BIOT NUMBER EFFECTS ON THE NUMERICAL STABILITY OF HEAT AND MASS TRANSFER PROBLEMS

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Abstract. *In many cases the mass and energy conservation equations are strongly coupled, specially at the boundaries of porous media. This strong coupling may easily cause numerical divergence when a not very robust solver is used. A dimensionless number that could numerically express when numerical instability arises is the Biot number for moisture diffusion. This number gives the relation between the convective and diffusive resistances, which means that, for high Bi_m numbers, the wall hygric resistance is much higher than the corresponding surface resistance. Another important dimensionless is the mass Fourier number, expressing a mesh-size parameter that could be related to a convergence error function. Therefore, we present a mathematical model to solve a heat and mass transfer problem by using two algorithms and carry out a sensitivity analysis of numerical performance in terms of: i) moisture and heat Biot Numbers; ii) moisture and heat Fourier Numbers; iii) Luikov number and iv) Posnov number. It is noted, for traditional algorithms such as TDMA, the risk of divergence arises very quickly as the moisture Biot number increases, showing a high sensitivity to this dimensionless parameter, which can be even higher than the one caused by the mass Fourier number.*

Keywords: *Biot number, Numerical stability, Coupled heat transfer, Porous media.*

1. Introduction

In many cases the mass and energy conservation equations are strongly coupled, specially at the boundaries of porous media. This strong coupling may easily cause numerical divergence when the solver is not robust.

A dimensionless number that could numerically express when numerical instability arises is the Biot number for moisture diffusion. This number gives the relation between the convective and diffusive resistances, which means that, for high Bi_m numbers, the wall hygric resistance is much higher than the corresponding surface resistance. Another important dimensionless is the mass Fourier number, expressing a mesh-size parameter that could be related to a convergence error function.

Dantas et al. (2003) presented the solution for an inverse problem of parameter estimation in a 1-D capillary porous medium, by using the well-known dimensionless Luikov's model. They showed that is possible to simultaneously estimate the Luikov number, the Kossovitch number and the Biot numbers for heat and moisture diffusion. In a previous work (Dantas et al., 2002), they concluded that those parameters could be accurately estimated by using only temperature measurements in the inverse analysis.

In order to analyze the dimensionless numbers influence, mainly the one caused by the Biot numbers, we have implemented a dynamic heat and moisture transfer model. For the walls, sensible and latent surface convection, absorbed solar radiation, heat and mass transfer through the wall, and vapor/liquid phase change are considered. The walls are described mathematically using the model of Philip and DeVries (1957) in which vapor and liquid flow under moisture content and temperature gradients. In this model, heat, vapor and liquid flow are taken to be simultaneous. Physical quantities, such as mass transport coefficients, thermal conductivity and specific heat, are variable and dependent of wall moisture content, but moderately temperature dependent.

In this papers, results in terms of numerical convergence errors in terms of Biot number for moisture diffusion are presented. Additionally, effects of Biot and Luikov numbers on the one-dimensional heat flux through a lime mortar wall are presented as well.

2. Mathematical Model

The governing partial differential equations are given by Equations (1) and (2). They were derived from conservation of mass and energy flow in an elemental volume of porous material.

Mass conservation equation

$$\rho_l \frac{\partial}{\partial t}(\theta) = -\frac{\partial}{\partial x}(j_l + j_v) \tag{1}$$

Energy conservation equation

$$\rho_0 c_m \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - L\frac{\partial}{\partial x}(j_V) \tag{2}$$

The parameter c_m is the specific heat, which is a function of θ. The quantity λ is the thermal conductivity of the medium in the absence of phase change. It usually depends strongly on θ and weakly on *T*.

