DYNAMIC SIMULATION OF FAN-COIL SYSTEMS

Rogério Marcos Barbosa
Pontifícia Universidade Católica do Paraná, Centro de Ciências Exatas e de Tecnologia, Laboratório de Sistemas Térmicos (www.pucpr.br/LST), Rua Imaculada Conceição, 1155, 80215-901, Curitiba/PR, Brasil.
nmendes@ccet.pucpr.br

Nathan Mendes
Pontifícia Universidade Católica do Paraná, Centro de Ciências Exatas e de Tecnologia, Laboratório de Sistemas Térmicos (www.pucpr.br/LST), Rua Imaculada Conceição, 1155, 80215-901, Curitiba/PR, Brasil.
nmendes@ccet.pucpr.br

Abstract. *Heating, Ventilation and Air Conditioning (HVAC) systems are responsible for a considerable amount of energy consumption, especially in office buildings. Normally, simulation of those systems, in building energy simulation programs, is made by using empirical steady-state correlations for the total cooling capacity, the sensible capacity and the Energy Efficiency Ratio (EER). However, the dynamic response of HVAC components has a direct impact on the energy consumption and thermal comfort evaluation so that a higher accuracy is needed. In this way, a dynamic model composed of 10 state variables is presented. The time derivative solution is analyzed for implicit and Crank-Nicolson schemes. Results in terms of psychrometrics state of air in different parts of a fan-coil system are presented. In addition, demand and energy consumption of electricity are also discussed.*

Keywords: *Fan-coil, non-steady simulation.*

1. Introduction

Energy and fuel required for HVAC systems have a direct impact on buildings operating costs and an indirect impact on the environment. In the 70´s, with the worldwide energy crisis, many research efforts started in order to reduce building energy consumption, with projects focused on demand-side management of fuel (DSM).

In Brazil, 48% of the total electricity is attributed to residential, commercial and industrial buildings (Lamberts et al., 1998) and, in commercial buildings, most of the consumed energy is due mainly to HVAC systems, which points out to an important research theme direction.

In the literature, several authors focus their research on steady behavior of HVAC systems with empirical, semi-empirical and mathematical approaches. Pereira and Mendes (2003) presented an empirical model to predict thermal performance of two direct-expansion air conditioners. Lebrun (2001) accomplished a simulation of a HVAC system with the help of an engineering equation solver. The HVAC system considered includes a set of four twin-screw chillers, four ice-storage tanks and five cooling towers. Knabe and Le (2001) introduces building simulation in conjunction with a HVAC system, especially a split system, considering the thermal and moisture behaviors of the perimeter walls.

Those above-mentioned works present a steady-state approach for HVAC systems analysis focused on the optimization, as they are expensive and heavy energy consumers.

Air conditioning systems can be built from a limited number of components: duct, mixing box, fan, heating coil, cooling coil, boiler, pump, pipe, chiller, heat pump, diffuser, damper and n-way diverging/converging junction. Mathematical models of these components are required at different levels of detail in order to support the range of possible design tasks. The simulation of HVAC systems is complicated by the fact that the working fluid comprises two phases, dry air and water vapor. Hence, normally, simulation of those systems, in building energy simulation programs, is made by using empirical steady-state correlations for the total cooling capacity, the sensible capacity and the Energy Efficiency Ratio (EER).

However, the dynamic response of HVAC components has a direct impact on the energy consumption and thermal comfort evaluation so that a higher accuracy is needed. In this way, a dynamic model composed of 10 state variables is presented. The time derivative solution is analyzed for implicit and Crank-Nicolson schemes. Results in terms of psychrometrics state of air in different parts of a fan-coil system are presented. In addition, demand and energy consumption of electricity are also discussed.

1. Mathematical Model

Fig. 1 shows a fan-coil representative scheme, which is made up of five components.

Component 1 is a mixing box that has the purpose of mixing the return air, which comes from the conditioned zone, with the external air.. Component 2 is a cooling coil, in which cold water circulates to cool down the moist air arrived from the mixing box. Component 3 represents a humidifier, which humidifies the air when it is necessary depending upon a relative humidity set point. Component 4 is a device for heating up air, which can be used when latent loads are high or for fine tuning of indoor temperature.. The last one (component 5) is the fan responsible for supplying the air into the zone.

