FAULT DETECTION OF ROTOR-SUPPORT-STRUCTURE INCLUDING FOUNDATION EFFECTS

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Abstract. Nowadays one of the largest concerns of the industry is to maintain its equipment in operation without the occurrence of a sudden break down. Due to this constant concern new techniques of fault detection and location in mechanical systems submitted to dynamic loads have been developed. Rotating systems have many applications in wide industrial contexts. Breakdown of this equipment provokes economic losses or leads to dangerous situations. To avoid such problems it is very important to use tools that can inform about the existence of faults, as well as, about the evolution of these in time. A review of the modeling process used for rotor-support-structure shows that the finite element method is the major method employed. In this paper, with the aid of well defined theoretical models, obtained using the finite element technique, and the state observer method for the identification and location of faults it is possible to monitor the parameters of a system rotor-support-structure, including the foundation effects. In order to improve safety, they must be supervised such that occurrence of failures or faults can be repaired as quickly as possible.

Keywords. *Observers of states, faults detection and location, rotor system, foundation effects.*

1. Introduction

In the last years, due to great concern of industry for the safety of its systems, new methodologies of fault detection and location have been developed in order to avoid false alarms and unnecessary stoppage of these systems during operation. It has been verified that in the practice certain parameters of the systems can vary during the process due to the specific characteristics of some materials or to the own natural wear of the components.

The state observers' technique can reconstruct the non-measured states or can estimate the values of points of difficult access in the system. Thus, the faults at these points can be detected without the knowledge of measured data, monitoring them through the reconstructions of their states (Luenberger, 1964). This technique consists of the development a model for the system to be analyzed and to compare the output at the observer with the output of the system.

In order to supervise the process, a set of observers is mounted where each observer is only dedicated to an instrument or physical parameter of this system. For fault detection in the system rotor-support-structure, an observer of global state is firstly projected. The global observer has the role of verifying if the system is working properly without indications of faults, because in this observer's assembly it uses the same system matrix of the mechanical system in the analysis. Thus, the global observer can detect a possible fault or irregularity in the system if the system's response is not coincident with the global observers response. Detecting a possible fault, the next step would be to locate such fault, and for this reason robust observers are used. Thus, a bank of observers is set up, where each observer is dedicated to a physical parameter of the system (Melo, 1998).

2. State Observer Formulation

Consider a linear time invariant system described by:

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [B]\{u(t)\} \tag{1a}$$
$$\{y(t)\} = [C_{me}]\{x(t)\} + [D]\{u(t)\} \tag{1b}$$

Where:

$[A] \in R^{nxn}$ Is the dynamical matrix;

$[B] \in R^{nxp}$ Is the input matrix;

$[C_{me}] \in R^{kxn}$ Is the measure matrix;

$[D] \in R^{kxp}$ Is the matrix of the direct inputs;

n is the order of the system, p the inputs $\{u(t)\}$, k the outputs $\{y(t)\}$.

One of the advantages of this representation type is that the state vector {x(t)} contains enough information to summarize completely the last behavior of the system, and the future behavior is governed by a simple differential equation of first order.

A state observer for system (1) is defined by:

$$\{\dot{\hat{x}}(t)\} = [A]\{\hat{x}(t)\} + [B]\{u(t)\} + [L]\left(\{y(t)\} - \{\hat{y}(t)\}\right) \tag{2a}$$

$$\{\hat{y}(t)\} = [C_{me}]\{\hat{x}(t)\} \tag{2b}$$

Where:

$[L]$ Is the observer matrix;

$\{\hat{y}(t)\}$ Is the output of the observer;

$\{\hat{x}(t)\}$ Is the states vector of the observer;

2.1.State Observer Method

Many control systems are based on the supposition that the full state vector is available for direct measurement, but in the practice, not always all the variables are available, and the variables that are not available for direct measurement must be estimated.

