BUBBLE DYNAMIC SEGMENTS NUMERIC MODELING IN HORIZONTAL GAS-LIQUID INTERMITTENT FLOW

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Abstract. *The intermittent gas-liquid slug flow is composed of a succession of elongated gas bubbles and liquid slugs which are under cinematic and dynamic interactions along the flow. These interactions are responsible for the flow intermittence profile, i.e., they are not periodically in time or in space. The purpose of this work is to present a one-dimensional dynamic tracking model. Lengths, velocities and frequencies estimates of the gas-liquid structures will be provided by the model. In the dynamic model the bubbles and the slugs are computational objects defined by discrete volumes. They interact each other due to the mass and momentum exchanges. The flow parameters are determined through the mass and momentum balance integrals for each gas bubble and liquid slug. The liquid slugs and the gas bubbles are formed in the duct inlet. The propagation of each liquid slug and gas bubble is dynamically tracked by the model,disclosing how the gas and liquid structures evolve along the line. The numeric results are compared with an analytical case.*

Keywords*. Horizontal slug flow, two phase flow, modeling*

1. Introduction

Intermittent gas-liquid flow is characterized by the alternating occurrence of a gas bubble and a liquid slug which carries liquid with some dispersed gas. The succession of these structures forms the gas-liquid slug flow pattern. It is a very common flow pattern occurring in a number of industrial two-phase pipe flows. The study and modeling of this type of flow pattern is demanded by the petroleum industry for the development of multiphase technology to transportation of oil and gas in off-shore production facilities.

Certainly the most used methodology for slug flow modeling is the so called steady state models reviewed in Taitel and Barnea (1990). The class of steady state models considers a quasi-stationary flow regime where the gas-liquid structures no longer change in time and space. The flow becomes periodic with alternating uniform gas bubbles and liquid pistons. The model is a set of algebraic equations derived from mass and momentum, plus constitutive equations for model's closure, applied to a single unit cell. The unit cell is a concept originally proposed by Dukler and Hubard (1975), to represent the gas and liquid structures. It divides the flow in two structures: one liquid slug, which carries liquid with some dispersed gas, and an elongated gas bubble flowing over a liquid film. Using the assumption of constant unit cell velocity, the slug flow steady state models are relatively easy to implement, and provide reliable results of the average pressures and flow rates. Since this class of methods strongly relies on closure laws, which in turn depend on experimental data, it has limitations for new scenarios such as change in pipe diameter, fluid properties, line pressure, among others factors. Also, for not taking into consideration the main characteristics of the slug flow pattern (the intermittence and the flow irregularity) they are not able to predict the cinematic and dynamic evolutions of the gas-liquid structures as they flow along the pipe in steady flow as well as the start up process of lines with slug flow.

To deal with intrinsically transient flow such as slug flow start up, Eulerian models have been proposed using the two-fluid model, the drift flux model and a no-pressure wave model, Masella et al (1998). These methods, in general, require further experimental data to feed into new constitutive laws to obtain the correct velocities for the propagation of the gas-liquid boundaries. They demand careful modeling of non-trivial terms such as gas-liquid drag laws as well as correction schemes to avoid numerical diffusion.

A less restrictive approach is to track each liquid slug and gas bubble along the pipeline. This class of methods is referred as slug tracking methods. They employ a Lagrangian approach to sharply follow the slug and the bubble front, thus avoiding numerical diffusion problems. Also they demand less constitutive equations and therefore are more suitable to use them to new scenarios. Despite of very attractive, there are few studies employing the slug tracking models. Perhaps a predecessor of this class of models is the slug length distribution prediction used by Barnea and Taitel (1993). A more sophisticated model is in Zheng et al. (1994), which is able to follow each slug and predict its growth, generation and dissipation. The model, however, was restricted to the case of constant properties of the liquid and the gas compressibility was ignored. The effect of gas compressibility on the Zheng slug tracking model is proposed in Taitel and Barnea (1998). De Hanau and Raithby (1995) also developed a transient model for the simulation of slug flow. But their analysis was aimed at adjusting steady-slug models to fit better the two-fluid model. Thus, it lacks the true feature of slug-tracking description. Also in 1995, Nydal and Banerjee (1995) presented an object oriented dynamic simulation of a slug tracking model. Unfortunately their presentation is brief and many details are omitted. Nevertheless, their results seem to be very effective in simulating many of real features of slug flow. Yet, they neglected the interfacial drag and acknowledged that the model cannot be applied to a perfectly horizontal pipe. More recently, Grenier (1997) propose a new scheme for a slug tracking model.

This work re-introduces the concepts set forth by Grenier (1997) to develop a one-dimensional Lagrangian method capable for tracking bubbles and slugs. It incorporates the interactions between succeeding cells and takes into consideration the intermittence and irregularity of this flow pattern. Such method shall be capable to provide lengths and velocities of each individual bubbles and slugs. The model is, at present time, tested against an analytical solution. In the near future, it will be tested against experimental data.