In order to write the components mathematical formulation, some considerations should be done: i) a mean air mass density is assumed for all components; ii) air pressure is kept constant and iii) air is a perfect gas.

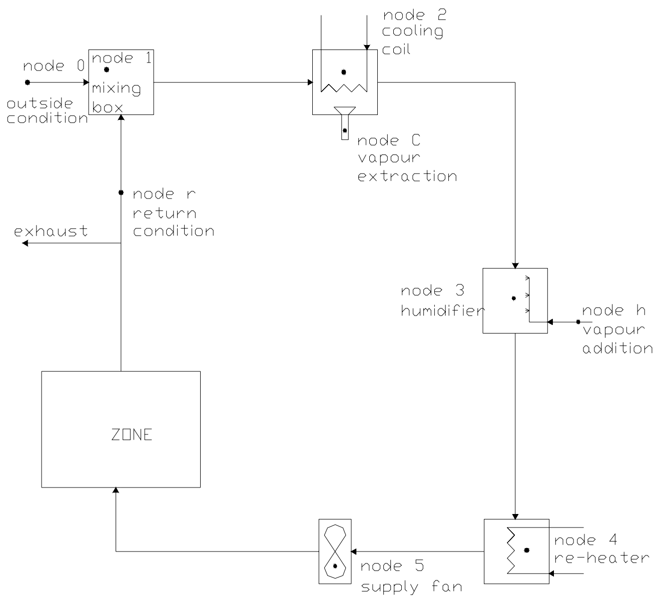

Figure 1 Schematic representation of a fan-coil system.

In the present work, a lumped formulation for calculating both air temperature and humidity ratio is considered for each fan-coil component. Eqs. (1-5) describe the energy balance equations, while Eqs. (8-16) the mass conservation equations for both water vapor and dry air. Hence, the system of equations is based on the lumped approach for the conservation equations applied to each control volume, similarly to what was presented by Clarke (2001) for a non-steady simulation in the ESP (Environmental System Performance) building simulation program.

2.1 Energy Conservation Equations

Applying the non-steady energy conservation equation for each component, at a given time ξ, one obtains:

For component 1:

$$\dot{m}_0 h_0 + \dot{m}_r h_r - \dot{m}_1 h_1 + q_{e1} = \left.\frac{d\left(\bar{\rho}_1 V_1 h_1\right)}{dt}\right|_{t=\xi} \qquad (1)$$

For component 2:

$$\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_c h_c + q_{e2} - q_{x2} = \left. \frac{d\left(\bar{\rho}_2 V_2 h_2\right)}{dt} \right|_{t=\xi} \tag{2}$$

For component 3:

$$\dot{m}_2 h_2 + \dot{m}_h h_h - \dot{m}_3 h_3 + q_{e3} = \left. \frac{d\left(\bar{\rho}_3 V_3 h_3\right)}{dt} \right|_{t=\xi} \tag{3}$$

For component 4:

$$\dot{m}_3 h_3 - \dot{m}_4 h_4 + q_{e4} + q_{x4} = \left. \frac{d\left(\bar{\rho}_4 V_4 h_4\right)}{dt} \right|_{t=\xi} \tag{4}$$

For component 5:

$$\dot{m}_4 h_4 - \dot{m}_5 h_5 + q_{e5} = \left. \frac{d\left(\bar{\rho}_5 V_5 h_5\right)}{dt} \right|_{t=\xi} \tag{5}$$

where $\dot{m}$ is the mass flow rate of the air/vapor mixture (kg/s), h the mixture specific enthalpy (J/kg), q_{ei} the *i-th* component's heat exchange with the surroundings (W), q_{x2} the cooling coil total heat transfer (W), q_{x4} the re-heat coil total heat transfer (W), $\bar{\rho}_i$ the volume weighted density of component *i* (kg m^{-3}), V_i the total volume of component i (m^3). The sub-scripts *o* and *r* are related to ambient and return air states respectively, while *c* relates to the cooler moisture extract and *h* to the humidifier moisture addition.