Therefore, control systems using state observers can reconstruct the non-measured states or to estimate the values of points of difficult access in the system. However, the necessary condition for this reconstruction is that all the states should be observable (Luenberger, 1964; D'Azzo and Houpis, 1988).

Figure 1 shows a logical diagram for faults detection and location in mechanical systems using the state observers' technique.

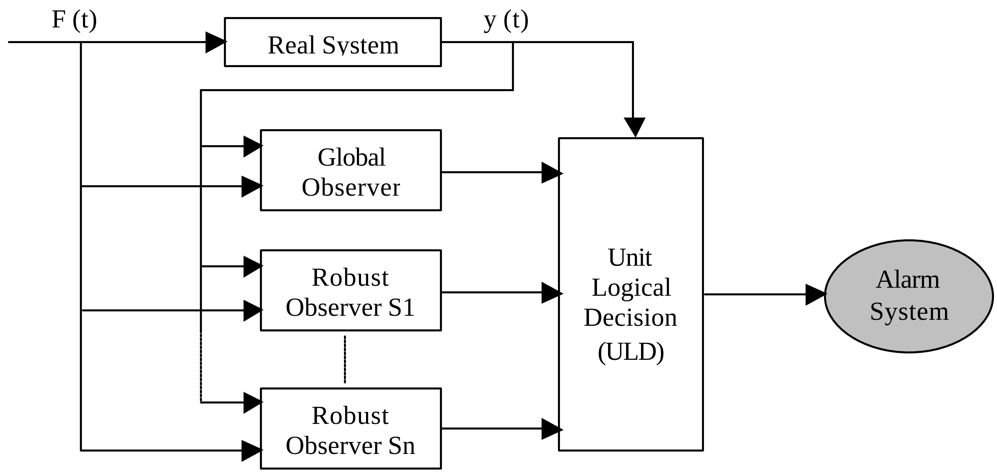

Figure 1 – Observation System

In the system of Fig. 1, when a certain component begins to fail, the state observer is capable to detect the influence of this fault quickly, because the observer is quite sensitive to any incipient irregularity that appears in the system. The state observer is a group of ordinary differential equations of first order that represents the same response as that of the real system, when it is working property. Therefore, the idea is to use this effect sensed for the state observer to detect and to locate a possible fault in a mechanical system.

In this set of observers, the global observer has the role of verifying if the system is working properly without indications of faults, because this observer uses the same system matrix of the mechanical system analysis. Thus, the global observer can detect a possible fault or irregularity in the system in analysis if the system's response is not coincident with the global observer's response.

If a possible fault is detected, the next step would be to locate such fault, and for this reason robust observers are used. In the robust observers' assembly are removed of system matrix of the system the parameters subject to the faults or the parameters subject to a reduction of its values. This way, the robust observer's response that to approach to the response of the system with faults it will be the responsible observer for the location of this possible fault of the system.

There are still the possibilities of one or more parameters fail at the same time. In this case, the solution in agreement with Melo (1998) would be to project robust state observers to all parameters subject to fails.

Finally, it is the Unit of Logical Decision (ULD) that collects and analyzes the difference between the real system and the mounted state observers, in order to detect and to locate faults or irregularities in the system. This unit also analyzes the progression of possible faults of the system, and activates, when it is necessary, an alarm system. This alarm system can be ready to be activated when happens a determined variation in a certain parameter.

2.2. State Observer Methodology

The figure 2 shows a block diagram of the developed methodology for faults detection and location in mechanical systems using state observer's technique. The stages of this block diagram go from finding a mathematical model of the system to the analysis of the response of the system and of the observers in the ULD. The commands used in this methodology belong to the package Matlab. In a general form, the developed methodology is:

- The measurements matrix [C_{me}] is defined so that the system is observable using this matrix;
- All the eigenvalues of the system in analysis should have its negative real parts for to guarantee stability and fast convergence.
- If the system isn't observable, new measures should be carried out until the system is observable;
- The matrix of the state observer [L] is obtained using MatLab' LQR command which is an implementation of the Ackerman's formula to calculate optimal gains [L] and to verify the stability of the system.