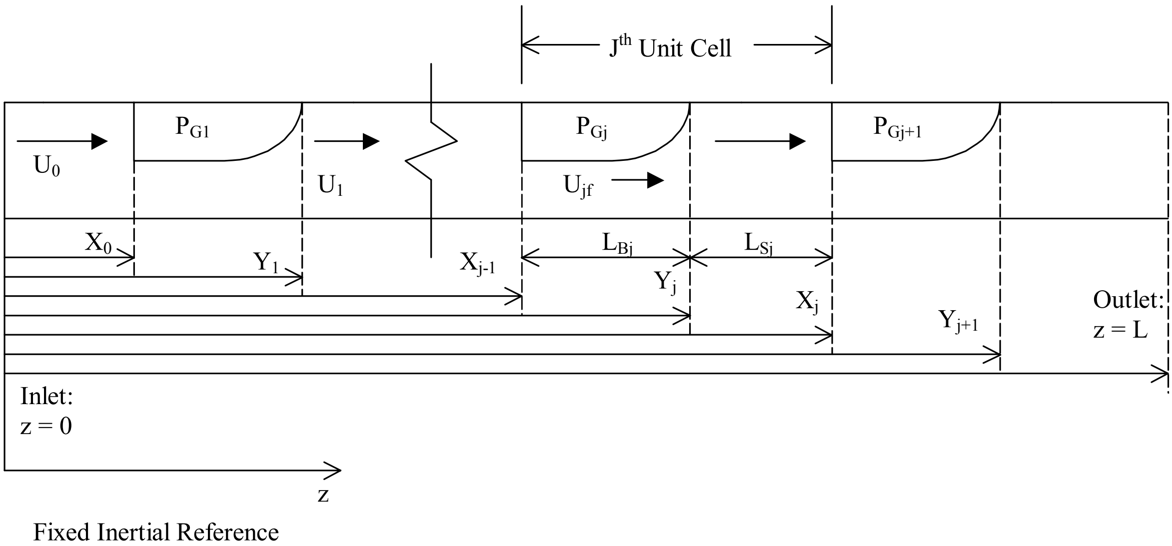

Figure 1. The unit cell, the mechanistic representation of slug flows and the coordinate system employed.

2. One-dimensional model

The model employs the unit cell concept. It is defined as a liquid piston followed by a gas bubble. The one-dimensional model views the intermittent flow as a succession of individual unit cells in a sense that its own properties such as length, velocities, frequency freely evolves along the pipe exchanging mass and momentum with the neighboring cells.

The identification of the bubble and liquid slug properties within the unit cell is done in nomenclature depicted in Fig. 1. The distance from the origin of the slug front and bubble front of the j^{th} unit cell are, respectively, X_j and Y_j. The bubble pressure of the j^{th} unit cell is P_{Gj} and it is constant through out the bubble. The liquid slug and liquid film velocities of the j^{th} unit cell are U_j and U_{fj}. Also, the lengths of the j^{th} slug and bubble are respectively identified as L_{Sj} and L_{Bj}.

Since the one-dimensional mass and momentum equations will be extensively used to develop the model they are convenient expressed in a generic form found in Eqs. (1) and (2). They express the mass and momentum balance in a control volume bounded by the pipe surface plus two other boundaries coincident with the pipe cross sectional area where the flow is allowed to cross. These cross flow boundaries are located at a distance z_1 and z_2 from the origin.

$$\frac{\partial}{\partial t}\left[(z_2 - z_1)\overline{R_k \rho_k}\right] + \left(R_k \rho_k V_{kr}\right)_{z=z_2} - \left(R_k \rho_k V_{kr}\right)_{z=z_1} = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\left[(z_2 - z_1)\overline{R_k \rho_k V_k}\right] + \left(R_k \rho_k V_k V_{kr}\right)_{z=z_2} - \left(R_k \rho_k V_k V_{kr}\right)_{z=z_1} = \left(P_k\right)_{z=z_1} - \left(P_k\right)_{z=z_2} + \overline{T}(z_1 - z_2) + \overline{I}(z_1 - z_2), \tag{2}$$

where k is the phase index denoting liquid or gas, L or G, R is the volumetric concentration of phase k, ρ is the phase density, V_k is the phase velocity taken from the inertial frame of reference; V_{kr} is the relative velocity between the phase k fluid and the boundary, P is the pressure experienced at position z, T and I represent the wall friction and the interfacial forces expressed in terms of force by unit volume. Furthermore, the over bar which appear on the transient terms means spaced averaged quantities over the control volume.

The tracking model has two equations, one expressing the momentum conservation across the liquid slug and another expressing the mass conservation across the gas bubble. These equations are developed in the next section considering the following hypothesis: i) the liquid has constant density; ii) the liquid slugs do not transport gas; iii) the flow is isothermal; iv) there are no pressure gradient along the gas bubble; v) the gas inside the bubble behaves as an ideal gas; vi) the gas liquid interface are plane and the bubble has a cylindrical shape; and vii) there are no interfacial forces.