The vapor and liquid flows are expressed in terms of transport coefficients, *D*, associated with the thermal and moisture gradients. According to Philip and DeVries (1957), the equations are:

For liquid flow

$$\frac{j_l}{\rho_l} = -D_{Tl}\frac{\partial T}{\partial x} - D_{\theta l}\frac{\partial \theta}{\partial x} \tag{3}$$

For vapor flow

$$\frac{j_v}{\rho_l} = -D_{Tv}\frac{\partial T}{\partial x} - D_{\theta v}\frac{\partial \theta}{\partial x} \tag{4}$$

3. Boundary Conditions

The associated conservation equations at the outside and inside wall surface are as follows. For the outside surface (x=0), it was considered the wall is exposed to short-wave radiation, convection heat and mass transfer, and phase change. Thus, the energy balance becomes

$$-\left(\lambda(T,\theta)\frac{\partial T}{\partial x}\right)_{x=0} - (L(T)j_v)_{x=0} = h\,(T_\infty - T_{x=0}) + \alpha q_r + L(T)h_m(\rho_{v,\infty} - \rho_{v,x=0}), \tag{5}$$

where: $h\,(T_\infty - T_{x=0})$ represents the heat exchanged with the outside air, described by the surface conductance h , $\alpha\, q_r$ is the absorbed short-wave radiation and $h_m(\rho_{v,\infty} - \rho_{v,x=0})$, the phase change energy term. The solar absorptivity is defined as α and the mass convection coefficient as h_m which is related to *h* by the Lewis' relation.

The mass balance at the outside surface (x=0) is described as,

$$-\frac{\partial}{\partial x}\left(D_\theta(T,\theta)\frac{\partial \theta}{\partial x} + D_T(T,\theta)\frac{\partial T}{\partial x}\right)_{x=0} = \frac{h_m}{\rho_l}(\rho_{v,\infty} - \rho_{x=0}). \tag{6}$$

where:

$D_\theta = D_{\theta l} + D_{\theta v}$;

$D_T = D_{Tl} + D_{Tv}$.

The same above equations apply to the inside surface (*x*=*L*), with the omission of short-wave related terms.

4. The MultiTriDiagonal-Matrix Algorithm

Mendes and Philippi (2003) presented a generic algorithm to solve strongly-coupled heat and mass transfer equations, which provides all the dependent variable profiles simultaneously at a given time step. The use of this algorithm avoids numerical divergence caused by the evaluation of coupled terms from previous iteration values. In this way, mathematical methods can become stable and closer to the nature of the physical phenomenon of combined heat and mass transfer.

Discretization of conservation equations in the physical domain leads to following system of algebraic equations, which can be written as

$$\mathbf{A_i}.\mathbf{x_i} = \mathbf{B_i}.\mathbf{x_{i+1}} + \mathbf{C_i}.\mathbf{x_{i-1}} + \mathbf{D_i}\ , \tag{6}$$

where $\mathbf{x}$ is a vector containing the m dependent variables φ_j,

$$\mathbf{x}_i = \left[\varphi_{1,i}\,\varphi_{2,i}\ldots\varphi_{m,i}\right]^t . \tag{7}$$

and, differently from the traditional TDMA, coefficients A, B and C are 2nd order tensors, in which each line corresponds to one dependent variable. The elements that do not belong to the main diagonal are the coupled terms for each conservation equation.

Vector $\mathbf{x}_i$ can be expressed as a function of $\mathbf{x}_{i+1}$,

$$\mathbf{x_i} = \mathbf{P_i}.\mathbf{x_{i+1}} + \mathbf{q_i} \tag{8}$$

where P_i is, now, a 2nd order tensor.