Since mass density of each component is calculated as:

$$\bar{\rho}_i = \sum_{j=1}^{N} (\rho_j V_j) \Big/ \sum_{j=1}^{N} (V_j) \tag{6}$$

where N is the number of distinct intra-component regions.

2.2 Mass Conservation Equations

Similarly to the energy conservation equations, the non-steady mass balance equations are written below.

For component 1:

$$\dot{m}_{0(d)} + \dot{m}_{r(d)} - \dot{m}_{1(d)} = 0 \Big|_{t=\xi} \tag{7}$$

$$\dot{m}_{0(d)} w_o + \dot{m}_{r(d)} w_r - \dot{m}_{1(d)} w_1 = 0 \Big|_{t=\xi} \tag{8}$$

For component 2:

$$\dot{m}_{1(d)} - \dot{m}_{2(d)} = 0\Big|_{t=\xi} \tag{9}$$

$$\dot{m}_{1(d)} w_1 - \dot{m}_{2(d)} w_2 - \dot{m}_c = \frac{d(\rho_L V_c)}{dt}\bigg|_{t=\xi} \tag{10}$$

For component 3:

$$\dot{m}_{2(d)} - \dot{m}_{3(d)} = 0\Big|_{t=\xi} \tag{11}$$

$$\dot{m}_{2(d)} w_2 - \dot{m}_{3(d)} w_3 + \dot{m}_h = \frac{d(\rho_L V_h)}{dt}\bigg|_{t=\xi} \tag{12}$$

For component 4:

$$\dot{m}_{3(d)} - \dot{m}_{4(d)} = 0\Big|_{t=\xi} \tag{13}$$

$$\dot{m}_{3(d)} w_3 - \dot{m}_{4(d)} w_4 = 0\Big|_{t=\xi} \tag{14}$$

For component 5:

$$\dot{m}_{4(d)} - \dot{m}_{5(d)} = 0\Big|_{t=\xi} \tag{15}$$

$$\dot{m}_{4(d)} w_4 - \dot{m}_{5(d)} w_5 = 0\Big|_{t=\xi} \tag{16}$$

where $\dot{m}_{i(d)}$ is the mass flow rate of dry air (kg/s) associated with component *i*, *w* the humidity ratio (kg/ kg), ρ_L the density of water remaining in the cooler or humidifier (kg/m^3), V_c the volume of this water, V_h the humidifier residual water volume, $\dot{m}_c$ the cooler vapor extraction rate (kg/s) and $\dot{m}_h$ the humidifier vapor addition rate (kg/s).

2. Discretization

The discretization of the differential conservation equations described in section 2 is presented below. These equations were integrated by using a weighting factor f, varying from *0* to *1*. When f is null, the method is explicit and when it is equal to one the method is implicit (Patankar, 1980). In this work, the discretized equations are written in terms of *f*, however, they are solved for fully-implicit and Crank-Nicolson (*f=0.5)* schemes

For the sake of clarity and due to space limitations, just the component 1 equations are shown.

For component 1:

$$\left(\frac{\bar{\rho}_1(t+\Delta t)V_1(t+\Delta t)}{\Delta t} + f\,\dot{m}_1(t+\Delta t)\right) h_1(t+\Delta t) =$$

$$\left(\frac{\bar{\rho}_1(t)V_1(t)}{\Delta t} - (1-f)\,\dot{m}_1(t)\right) h_1(t) + f\left[\dot{m}_o(t+\Delta t)h_o(t+\Delta t) + \dot{m}_r(t+\Delta t)h_r(t+\Delta t) + q_e(t+\Delta t)\right]$$

$$+(1-f)\left[\dot{m}_o(t)h_o(t) + \dot{m}_r(t)h_r(t) + q_e(t)\right]$$

Fig. 2 shows the matrices for the set of governing equations that model the system illustrated in Fig. 1. Then, the component 1 discretized equation gives