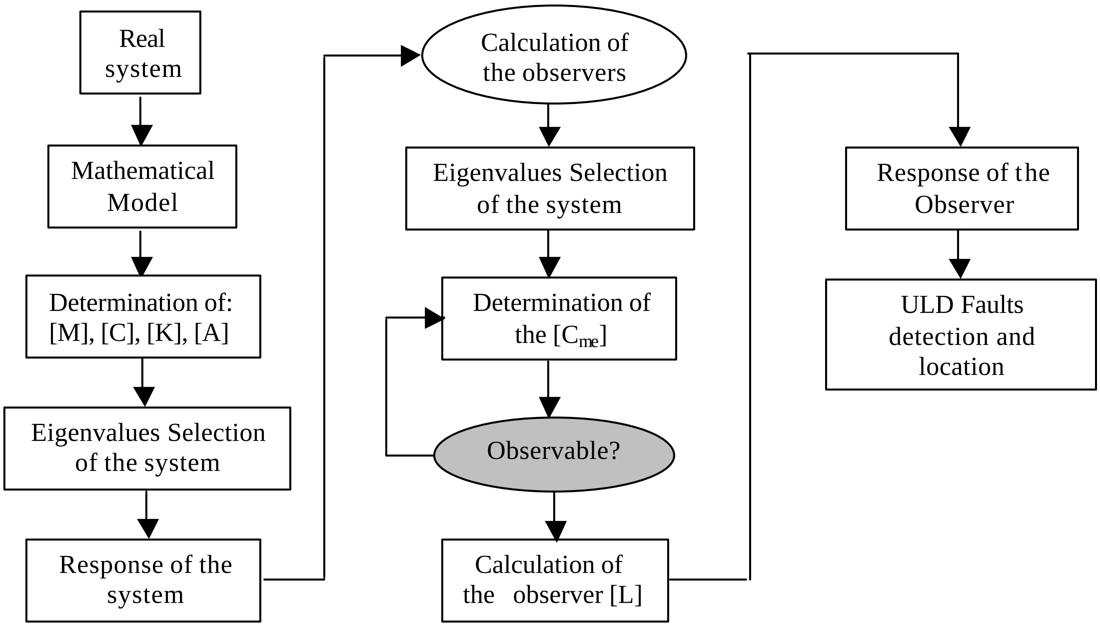

Figure 2. Block diagram of the developed methodology.

2.3.Modeling of the rotor-suport-structure

The mathematical model considers the whole system (rotor-supports-foundation) divided in two different subsystems: rotor-supports-subsystem and foundation subsystem. In Figure 3 the interaction forces between the foundation and the oil film ($R_f(t)$) are shown (Cavalca, 1993).

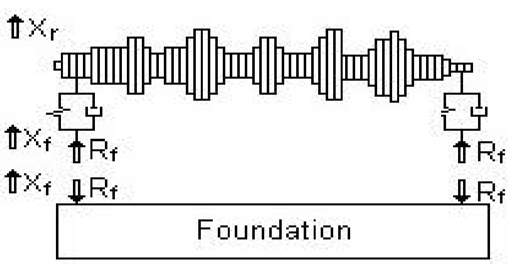

Figure 3. Rotor bearing subsystem and foundation subsystem

Defining the vector containing both coordinates $\{x_r(t)\}$ of the rotor and $\{x_f(t)\}$ of the foundation connecting points, the equations of motions for the rotor-supports subsystem are:

$$[M]\{\underline{\ddot{x}}(t)\}+[C]\{\underline{\dot{x}}(t)\}+[K]\{\underline{x}(t)\}=\{F(t)\} \tag{3}$$

Where:

$$\{\underline{x}(t)\}=\begin{bmatrix}\underline{x}_r(t)\\ \underline{x}_f(t)\end{bmatrix} \text{ and } \{\underline{F}(t)\}=\begin{bmatrix}\underline{F}_r(t)\\ \underline{R}_f(t)\end{bmatrix}$$