2.1 The momentum balance for the liquid slug

This section develops the liquid slug momentum equation. A moving control volume for the j^{th} liquid slug is set up coincident with the pipe boundary, with the front and the rear positioned at X_j and Y_j respectively. The mass conservation, Eq. (1), for the liquid slug (R_L =1) turns to be:

$$\rho_L \frac{d}{dt}\left[\left(x_j - y_j\right)\right] + \rho_L\left[\left(U_j\right)_{z=x_j} - \frac{dx_j}{dt}\right] - \rho_L\left[\left(U_j\right)_{z=y_j} - \frac{dy_j}{dt}\right] = 0,$$

(3)

which after algebraic simplifications ends up to:

$$\left(U_j\right)_{z=x_j} = \left(U_j\right)_{z=y_j} = \overline{U}_j = U_j,$$

(4)

meaning that the front and the rear liquid slug velocity are coincident, therefore the positioning sub-index x_j and y_j will be dropped and the j^{th} liquid slug velocity will be denoted simply by U_j hence forth.

The momentum balance, Eq. (2), for the liquid slug considering no interfacial forces, becomes:

$$\frac{d}{dt}\left[\left(x_j - y_j\right)\rho_L U_j\right] = \rho_L U_j\left(\frac{dx_j}{dt} - U_j\right) - \rho_L U_j\left(\frac{dy_j}{dt} - U_j\right) + \left(P_{Lj}\right)_{z=y_j} - \left(P_{Lj}\right)_{z=x_j} + \left(x_j - y_j\right)\overline{T},$$

(5)

which, after the simplifications, reduces to:

$$\left(x_j - y_j\right)\rho_L \frac{dU_j}{dt} = \left(P_{Lj}\right)_{z=y_j} - \left(P_{Lj}\right)_{z=x_j} + \left(x_j - y_j\right)\overline{T}.$$

(6)

Equation (6) states the momentum balance in terms of the liquid slug acceleration term on the LHS and the pressure difference along the slug front and rear and also from the wall shear stress terms on the RHS.

2.1.1 The pressure link

In order to give numerical stability to the method, the pressure difference appearing in Eq. (6) is expressed in terms of the pressure difference at the two neighboring bubbles to the liquid slug. This procedure allows the pressure node to be staggered in relation to the velocity node. The next step express the front and the rear slug pressure in terms of the gas pressure occurring to the leading and trailing bubble to the slug.

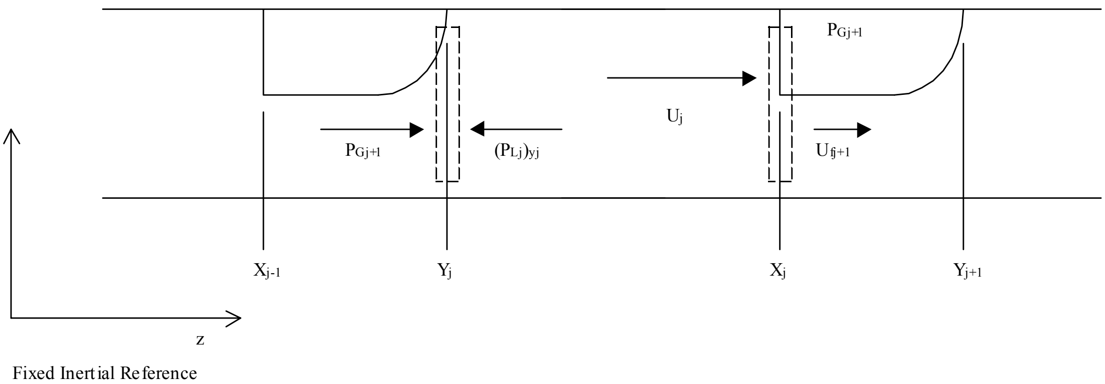

Figure 2. Mass and momentum balances across the boundaries.

One piece of information stems from the mass and momentum balance across the slug front, sketched in Fig. 2. The gas-liquid interface is a discontinuity coincident with the boundary of the unit cell j^{th} to the j^{+1th}. Applying Eq. (1) to the control volume depicted in Fig. 2 the mass conservation turns to:

$$\left(R_{fj+1}\right)_{z=x_j}\left[\frac{dx_j}{dt}-\left(U_{fj+1}\right)_{z=x_j}\right]=\left[\frac{dx_j}{dt}-U_j\right], \tag{7}$$

where R_f means the liquid hold up of the liquid film $\left(R_{fj+1}\right)_{z=x_j}$ which is the same as $1-\left(R_{Gj+1}\right)_{z=x_j}$. The momentum balance across the slug front arises from Eq. (2) as:

$$\left(R_{fj+1}\right)_{z=x_j}\rho_L\left(U_{fj+1}\right)_{z=x_j}\left[\left(U_{fj+1}\right)_{z=x_j}-\frac{dx_j}{dt}\right]A_T-\rho_L\left(U_j\right)_{z=x_j}\left[\left(U_j\right)_{z=x_j}-\frac{dx_j}{dt}\right]A_T=\left[\left(P_{Lj}\right)_{z=x_j}+\rho_L g\frac{D}{2}\right]A_T-, \\ \left[\left(P_{Gj+1}\right)_{z=x_j}+\rho_L g\left(\xi_{j+1}\right)_{x=x_j}\right]A_T \tag{8}$$

where D is the pipe diameter, A_T is the pipe cross section area and ξ is the film height (such that the pressure along this line times the film cross section area results on the net force exerted by the pressure on the liquid film). The positioning sub-index for the gas pressure was dropped since the pressure along the bubble is considered to be constant, that is: $(P_{Gj+1})_{Xj}=(P_{Gj+1})_{Yj+1}=P_{Gj+1}$.