$$a_{11}h_1(t+\Delta t) = b_{11}h_1(t) + c_1 \tag{17}$$

$$\begin{array}{c} \text{A} \\ j = \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \\ \begin{matrix} 1 \\ 2 \\ i=3 \\ 4 \\ 5 \end{matrix} \begin{bmatrix} x & & & & \\ x & x & & & \\ & x & x & & \\ & & x & x & \\ & & & x & x \end{bmatrix} \end{array} \; X \; \begin{array}{c} h_i(t+\Delta t) \\ \begin{bmatrix} x \\ x \\ x \\ x \\ x \end{bmatrix} \end{array} = \begin{array}{c} \text{B} \\ \begin{bmatrix} x & & & & \\ x & x & & & \\ & x & x & & \\ & & x & x & \\ & & & x & x \end{bmatrix} \end{array} \; X \; \begin{array}{c} h_i(t) \\ \begin{bmatrix} x \\ x \\ x \\ x \\ x \end{bmatrix} \end{array} + \begin{array}{c} \text{C} \\ \begin{bmatrix} x \\ x \\ x \\ x \\ x \end{bmatrix} \end{array}$$

$\downarrow$

entry a_{55} removed to c_5 in absence of a zone matrix

Figure 2. Fan-coil system energy balance matrix equation, Ah(t+Δt) = Bh(t) + C.

Fig. 2 subscripts of *a b* coefficients refer to the *i-th* row and *j-th* column positions:

$$a_{11} = \left(\frac{\bar{\rho}_1(t+\Delta t)V_1(t+\Delta t)}{\Delta t} + f\,\dot{m}_1(t+\Delta t) \right) \qquad b_{11} = \left(\frac{\bar{\rho}_1(t)V_1(t)}{\Delta t} - (1-f)\,\dot{m}_1(t) \right)$$

$$c_1 = f\left[\dot{m}_o(t+\Delta t)h_o(t+\Delta t) + \dot{m}_r(t+\Delta t)h_r(t+\Delta t) + q_e(t+\Delta t)\right] + (1-f)\left[\dot{m}_o(t)h_o(t) + \dot{m}_r(t)h_r(t) + q_e(t)\right]$$

For the other components, the differential governing equations are similarly discretized, andthe linear system of equations shown in Fig. 2 is accomplished.

The next step is solving the mass balance equations, which gives for the component 1 the following equations:

<u>Dry air</u>:

$$f\,\dot{m}_{1(d)}(t+\Delta t) = -(1-f)\dot{m}_{1(d)}(t) + f\left[\dot{m}_{o(d)}(t+\Delta t) + \dot{m}_{r(d)}(t+\Delta t)\right] + (1-f)\left[\dot{m}_{o(d)}(t) + \dot{m}_{r(d)}(t)\right]$$

<u>Water vapor</u>

$$f\left[\dot{m}_{1(d)}(t+\Delta t)\,w_1(t+\Delta t)\right] = -(1-f)\,\dot{m}_{1(d)}(t)\,w_1(t) + f\left[\dot{m}_{o(d)}(t+\Delta t)\,w_o(t+\Delta t) + \dot{m}_{r(d)}(t+\Delta t)\,w_r(t+\Delta t)\right]$$

$$+ (1-f)\left[\dot{m}_{o(d)}(t)\,w_o(t) + \dot{m}_{r(d)}(t)\,w_r(t)\right]$$

which gives, according to Fig. 3,

$$d_{11}\,\dot{m}_{1(d)}(t+\Delta t) = e_{11}\,\dot{m}_{1(d)}(t) + y_1$$

and

$$d_{22}[\dot{m}_{1(d)}(t+\Delta t)\,w_1(t+\Delta t)] = e_{22}[\dot{m}_{1(d)}(t)\,w_1(t)] + y_2$$

where

$d_{11} = f$ $\qquad\qquad$ $e_{11} = -(1-f)$

$$y_1 = f\left[\dot{m}_{o(d)}(t+\Delta t) + \dot{m}_{r(d)}(t+\Delta t)\right] + (1-f)\left[\dot{m}_{o(d)}(t) + \dot{m}_{r(d)}(t)\right]$$

