2w}{\pi c}\sqrt{\frac{t}{a}}\tan\left(\frac{\pi c}{2w}\sqrt{\frac{a}{t}}\right)} \tag{33}$$

Using the above equation, Newman-Raju's SIF solutions are improved to better model the effects of the bending moment on the single edge-cracked plate.

The transition from 2D to 1D crack growth is then modeled considering **a = t** in Eqs. (26-27), (29-31), and (33). As in Section 3.1, the ratio **c/t** is replaced here by a function **r'(c/t, a'/t)** that guarantees continuity between Eqs. (26) and (32). For corner quarter-elliptical cracks, the value of α in Eq. (8) is 1.73, and thus the function **r'** is represented as

$$r' = 1.73 \cdot \left(\frac{c}{1.73t}\right)^{\frac{2.3 - a'/t}{1.3}} \tag{34}$$

The transition from part-through to through cracks (**t < a' < 2.3t**) is then modeled as

$$K_I'(c) = \sigma\sqrt{\pi c} \cdot F_{q,w}' \cdot \frac{M_q'}{\sqrt{Q'}} \cdot \frac{1}{r'} \cdot F_{q,c}' \tag{35}$$

$$K_I'(a) = \sigma\sqrt{\pi t} \cdot F_{q,w}' \cdot \frac{M_q'}{\sqrt{Q'}} \cdot F_{q,a}' \tag{36}$$

where

$$F_{q,w}'(\tfrac{c}{w}) = \sec\left(\frac{\pi c}{2w}\right) \cdot \left[0.752 + 2.02\frac{c}{w} + 0.37\left(1 - \sin\frac{\pi c}{2w}\right)^3\right]\sqrt{\frac{2w}{\pi c}\tan\frac{\pi c}{2w}} \tag{37}$$

$$M_q'(r') = \begin{cases} 0.14 + 0.22\dfrac{1}{r'} + \dfrac{1.06}{0.3 + 1/r'} + 14.8\left(1 - \dfrac{1}{r'}\right)^{15}, & r' \geq 1 \\ 1.08r' - 0.03(r')^2 + 0.125(r')^{2.5}, & r' < 1 \end{cases} \tag{38}$$

$$Q'(r') = \begin{cases} 1 + 1.464(1/r')^{1.65}, & r' \geq 1 \\ 1 + 1.464(r')^{1.65}, & r' < 1 \end{cases} \tag{39}$$

$$F_{q,c}'(r') = \begin{cases} 1.48, & r' \geq 1 \\ 1.08 + 0.4(r')^2, & r' < 1 \end{cases} \tag{40}$$

$$F_{q,a}'(r') = \begin{cases} 1.23, & r' \geq 1 \\ 1.08 + 0.15(r')^2, & r' < 1 \end{cases} \tag{41}$$

When the imaginary crack depth **a'** reaches **2.3t**, Eq. (32) is used to model 1D crack growth.

Figures (9-10) show the geometry factors of the stress intensity factor expressions in the **c** and **a** directions, calculated using the presented expressions for corner quarter-elliptical cracks.

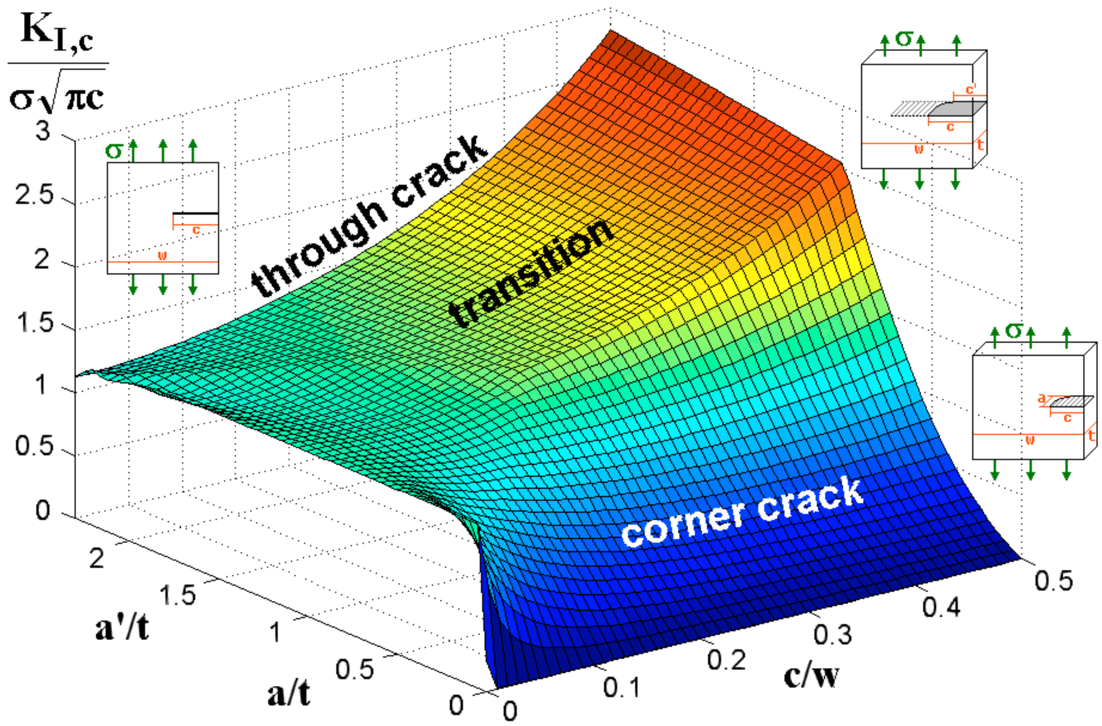

Figure 9. Geometry factor in the **c** direction for a corner crack on a rectangular plate (**w/t = 5**).