Using Eqs. (8) and (7) and taking into consideration that we are dealing with a differential control volume, it is possible to express the pressure at the slug front in terms of the pressure of its leading bubble:

$$\left(P_{Lj}\right)_{z=x_j}=-\rho_L g\frac{D}{2}+\left[P_{Gj+1}+\rho_L g\left(\xi_{j+1}\right)_{z=x_j}\right]+\rho_L\frac{\left[1-\left(R_{fj+1}\right)_{z=x_j}\right]}{\left(R_{fj+1}\right)_{z=x_j}}\left[\frac{dx_j}{dt}-U_j\right]^2. \tag{9}$$

The following step express the pressure at the slug tail as a function of the pressure of its trailing bubble. Considering the control volume at the slug tail sketched in Fig. 2, the momentum balance in Eq. (2) reduces to:

$$\left(P_j\right)_{x=y_j}=P_j+\rho_L g\left(\xi_j\right)_{x=y_j}-\rho_L g\frac{D}{2}, \tag{10}$$

upon the consideration that at the bubble nose the velocity gradient is negligible and there are no interfacial forces.

Collecting the pressures at the slug front and rear, Eqs. (9) and (10), and substituting them into Eq. (6) one gets:

$$\rho_L\left(x_j-y_j\right)\frac{dU_j}{dt}=P_{Gj}-P_{Gj+1}+\left(x_j-y_j\right)\overline{T_{Lj}}-\rho_L\frac{\left[1-\left(R_{fj+1}\right)_{z=x_j}\right]}{\left(R_{fj+1}\right)_{z=x_j}}\left[\frac{dx_j}{dt}-U_j\right]^2+\rho_L g\left[\left(\xi_j\right)_{z=y_j}-\left(\xi_{j+1}\right)_{z=x_j}\right]. \tag{11}$$

Considering the assumption that the bubble has a cylindrical form the last term on the RHS of Eq. (11) is dropped. The forth term on the RHS of Eq. (11) has a cumbersome form for computing. But comparing it against the inertia term on the LHS, it appears negligible. The analysis is based on time scale for tracking a liquid slug proportional to the ratio between the pipe length to the slug velocity. After these considerations Eq. (11) reduces to:

$$\rho_L\left(x_j-y_j\right)\frac{dU_j}{dt}=P_{Gj}-P_{Gj+1}+\left(x_j-y_j\right)\overline{T_{Lj}} \tag{12}$$

Equation (12) represents the momentum balance to the liquid slug in terms of the pressure difference between the leading and trailing bubble. In comparing Eq. (12) to Eq. (6) one find the pressure difference between two consecutive bubbles is equal to the pressure difference existing at the front ant the rear of the liquid slug between them.

2.2 Mass balance for the gas bubble

The mass balance for the gas reduces to the statement that the mass of gas is constant for a single bubble,

$$\frac{d}{dt}\left[\left(y_j - x_{j-1}\right)\overline{R_{Gj}\rho_{Gj}}\right] = 0 \cdot$$

(13)

It is a consequence of no dispersed gas within the liquid slug assumption. Expanding the terms of Eq. (13) and re-grouping them one finds:

$$\frac{d}{dt}\left[\overline{R_{Gj}}\left(y_j - x_{j-1}\right)\right] + \left(y_j - x_{j-1}\right)\frac{\overline{R_{Gj}}}{\rho_{Gj}}\frac{d\rho_{Gj}}{dt} = 0 \cdot$$

(14)

Recognizing the first term of Eq. (14) as a geometrical quantity which defines the bubble volume time rate, it is possible to express it in terms of the liquid velocity upstream and downstream of the bubble. The complementary of the bubble volume is the liquid film volume beneath the gas bubble. Looking at the mass balance for the liquid film beneath the gas bubble, Eq. (1) becomes:

$$\frac{d}{dt}\left[\left(y_j - x_{j-1}\right)\left(1-\overline{R_{Gj}}\right)\right] = \left(1-R_{Gj}\right)_{y_i}\left[\frac{dy_j}{dt} - \left(U_{jf}\right)_{y_j}\right] - \left(1-R_{Gj}\right)_{x_{i-1}}\left[\frac{dx_{j-1}}{dt} - \left(U_{jf}\right)_{x_{j-1}}\right]$$

(15)