$d_{22} = f$ $\qquad\qquad$ $e_{22} = -(1-f)$

$$y_2 = f\left[\dot{m}_{o(d)}(t+\Delta t)\, w_o(t+\Delta t) + \dot{m}_{r(d)}(t+\Delta t)\, w_r(t+\Delta t)\right] + (1-f)\left[\dot{m}_{o(d)}(t)\, w_o(t) + \dot{m}_{r(d)}(t)\, w_r(t)\right]$$

$$\underbrace{\begin{bmatrix} 1 & & & & & & & & & \\ & 1 & & & & & & & & \\ 1 & & -1 & & & & & & & \\ & 1 & & -1 & & & & & & \\ & & 1 & & -1 & & & & & \\ & & & 1 & & -1 & & & & \\ & & & & 1 & & -1 & & & \\ & & & & & 1 & & -1 & & \\ & & & & & & 1 & & -1 & \\ & & & & & & & 1 & & -1 \end{bmatrix}}_{D,\ i = 1 \ldots 10,\ j = 1 \ldots 10} \mathrm{X} \underbrace{\begin{bmatrix} \dot{m}_{1(d)} \\ \dot{m}_{1(d)} w_1 \\ \dot{m}_{2(d)} \\ \dot{m}_{2(d)} w_2 \\ \dot{m}_{3(d)} \\ \dot{m}_{3(d)} w_3 \\ \dot{m}_{4(d)} \\ \dot{m}_{4(d)} w_4 \\ \dot{m}_{5(d)} \\ \dot{m}_{5(d)} w_5 \end{bmatrix}}_{\phi_i(t+\Delta t)} =$$

$$\underbrace{\begin{bmatrix} -1 & & & & & & & & & \\ & -1 & & & & & & & & \\ -1 & & 1 & & & & & & & \\ & -1 & & 1 & & & & & & \\ & & -1 & & 1 & & & & & \\ & & & -1 & & 1 & & & & \\ & & & & -1 & & 1 & & & \\ & & & & & -1 & & 1 & & \\ & & & & & & -1 & & 1 & \\ & & & & & & & -1 & & 1 \end{bmatrix}}_{E} \mathrm{X} \underbrace{\begin{bmatrix} \dot{m}_{1(d)} \\ \dot{m}_{1(d)} w_1 \\ \dot{m}_{2(d)} \\ \dot{m}_{2(d)} w_2 \\ \dot{m}_{3(d)} \\ \dot{m}_{3(d)} w_3 \\ \dot{m}_{4(d)} \\ \dot{m}_{4(d)} w_4 \\ \dot{m}_{5(d)} \\ \dot{m}_{5(d)} w_5 \end{bmatrix}}_{\phi_i(t)} + \underbrace{\begin{bmatrix} X \\ X \\ 0 \\ X \\ 0 \\ X \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{y}$$

Figure 3 Fan-coil system mass balance matrix equation, $D\phi(t+\Delta t) = E\phi(t) + y$.

The other components discretized equations allow establishing the system of linear equations shown in Fig. 3.

In order to solve the linear system of equations represented by Figs. 2 and 3, a *C* program was written by using the Gauss Jordan's method (Carvalho et al., 1987).

3. Results

The results presented in this section were gotten from simulations for a summer period by using three fan-coil components: mixture box, cooling coil and re-heater. In a first analysis, the cooling coil surface temperature was above the dew point temperature so that avoiding condensation in a non-steady flow.

Figs. 4 and 5 show the psychrometrics input data. A sinusoidal variation for temperature and relative humidity were considered for both external and return air flows.

Table 1 presents the necessary boundary conditions to carry out 10-s time step simulations. In this case, the sensible and latent zone loads were assumed to be constant. The energy and mass balance equations were integrated by using the Crank-Nicolson scheme (f=0.5).

Table 1. Input data for a summer period.

Zone sensible load (kW)	92
Zone latent load (kW)	8
Temperature set point (ºC)	24
Supply air temperature (ºC)	16,3
Mass flow rate of external air (kg/s)	6
Mass flow rate of return air (kg/s)	6
Cooling power (kW)	175

The supply air temperature was calculated based on both temperature set point and zone sensible load.