[*M*], [C] and [K] are the respectively the mass, damping and stiffness matrices containing the mass matrix for each finite element of the rotor and the equivalent damping and stiffness matrices of to the oil film for each bearing. In the equation. 3, $\{\underline{F}_r(t)\}$ represents the vector of the external generalized forces applied to the rotor and $\{\underline{R}_f(t)\}$contains the forces transmitted between rotor and foundation in each connecting node. Forces $\{\underline{R}_f(t)\}$ are unknown and depend on both rotor and foundation dynamic behavior. By matrix partitioning techniques the Eq.3 can be rewritten as:

$$\begin{bmatrix}[M_{rr}] & [M_{rf}]\\ [M_{fr}] & [M_{ff}]\end{bmatrix}\begin{Bmatrix}\underline{\ddot{x}}_r(t)\\ \underline{\ddot{x}}_f(t)\end{Bmatrix}+\begin{bmatrix}[C_{rr}] & [C_{rf}]\\ [C_{fr}] & [C_{ff}]\end{bmatrix}\begin{Bmatrix}\underline{\dot{x}}_r(t)\\ \underline{\dot{x}}_f(t)\end{Bmatrix}+\begin{bmatrix}[K_{rr}] & [K_{rf}]\\ [K_{fr}] & [K_{ff}]\end{bmatrix}\begin{Bmatrix}\underline{x}_r(t)\\ \underline{x}_f(t)\end{Bmatrix}=\begin{bmatrix}\underline{F}_r(t)\\ \underline{R}_f(t)\end{bmatrix} \tag{4}$$

The equations of motion of the foundation subsystem can be written by means of a modal approach:

$$[m_f]\{\underline{\ddot{q}}(t)\}+[c_f]\{\underline{\dot{q}}(t)\}+[m_f]\{\underline{q}(t)\}=-[\phi^t]\{\underline{R}_f(t)\} \tag{5}$$

Where $\{\underline{q}(t)\}$is the vector of the foundation modal coordinates corresponding to the coordinates $\underline{x}_f(t)$ of the connected points.

$$\{\underline{x}_f(t)\}=[\phi]\{\underline{q}(t)\} \tag{6}$$

Where [*f*] is the matrix containing the foundation normal modes, which are evaluated in correspondence to the connecting nodes. The [*f*] matrix is rectangular with as many columns as the foundation vibration modes considered and as many rows as the degrees of freedom associated to the connecting nodes.

It is possible to define the connecting forces vector $\{\underline{R}_f(t)\}$ between rotor and supporting structure as a function of $\{\underline{x}_f(t)\}$ applying $\{\underline{q}(t)\}=[\phi]^{-1}\{\underline{x}_f(t)\}$, thus:

$$[M_f]\{\underline{\ddot{x}}_f(t)\}+[C_f]\underline{\dot{x}}_f(t)+[K_f]\underline{x}_f(t)=-\underline{R}_f(t) \tag{7}$$

Where:

$$[M_f]=[f^t]^{-1}[m_f][f]^{-1}$$
$$[C_f]=[f^t]^{-1}[c_f][f]^{-1}$$
$$[K_f]=[f^t]^{-1}[k_f][f]^{-1}$$

This transformation is possible only if the [*f*] matrix is square. In this case, it is necessary to consider a number of natural frequencies equal to the number of degrees of freedom of the connecting nodes. Substituting Eq.7 into Eq.4, gives the equation of motion of the complete system including foundation, when subject an external excitation.