The RHS of Eq. (15) express the volumetric flux crossing in and out the liquid film boundary. But the volumetric fluxes at the bubble tail and bubble nose match the volumetric flux of the neighboring liquid slugs, therefore

$$\left(1-R_{Gj}\right)_{y_i}\left[\frac{dy_j}{dt} - \left(U_{jf}\right)_{y_j}\right] = \frac{dy_j}{dt} - U_j \quad \text{and} \quad \left(1-R_{Gj}\right)_{x_{i-1}}\left[\frac{dx_{j-1}}{dt} - \left(U_{jf}\right)_{x_{j-1}}\right] = \frac{dx_j}{dt} - U_{j-1}$$

(16)

The bubble volume time rate is then written substituting Eq. (15) in Eq. (16) and ordering the terms:

$$\frac{d}{dt}\left[\overline{R_{Gj}}\left(y_j - x_{j-1}\right)\right] = U_j - U_{j-1}$$

(17)

It states that the volume rate of change inside the bubble depends on the difference of liquid velocities in the neighboring slugs. Equation (17) enables to express the gas conservation equation, Eq. (14) in terms of the liquid slug velocities. Furthermore, using the ideal gas law, the gas density is replaced by the gas pressure and Eq. (14) is re-written as:

$$U_j = U_{j-1} - \frac{L_{Bj}\overline{R_{Gj}}}{P_{Gj}}\frac{dP_{Gj}}{dt} \cdot$$

(18)

where $L_{Bj} = y_j - x_{j-1}$ is the bubble length.

2.3 Constitutive equations

Equations (12) and (18) constitute the one-dimensional model. They solve for U_j and P_{Gj} but need further information regarding the wall friction force, T, the gas volumetric concentration of the bubble, R_G, and the distances X_j and Y_j. This information is supplied through constitutive equations.

The wall friction term is estimated using the Blasius' law, as shown in Eq. (19):

$$\overline{T_{Lj}} = -\frac{2}{D} f_j \rho_L \left|U_j\right| U_j \qquad \text{where} \quad \mathrm{f_j} = 0.079\,\mathrm{Re}_j^{-0.25} \quad \textit{being} \quad \mathrm{Re_j} = \frac{\rho_L U_j D}{\mu_L} \cdot$$

(19)

The gas bubble volumetric concentration, R_G, is estimated considering a fully developed slug flow where the gas bubble and the liquid slug travel at the same speed. Additionally, it is considered that the slug is non-aerated. Under these conditions, the gas volumetric flux is estimated by the gas flux transported by the bubble weighted by the bubble length to the unit length ratio:

$$Q_G = V_B \left\langle \overline{R_{Gn}} \right\rangle A_T \left(\frac{L_B}{L_B + L_S} \right),$$

(20)

where V_B is the bubble front velocity defined in Eq. (22) and A_T is the pipe cross section area. Employing the definition of gas superficial velocity, $j_G = Q_G/A_T$ and naming β as the length ratio, $\beta = L_B/(L_B+L_S)$, the averaged bubble volumetric concentration, $\overline{R_G}$, is them estimated as:

$$\frac{j_G}{\langle V_B \rangle} \frac{1}{\beta} = \left\langle \overline{R_{Gn}} \right\rangle.$$

(21)

The $\overline{R_G}$ value is kept constant through out the calculations, that is, its value is not updated neither in space nor in time.

The bubble speed, V_B, comes from the kinematic relation proposed by Moissis and Grifth (1962):

$$V_{Bj} \equiv \left(\frac{dy_j}{dt} \right) = C_0 U_j \left[1 + h(L_S) \right],$$

(22)

where C_0 is a constant, $C_0 = 1.2$ and $h(L_S)$ is a decaying function dependent on the slug length ahead of the bubble, here defined as:

$$h(L_{Sj}) = 8 \exp\left(-1.06 \frac{L_{Sj}}{D} \right).$$

(23)

The bubble front position, Y_j, comes from the time integration of Eq. (22). At last, the slug front position, X_{j-1}, derives from the bubble expansion. Since there is no gas within the liquid slug, the mass of gas trapped inside a single bubble remains constant through out the pipe:

$$(y_j - x_{j-1}) P_{Gj} \equiv L_{Bj} P_{Gj} = const.$$

(24)

2.4 Boundary conditions

The slug tracking model uses a methodology continuous in time but discrete in space. The spatial discretization allows the convenient treatment of the intermittent nature of the slug flow regime. The set of differential Eqs. (12) and (18) is first order in time and space demanding for each solved variable, $U_j(t,z)$ and $P_{Gj}(t,z)$, one boundary condition in time and other in space.

The time domain starts at time t = 0, and stops at a time t = t. The space domain starts at z = 0 coincident with the domain inlet and finishes at z = L at the pipe outlet, see Fig. 1 for reference. The initial conditions are set considering at t = 0 a single phase flow of liquid. The initial fields U_j and P_{Gj} are determined by:

$$U_j(0,z) = j_L \quad \text{and} \quad P_{G_j}(0,z) = 2\frac{(L-z)}{D} \cdot f \cdot \rho_L \cdot j_L^2 + P_{atm},$$

(25)

where j_L is the averaged liquid velocity inside the pipe, L is the duct total length and t is the time.