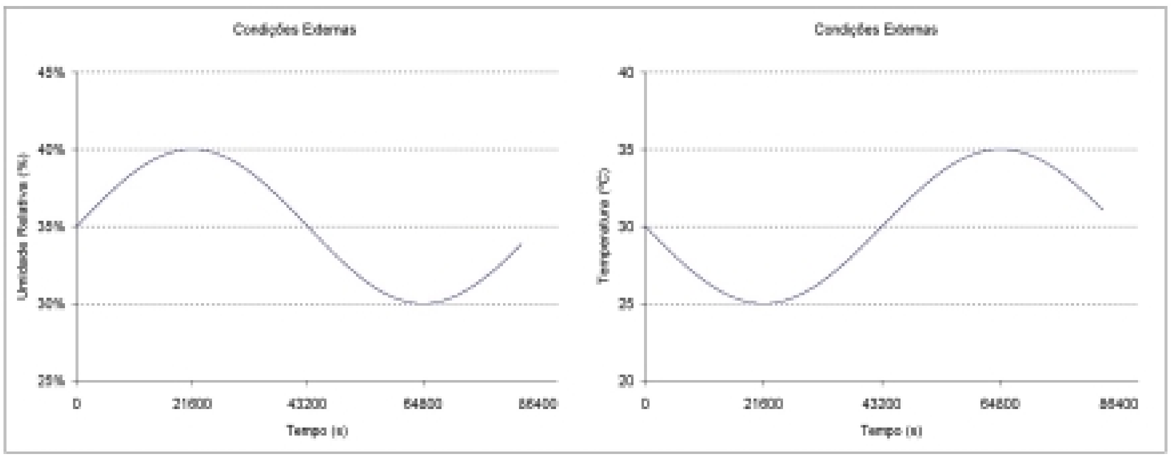

Figure 4. Relative humidity and temperature evolution in time of external air

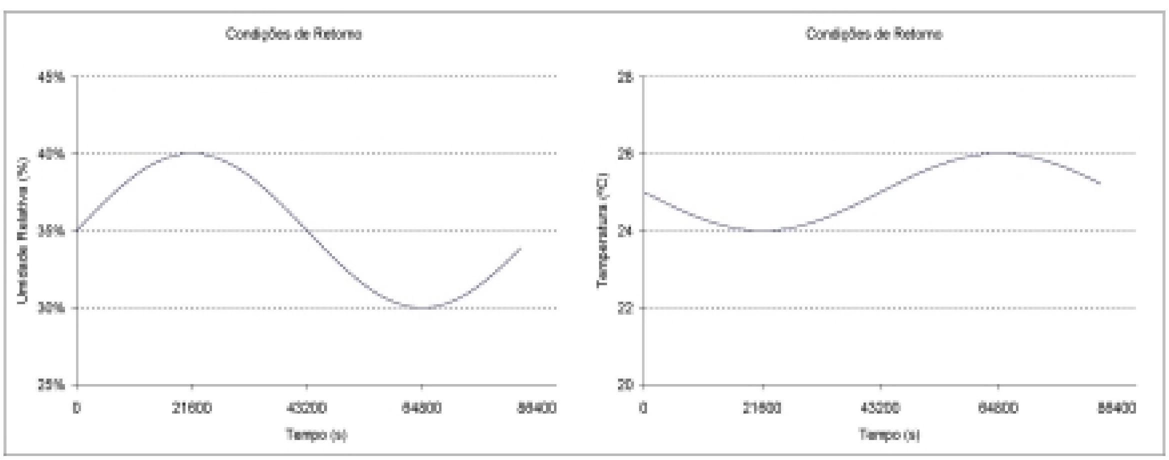

Figure 5 Relative humidity and temperature evolution in time of return air.