$$\begin{bmatrix}[M_{rr}] & [M_{rf}]\\ [M_{fr}] & [M_{ff}]+[M_f]\end{bmatrix}\begin{Bmatrix}\underline{\ddot{x}}_r(t)\\ \underline{\ddot{x}}_f(t)\end{Bmatrix}+\begin{bmatrix}[C_{rr}] & [C_{rf}]\\ [C_{fr}] & [C_{ff}]+[C_f]\end{bmatrix}\begin{Bmatrix}\underline{\dot{x}}_r(t)\\ \underline{\dot{x}}_f(t)\end{Bmatrix}+\begin{bmatrix}[K_{rr}] & [K_{rf}]\\ [K_{fr}] & [K_{ff}][K_f]\end{bmatrix}\begin{Bmatrix}\underline{x}_r(t)\\ \underline{x}_f(t)\end{Bmatrix}=\begin{bmatrix}\underline{F}_r(t)\\ \underline{R}_f(t)\end{bmatrix} \tag{8}$$

2.4. Numerical Example

A numerical example is given in this section starting from the developed methodology.

Considering a system of rotors, modeled by Finite Element Technique using beam elements. The disc is taken into account as additional mass, neglecting their stiffness and considering only the effect of inertial mass. Fig. 4 shows a scheme of the rotor model with the location of bearings and foundation parts.

The system was excited only initial conditions of the displacement and was disrespected the damping of the system. The physical parameters values adopted for the system are shown in the table 1.

Table 1. Physical parameters of the system

Bearings	K_{xx}=1 E9 N/m
Disk	M_d=0.4 kg, I_d=1.6E-04 kgm^2
Shaft	E=2E11 N/m^2, I=4E-12 m^4, $L_1 = L_2 = L_3 = L_4 = L_5$=0.1m, ρ=7850 kg/m^3
Foundation	k_f = 1E9 N/m., M_f = 30 Kg.

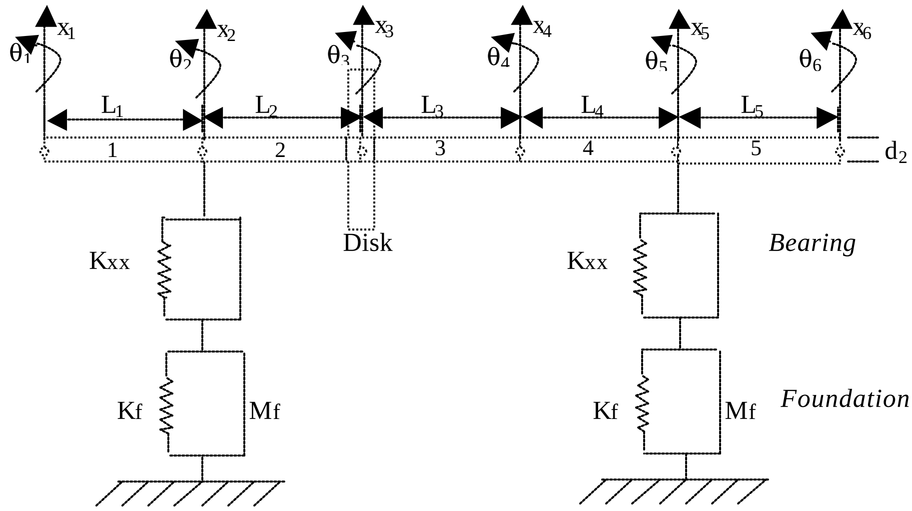

Figure 4. Representative scheme of rotor system to finite element mounted on lumped foundation

For this example, the natural frequencies are shown on the table 2.

Table 2. Natural frequencies of the system

ω_1 = 1.0041E04 rd/s	ω_7 = 4.0732E03 rd/s
ω_2 = 8.8248E03 rd/s	ω_8 = 1.1340E03 rd/s
ω_3 = 4.7949E03 rd/s	ω_9 = 6.0757E02 rd/s
ω_4 = 3.5816E03 rd/s	ω_{10} = 3.6682E02 rd/s
ω_5 = 2.9164E03 rd/s	ω_{11} = 1.9720E02 rd/s
ω_6 = 4.0618E03 rd/s	ω_{12} = 1.0729E01 rd/s

For this example, the parameters subject to fail are the diameters of each element of the rotor. The interval of time used for simulation went from 0 to 0.2 seconds, and was taken the number of sampled points equal 256.