Replacing equation (8), evaluated at point I-1 in (6), the following equation is obtained

$$\left(\mathbf{A_i} - \mathbf{C_i}.\mathbf{P_{i-1}}\right).\mathbf{x_i} = \mathbf{B_i}.\mathbf{x_{i+1}} + \mathbf{C_i}.\mathbf{q_{i-1}} + \mathbf{D_i}\ , \tag{9}$$

Writing equation (9) in an explicit way for $\mathbf{x}_i$,

$$\mathbf{x_i} = \left[\left(\mathbf{A_i} - \mathbf{C_i}.\mathbf{P_{i-1}}\right)^{-1}.\mathbf{B_i}\right].\mathbf{x_{i+1}} + \left(\mathbf{A_i} - \mathbf{C_i}.\mathbf{P_{i-1}}\right)^{-1}\left(\mathbf{C_i}.\mathbf{q_{i-1}} + \mathbf{d_i}\right), \tag{10}$$

Thus, a Comparison between equations (10) and (8) gives the following new recursive expressions,

$$\mathbf{P_i} = \left[\left(\mathbf{A_i} - \mathbf{C_i}.\mathbf{P_{i-1}}\right)^{-1}.\mathbf{B_i}\right] \tag{11}$$

and

$$\mathbf{q_i} = \left(\mathbf{A_i} - \mathbf{C_i}.\mathbf{P_{i-1}}\right)^{-1}\left(\mathbf{C_i}.\mathbf{q_{i-1}} + \mathbf{D_i}\right). \tag{12}$$

Once those matrix coefficients are calculated, the back substitution, in reverse order, makes that all vector $\mathbf{x_i}$ elements appear quite mechanically.

The use of this new algorithm makes the systems of equations to be more diagonally dominant and its use is illustrated in the sections below, for the study of heat and mass transfer through a porous medium.

5. Nondimensionalizing the Governing Equations

Considering constant all the coefficients, the energy and mass conservation equations can be written as:

$$\frac{\partial T}{\partial t} = \alpha_{ap} \frac{\partial^2 T}{\partial x^2} + \frac{L\rho_l}{C} D_{\theta V} \frac{\partial^2 \theta}{\partial x^2} \tag{13}$$

and

$$\frac{\partial \theta}{\partial t} = D_\theta \frac{\partial^2 \theta}{\partial x^2} + D_T \frac{\partial^2 T}{\partial x^2}\,. \tag{14}$$

where: α_{ap} is the thermal diffusivity (λ_{ap}/C);

C is the thermal capacity (ρc_m).

In many Engineering applications, the second right-hand term of the energy conservation equation which is directly associated to the phase change term can be disregarded when compared to the heat diffusion term. In this way, the energy conservation equation is reduced to:

$$\frac{\partial T}{\partial t} = \alpha_{ap} \frac{\partial^2 T}{\partial x^2}\,. \tag{15}$$

Considering the following dimensionless groups,

$$T^* = \frac{T - T_\infty}{T_0 - T_\infty};\, t^* = \frac{t}{\tau};\, x^* = \frac{x}{L};\; \theta^* = \frac{\theta}{\eta}, \tag{16}$$

where T_0 is the highest temperature reached and T_∞ the lowest one, and assuming a time constant - τ. - associated to the development rapidness of moisture content profile ($\tau = \frac{L^2}{D_\theta}$), the governing equations are written as:

$$Lu \frac{\partial T^*}{\partial t^*} = \alpha_{ap} \frac{\partial^2 T^*}{\partial x^{*2}} \tag{17}$$

and

$$\frac{\partial \theta^*}{\partial t^*} = \frac{\partial^2 \theta^*}{\partial x^{*2}} + P_n \frac{\partial^2 T^*}{\partial x^{*2}}\,, \tag{18}$$

where: $Lu = \frac{D_\theta}{\alpha}$ and $Pn = \frac{D_T(T_0 - T_\infty)}{D_\theta \eta}$.

The Luikov and Posnov numbers are important parameters to analyze the coupling intensity of simultaneous heat and moisture transfer problems in porous media. The Luikov number represents the evolution rapidness between moisture content and temperature spatial distributions. Thereupon, for low Luikov numbers, temperature profiles are developed much more rapidly than moisture content ones.