The boundary condition for pressure is set at the outlet, z = L, while for velocity at the inlet, z = 0. At the domain inlet, the liquid slug velocity is set equals to the mixture velocity, j, whilst at the domain outlet, the pressure is fixed to the atmospheric pressure, Patm:

$$U_j(t,0) = j \quad \text{and} \quad P_{G_j}(t,L) = P_{atm}.$$

(26)

The inlet velocity is a valid approximation for non-aerated liquid pistons where the averaged liquid piston velocity is equal to the mixture velocity, j defined as the sum of the gas and the liquid velocities, $j = j_G+J_L$ (Taitel and Duckler 1976).

The boundary conditions also help to define the way the liquid slugs and gas bubbles are built at the inlet and extinguished at the outlet. The liquid slugs and the gas bubbles are generated in an alternating pattern at the inlet. During the formation of a gas bubble or of a liquid slug, the liquid slug velocity is equal to the averaged mixture velocity, j. At the outlet, either the liquid slug or the gas bubble is freely discharged at the atmosphere, that is, their pressure is fixed at P_{atm}. In particular, when a gas bubble is exiting the duct, its tail speed is assumed to be the same its front had just before leave the duct.

2.5 Front tracking initial conditions and bubble coalescence

The liquid slug and gas bubble fronts are tracked by the variables X and Y, respectively. They are not solved explicitly by Eqs. (12) and (18) but implicitly through the bubble mass conservation and propagation velocity. Likewise the U and P, they need initial values which are related at the instantaneous formation process of either a liquid slug or a gas bubble at the inlet. Considering a bubble initiates its formation at an instant $t = t_{Bi}$ and is completed at an instant $t = t_{Bf}$, the bubble tail X_0 and the bubble front Y_1, are evaluated accordingly to:

$$X_0(t)=0 \quad \text{and} \quad Y_1(t)=\int_{t_{Bi}}^{t} V_B(t)dt \quad \text{for} \quad t_{Bi} \le t \le t_{Bf} \tag{27}$$

similarly for a liquid slug forming and ending at the instants t_{Si} and t_{Sf}, have its slug front X_0 evaluated accordingly to:

$$X_0(t)=\int_{t_{Si}}^{t} j\cdot dt \quad \text{for} \quad t_{Si} \le t \le t_{Sf} \quad , \tag{28}$$

where j is the mixture velocity imposed at the domain inlet.

In the analysis the slug lengths or the bubble lengths are introduced at the pipe entrance either by random draw or taken from experimental data base in which individual values of lengths are available. Starting with Eq. (21), the length of the gas bubble behind each generated liquid slug is assumed to be associated with the slug length by the relation:

$$L_{B1} = L_{S1}\left[\frac{V_B \overline{R_G}}{j_G}-1\right]^{-1} \tag{29}$$

The initial and final times for the slug and bubble formation, t_{Si}, t_{Sf}, t_{Bi} and t_{Bf}, are evaluated with Eq. (29). Furthermore, Eqs. (29) and (26) assure that the gas superficial velocity j_G and the total mixture velocity, j are both satisfied at the pipe entrance. The first is matched by the proper choice of the bubble length while the last is the boundary condition for U.

At last but not the least, the kinematic law stated in Eq. (22) allows that trailing bubbles that are faster than the leading ones overtake the leading bubbles. During the merging process the liquid slug length as well as the bubble length increases. This bubble merging phenomena is also called by bubble coalescence. The coalescence occurs when the $L_{Sj} \to 0$, that is, when $Y_j - X_j = 0$. In this case, all the unit cells ahead of the coalescence must be re-indexed and the new bubble resulting from the coalescence must have the length of the two coalesced bubbles.

3. Implementation of a numeric solution model

The Eqs. (12) and (18) are numerically solved simultaneously for P_{Gj} and U_j. They are set in a matrix form using an explicit time integration scheme. The discretized forms of Eqs. (12) and (18) are :

$$\begin{cases} -U_{j-1}^N + \dfrac{L_{Bj}^O \overline{R_{Gj}}^O}{H_j^O \Delta t} H_j^N + U_j^N = \dfrac{\overline{R_{Gj}}^O L_{Bj}^O}{\Delta t} \\ -H_j^N + \left(\dfrac{2}{D} L_{Sj}^O f_j^O U_j^O + \dfrac{L_{Sj}^O}{\Delta t}\right) U_j^N + H_{j+1}^N = \dfrac{L_{Sj}^O U_j^O}{\Delta t} \end{cases} , \tag{30}$$

where H_j represents the ratio P_{Gj}/ρ_L thus avoiding working with big numbers such absolute pressures of the order of 10^5. Furthermore, Δt denotes the time step and the super-index 'o' or 'n' denotes old or new time. The positioning of the bubble and slug fronts come from Eqs. (22) and (24) which discrete in time become:

$$y_j^N = y_j^O + C_0 U_j^N \Delta t\left[1 + h^O\left(x_j^O - y_j^O\right)\right]$$
$$x_j^N = y_{j+1}^N - \frac{H_{Gj+1}^O}{H_{Gj+1}^N}\left(y_{j+1}^O - x_j^O\right) \cdot$$