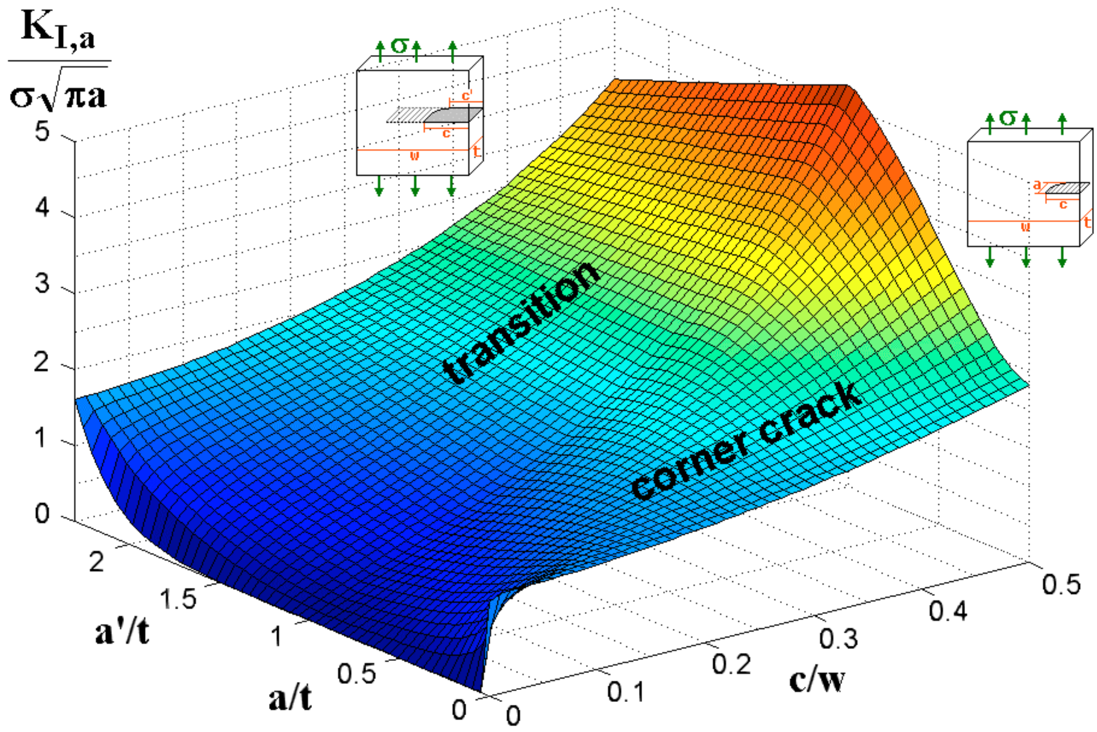

Figure 10. Geometry factor in the **a** direction for a corner crack on a rectangular plate (**w/t = 5**, assuming **a = t** if **a' > t**).

Finally, consider an example where the SIF in the width $\mathbf{K_I(c)}$ and depth $\mathbf{K_I(a')}$ directions have similar values (which happens e.g. for circular crack fronts), resulting in crack growth rates **dc/dN** and **da'/dN** that are approximately the same. Then, deriving Eq. (2) with respect to the number of cycles **N**, it is easy to show that, if **c'** is much smaller than **c**, then the back face growth rate **dc'/dN** is such that

$$\frac{dc'}{dN} = \frac{da'}{dN} \cdot \frac{c^2}{c't} \cdot \left[1 - \left(\frac{c'}{c}\right)^2\right]^{3/2} \cong \frac{da'}{dN} \cdot \frac{c}{t} \cdot \frac{c}{c'} >> \frac{da'}{dN} \cong \frac{dc}{dN} \quad \Rightarrow \quad \frac{dc'}{dN} >> \frac{dc}{dN} \tag{42}$$

Therefore, as long as **c'** is much smaller than **c** (which happens in the beginning of the transitioning period), Eq. (42) implies that the back face may experience a much larger growth rate than the front surface. This result is well in agreement with the "catch up effect" described by Grandt et al. (1984), which is reproduced by the proposed methodology.

4. Conclusions

In this work, the transition of part-through cracks (such as surface or corner flaws) to through cracks is modeled based on Newman-Raju's SIF equations for 2D cracks. The proposed methodology is applied to both surface and corner cracks on rectangular plates. The resulting continuous expressions are shown to better model not only the back face magnification and crack shape effects, but also the influence of the specimen width and front surface on the growth of part-through cracks, with an estimated accuracy of 3%. In addition, the proposed equations guarantee continuity of the SIF expressions between the transitioning period and the through-crack growth regime, as well as being able to reproduce the observed "catch up effect".

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