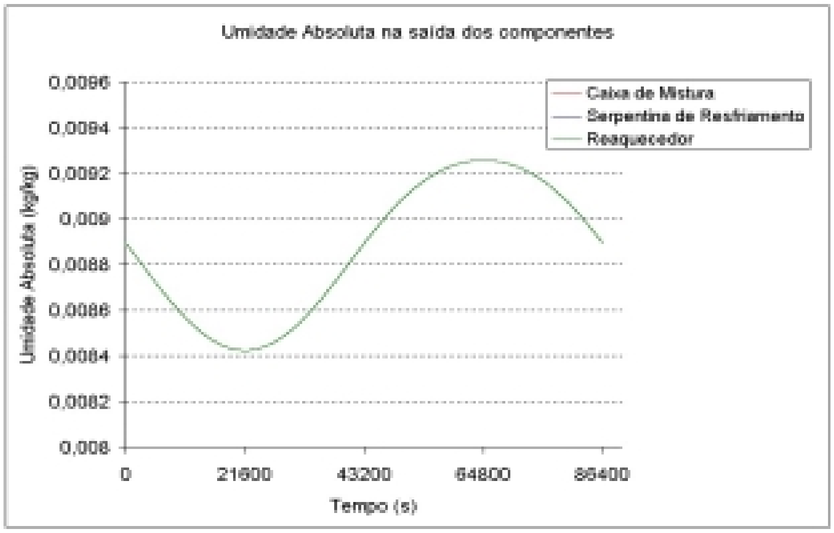

Figure 6 Humidity ratio evolution in time in each fan-coil component.

Figure 6 illustrates the temporal variation of humidity ratio in each fan-coil component. Actually, as expected, the humidity is the same in all components as no condensation is occurring on the cooling coil and humidifier is off. The Fig.-6 sinusoidal shape is due to the combined variation of external and return air flows.

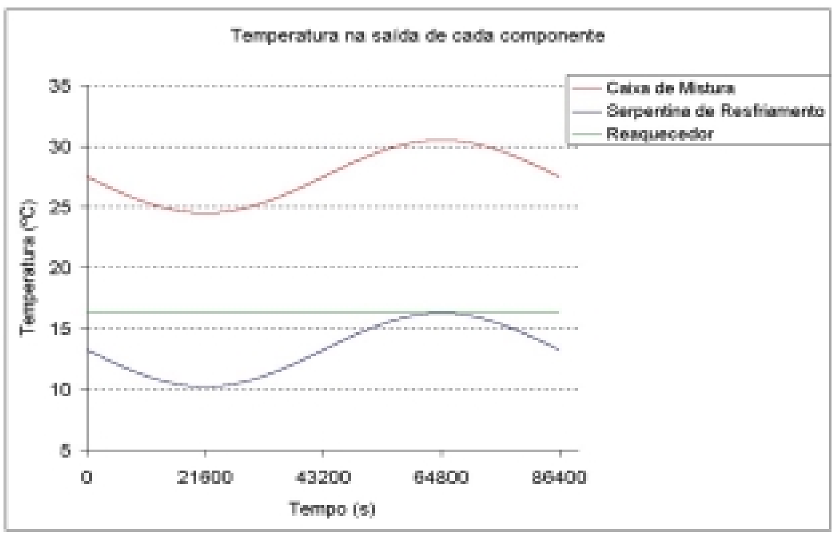

Figure 7 Air temperature in each fan-coil component.

Figure 7 presents the air temperature variation in each fan-coil component. The red curve represents the temperature variation within the mixing box. As one could expect, this variation is between those of external and return air flows, shown in Figs. 4 and 5. The blue curve shows the temperature of air leaving the cooling coil, which is the lowest temperature found in the system. As this temperature is below the calculated supply air temperature, it is necessary to heat up air in order to make it reach the temperature of 16,3 ºC. The green curve shows the air temperature at re-heater exit, which is exactly the necessary supply air temperature, thanks to the re-heater fine tuning.