(31)

4. Test case

The slug tracking model is tested against an idealized horizontal flow case where at the pipe entrance the flow is periodic, i.e., the lengths of the slugs and bubbles have all the same size. Also no overtaking mechanism is considered and the bubble velocity is a linear function of the liquid slug velocity ahead of it. Under these constraints it is possible to find an analytical solution for this case. The usefulness of this analytical solution is two-fold: comparison standard against the numerical solution, seeking numerical algorithm consistency, and to test the model sensitivity to the time step. As numerical consistency is understood, a check is performed in the following routines: linear solver, bubble and slug indexing routines and the numerical probes which pick-up the velocities, lengths and period of the passing by flow structures. Also, it is necessary to investigate the sensitivity of the numerical solution to the time step. Since the method advances in time in an explicit way, the time and space steps are controlled by the Courant stability criteria, $\Delta x/\Delta t \le C$ and $\Delta y/\Delta t \le C$ where C is characteristic velocity.

The case is set up for a horizontal pipe with an internal diameter d = 26 mm and length L = 20 meters. The pair of fluids is air treated as an ideal gas and ordinary tap water, both at ambient temperature of 23°C. The slugs are formed at the pipe entrance and at the pipe exit the mixture is freely discharged at the atmosphere. For this case are considered known the following values: the liquid superficial velocity, jL = 0.50 m/s; the gas superficial velocity at the pipe outlet, jG(z=L)=0.50 m/s; the averaged bubble void fraction, R_G = 0.54; the slug length at the pipe entrance, LS(0)=0.213 m and finally the pressure drop along the line given by:

$$P_G(z) = \lambda(L - z) + P_{atm},$$

(32)

where Patm = 94700 Pa and λ is the pressure gradient, λ = 110 Pa/m.

For non-aerated liquid slugs, all the gas content is transported by the gas bubble, therefore the mass of gas of a single bubble is constant trough out the pipe. Taking into account that the flow is isothermal, the bubble length and pressure as well as the gas superficial velocity and the pressure bear the relations:

$$\mathrm{L_B}(z)P_G(z) = \mathrm{L_B}(0)P_G(0),$$

(33)

$$\mathrm{j_G}(z)P_G(z) = \mathrm{j}_G(0)P_G(0),$$

(34)

here, the bubble length at the formation, LB(0), comes from Eq. (29) repeated here for convenience:

$$L_{B1} = L_{S1}\left[\frac{V_B\overline{R_G}}{j_G} - 1\right]^{-1}$$

(29)

Finally the mixture superficial velocity is:

$$\mathrm{j}(z) = \mathrm{j_L} + \frac{\mathrm{j}_G(0)P_G(0)}{P_G(z)} \equiv U(z)$$

(35)

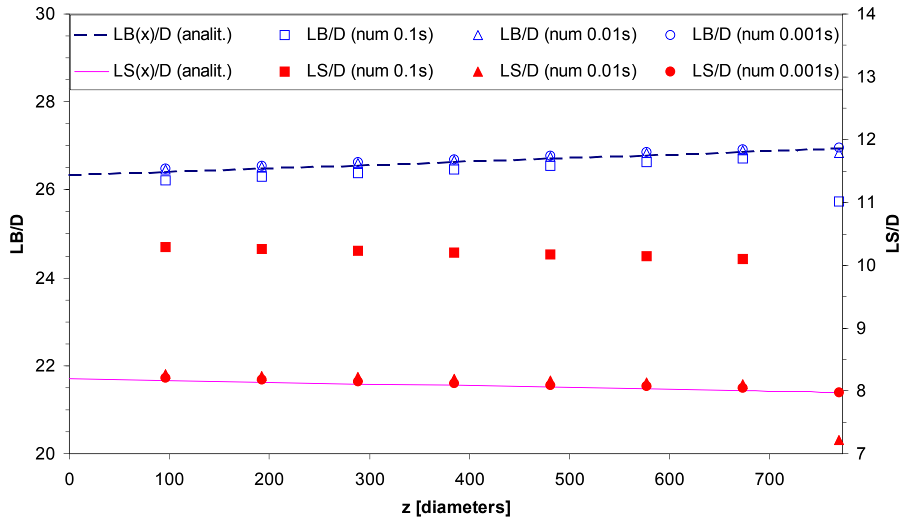

Figure 3 . Analytical and numerical solutions showing LB/D and LS/D for the periodic case.