The matrices [A] and [B] are given for:

$$[A]_{2nx2n} = \left[\begin{array}{c:c} [0]_{nxn} & [I]_{nxn} \\ \hdashline -[M]^{-1}[K] & -[M]^{-1}[C] \end{array}\right] \tag{9}$$

$$[B] = \left[\begin{array}{c} [0]_{nxn} \\ \hdashline [M]^{-1}_{nxn} \end{array}\right] \tag{10}$$

In this example, considered that the parameter d_2 suffered a loss 20% of its real value.

Therefore, using the developed methodology, the objective is to detect and to locate this fault in the analyzed system in question.

In the fig. 5 are presented the graphics 1 to 7, in the ordinate, the values of displacement $\{x_1(t)\}$ of the system (simulated) and the values reconstructed $\{\hat{x}_1(t)\}$ for the state observers, and in the abscissas, the time in seconds.

Firstly, as can be observed in the graphic 1, both curves are coincident, i.e., the global observer doesn't detect any irregularity in the simulated mechanical system.

In order to simulate a possible fault, 20% of the real value of the diameter of second element d_2 of the simulated system was removed. Thus, it is observed in graphic 2 that the curves are not more coincident, i.e., the global observer gets to detect a possible fault in the simulated system.

Detecting this fault, the next step is to locate this fault. For this, a set of robust observers to the possible parameters of system subject to fails was mounted, as can be seen in graphics 3 to 7.

It was verified that only in graph. 4 the curves are coincident, i.e., the robust observer mounted with 20% of fault in d_2 gets to locate the fault in the simulated system, that it was induced by the loss of 20% of d_2.