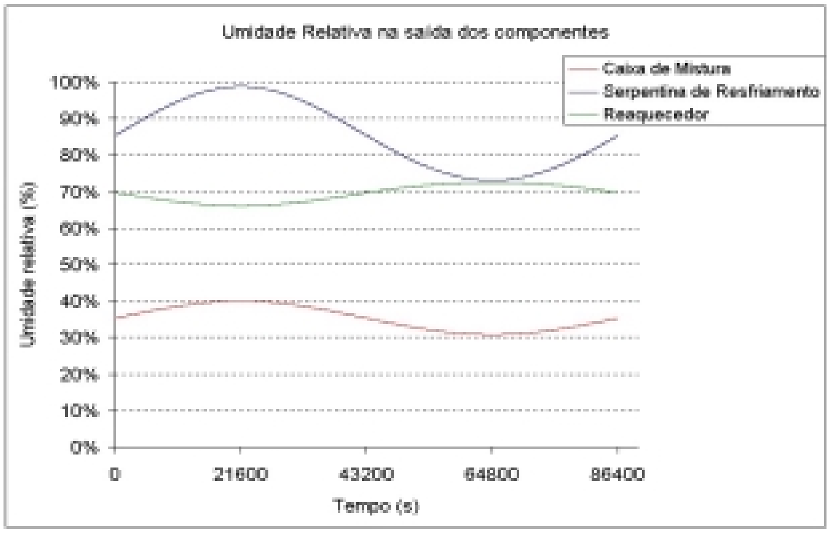

Figure 8 Air relative humidity in each fan-coil component..

Figure 8 shows air relative humidity behavior within each fan-coil component. High relative humidity at the cooling coil is noticed which is explained by the strong temperature decrease in the absence of condensation. On the other hand, in the re-heater, the air relative humidity drops substantially due to the pure heating process that takes place in that component.

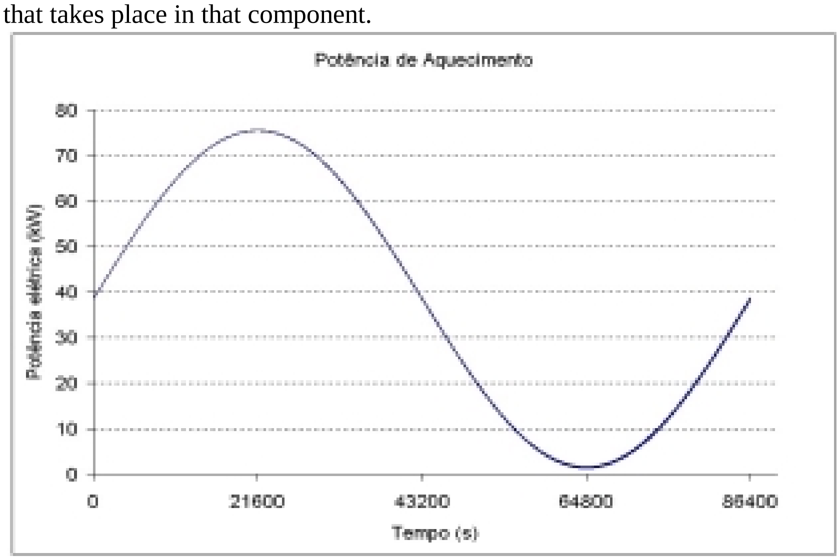

Figure 9 Heating power.

Figure 9 shows the heating power variation necessary to heat up air from cooling coil so that reaching the necessary supply air temperature.

The weighting factor *f* has also been analyzed. Simulations with *f* smaller than 0.5 could not been carried out due to overflow errors for the 10-s time step, or even smaller than that. On the other hand, the higher weighting factor the lower the computer run time. Temperature and humidity ratio did not show any sensitivity by varying the weighting factor from 0.5 to 1. Therefore, the use of a fully-implicit scheme ($f = 1$) is recommended for simulating this fan-coil model presented in this work.

4. Conclusions

In this paper we have described a mathematical model and a discretization process to simulate a fan-coil system. We have noticed that simulation of those systems is important to help engineers on taking decision and on estimating energy costs.

The fan-coil mathematical model was described, as proposed by Clarke (2001), composed by a mixing box, a cooling coil, a humidifier, a re-heater and a supply fan. In this model, a non-steady state simulation can be carried out according to imposed boundary conditions.

The system of differential equations was linearized in a literal way as a function of a factor *f*, which determines if the system will be solved by an explicit way (f=0), implicit (f=1) or by the Crank-Nicolson scheme (f=0.5). It was noticed that for *f* greater than 0.5 the numerical solution converges and lead to the same results for a 10-s time step. Then, as a higher weighting factor speeds up simulations, we recommend the use of the fully-implicit approach.

For future work, we recommend replacing the cooling coil model by a cooling and dehumidifying coil model, which will allow taking the non-steady condensation phenomenon into account. Also, this model can be extended to a primary refrigeration system, i.e., chiller, and integrated to the Domus building simulation program.

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