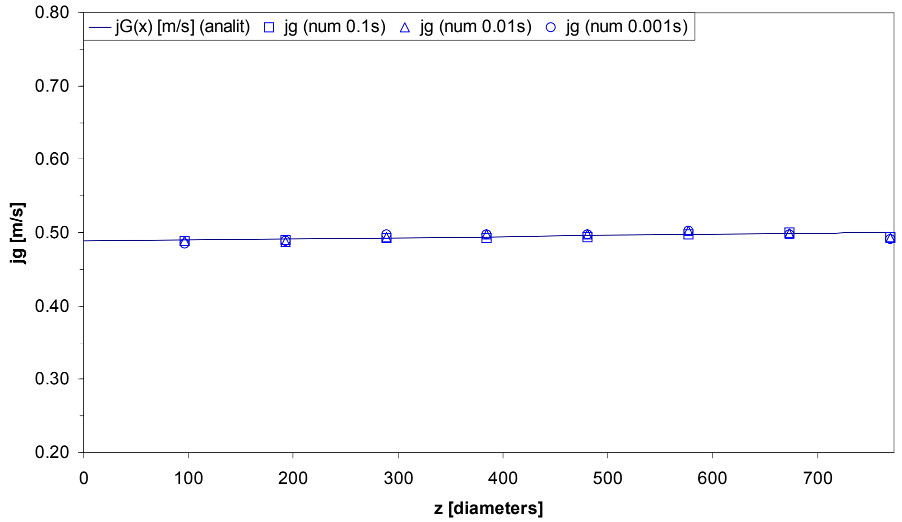

Figure 4. Analytical and numerical solutions showing j_G for the periodic case.

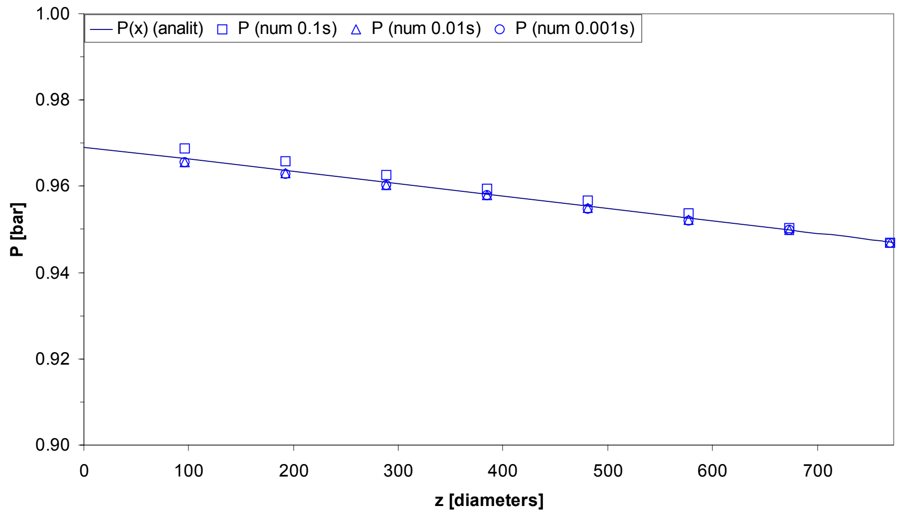

Figure 5. Analytical and numerical solutions showing P for the periodic case.

The results from the analytical and numerical solutions are now compared. Also the numerical solution sensitivity to the time step is analyzed. Figure 3 shows the bubble length (blue color) and slug length (red color) as a function of the pipe distance from the gas-liquid mixer in terms of the pipe diameter. The continuous lines represent the analytical solution of the periodic test case while the symbols are the numerical outcome. Figures 3 also portraits the numerical solution sensitivity to three time steps: 0.1 s; 0.01s and 0.001 s. As far as the bubble length is concerned, there is no apparent dependence of the bubble length on the time steps. Furthermore, the numerical solution matches the analytical solution for all tested time steps (except for z = L at t ≥ 0.1s, due to the fact that the bubble front exiting is not accurately tracked at such time steps). On the other hand, the slug length does show a dependence on the time step. Figure 3 show a large deviation from the numerical to the analytical solution when the time step is of 0.1 s. Decreasing the time step to 0.01s or to 0.001 s make the numerical solution match the analytical solution.

Figure 4 displays the gas superficial velocity, J_G, as a function of the axial pipe distance from the mixer expressed in terms of the pipe diameter, D. The numerical solution does not show a great dependence on the time step for the three tested time step values. However, it can be seen that for a time step of 0.1s the numerical solution is less accurate when compared with the 0.01s and 0.001s time steps, just as expected.

Finallly, fig. 5 shows the absolute pressure drop along the axial pipe distance from the mixer expressed in terms of the pipe diameter, D. The use of a time step of 0.1s shows numerical solution not coincident with the analytical solution. As the time step is decreased to 0.01 s or even more, to 0.001 s, both numerical solution are coincident and also matches the analytical solution.

5. Conclusions

The numerical routine was successfully tested against the bench mark solution from the periodic slug flow. The numerical solution show a dependence on the used time step as expected. For time step values of 0.1 s the numerical estimates of the slug length and pipe pressure drop do not match the analytical solution. As the time step value decreases to 0.01 s, the numerical estimates match the analytical solution. A further decrease on the time step value to 0.001 s do not show any significant change on the numerical output revealing that the numerical solution has reached a converged and stable value coincident with the analytical solution. The agreement between the numerical and analytical solution lends reliability to the developed numerical routine. The next step in this work will incorporate the overtaking mechanism to the model and extend its capabilities to predict bubble coalescence. The results will also be compared against experimental data.

6. Acknowledgement

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