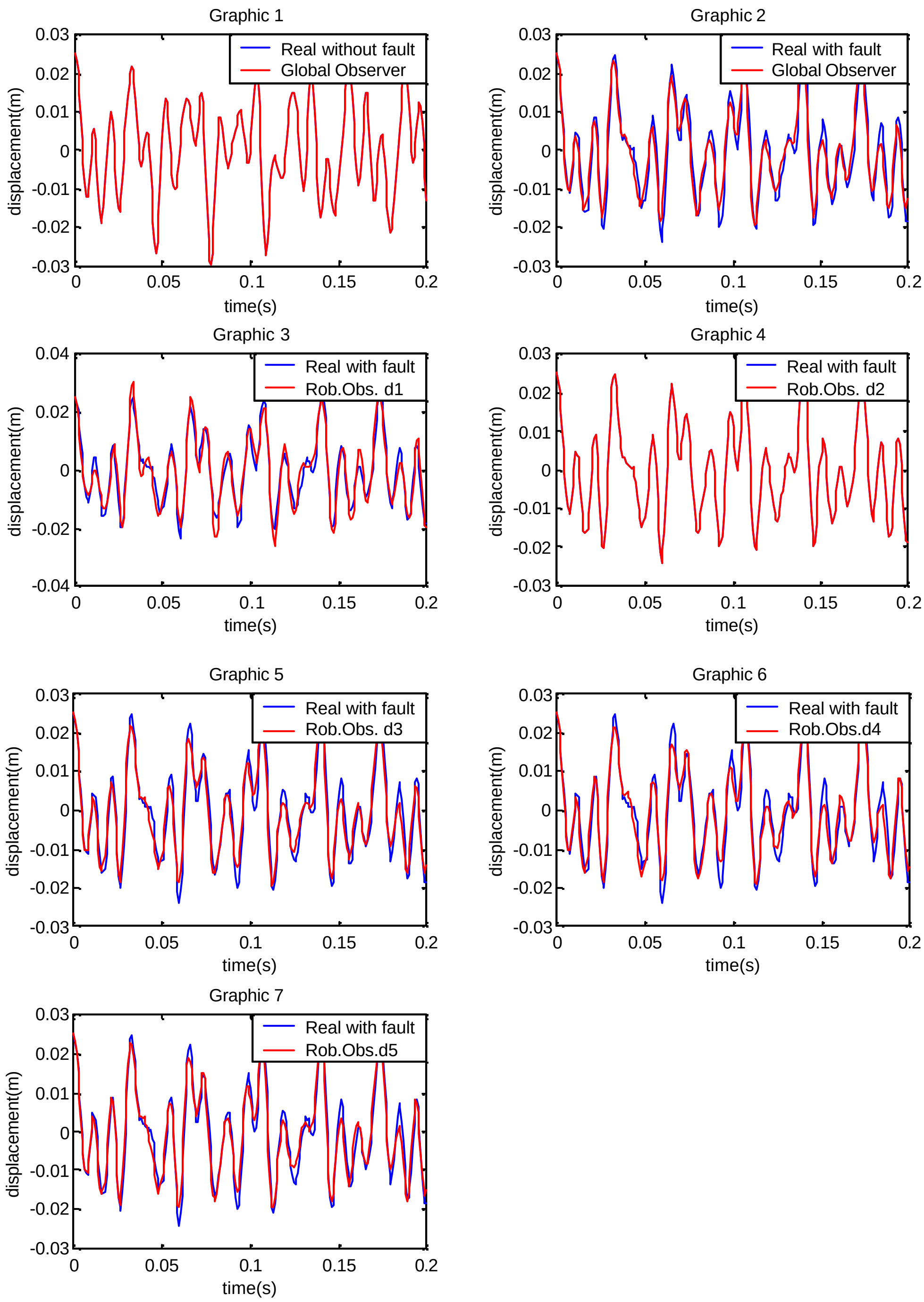

Figure 5. Results obtained

It can be seen from table 2, that the fault can be detected and located. By comparing the global system without fault with the global observer (second line with the second column of the table 2) it is verified that the order to difference of RMS (Root mean square) is 10E-10, shown that the curves are practically coincident, indicating that do not have any irregularity in the system. The fault is detected when it is compared the real system with fault with the global observer, the RMS difference is 10E-03, shown that the system has some irregularity (see second line with third column), to locate such irregularity, the global fault system is compared with the robust observer dedicated to a fault parameter. Under these conditions, the difference of RMS is order to 10E-11 (fourth line with thirty column) locating the fault.

Table 2. Difference of RMS.

	Real System without fault (^{1} RMS)	**Real System with fault of 20% d_2 (^{1} RMS)**
Global Observer	**1.0348E-10**	**1.1273E-03**
Robust observer (d_1)	**1.1899E-03**	**3.3641E-04**
Robust observer (d_2)	**2.7377E-04**	**4.6143E-11**
Robust observer (d_3)	**6.2635E-05**	**9.5473E-04**
Robust observer (d_4)	**1.0174E-03**	**7.9035E-04**
Robust observer (d_5)	**8.5298E-04**	**6.7237E-04**

2.5. Conclusion

In this paper a methodology has been developed using the state observers' technique for the detection and location of faults in rotor system considering their foundation. The technique employed can reconstruct non measured states or to estimate the values of points of difficult access in a system, detecting faults in these points without the knowledge of measured data, monitoring them through the reconstructions of its states. The methodology developed is only satisfied, if the analyzed system in question is observable using the number of measures carried out. In case this does not happen, new measures should be carried out until the system to be observable. Besides, the observers' eigenvalues should be selected to be a little more to the left in the complex plan of the eigenvalues of the observed system in order to guarantee the stability and the fast convergence of the process.

The numerical simulation has show the efficiency this technique, presenting a methodology for the detection and location of faults in rotor systems considering their foundation using state observers, which obtained quite satisfactory results, confirming the reliability of this methodology. This way, this methodology can be implemented for several types of mechanical systems.

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