The Posnov number reports the importance of temperature gradients on the moisture transport through porous materials. Consequently, for high Posnov numbers, the moisture transport occurs predominantly due to temperature gradients. Therefore, This dimensionless number significance arises when $\frac{\partial \theta}{\partial x} \to 0$, which may happen in thick walls composed of low Luikov number materials.

Discretizing the nondimensional governing equations, the following mass conservation equation is obtained:

$$\left(1+2\mathrm{Fo_m}\right)\theta(\mathrm{i})+2\mathrm{Fo_m PnT(i)}=$$
$$\mathrm{Fo_m}\theta(\mathrm{i}+1)+\mathrm{Fo_m PnT(i+1)}+\mathrm{Fo_m}\theta_{\mathrm{i}-1}+\mathrm{Fo_m PnT(i-1)}+\theta^0(\mathrm{i}) \tag{19}$$

Similarly for the energy conservation equation, viz.:

$$\left(Lu+2Fo_m\right)T(i)=Fo_mT(i+1)+Fo_mT(i-1)+LuT^0(i). \tag{20}$$

Therefore, the following MTDMA matrix coefficients are obtained for the inner nodes of the physical domain:

$$A(i)=\begin{bmatrix}1+2Fo_m & 2Fo_mPn\dfrac{\eta}{\Delta T_{ref}}\\ 0 & Lu+2Fo_m\end{bmatrix}, B(i)=\begin{bmatrix}Fo_m & Fo_mPn\dfrac{\eta}{\Delta T_{ref}}\\ 0 & Fo_m\end{bmatrix},$$
$$C(i)=\begin{bmatrix}Fo_m & Fo_mPn\dfrac{\eta}{\Delta T_{ref}}\\ 0 & Fo_m\end{bmatrix}, D(i)=\begin{bmatrix}\theta^0(i)\\ LuT^0(i)\end{bmatrix}, \tag{21}$$

where: $Fo_T=\dfrac{\alpha\Delta t}{\Delta x^2}$; $Fo_M=\dfrac{D_\theta\Delta t}{\Delta x^2}$; $Bi_M=\dfrac{h_m\Delta x}{D_\theta}$ and $Lu=\dfrac{Fo_M}{Fo_T}=\dfrac{D_\theta}{\alpha}$.

The Biot numbers for heat and mass express the relative importance of thermal and hygric resistances, respectively, within a solid material. In this way, for a high Biot number for mass (Bi_m), the convective transport of moisture at the boundaries is much higher the diffusive transport through the porous medium. Therefore, a high Bi_m wall has a large relative hygric resistance. Analogously, the same analysis can be done for heat flux using the Bi_T number.

The Fourier numbers for heat and mass are nondimensionalized forms for the time variable. The ratio between Fo_m and Fo_T gives the Luikov number, i.e., the relative development rapidness between the moisture content and the temperature profiles.

6. Nondimensionalizing the Boundary Condition Equations

In the same way, the boundary conditions are leaded to:

- Mass conservation equation (x=0):

$$(\theta(0)-\theta^0(0)).\frac{\Delta x}{2\Delta t}=D_{Te}\left(\frac{T^0(1)-T^0(0)}{\delta x_e}\right)+D_{\theta e}\left(\frac{\theta(1)-\theta(0)}{\delta x_e}\right)+\frac{h_{m,ext}}{\rho_l}\left(\rho_{v,ext}-\rho(0)\right). \tag{22}$$

The vapor concentration difference, $\Delta\rho_v$, in Eq. 22, is normally determined by using the values of previous iterations for temperature and moisture content, generating additional instability. Due to the numerical instability created by this source term, the solution of the linear set of discretized equations normally requires the use of very small time steps, which can be exceedingly time consuming, especially in long-term whole-building hygrothermal simulations or soil simulations or even in drying process analysis.

In order to rise the simulation time step, Mendes et al. (2002) presented a mathematical procedure to calculate the vapor flow, independently of previous values of temperature and moisture content. In this way, the term ($\Delta\rho_v$) was linearized as a linear combination of temperature and moisture content, viz.,

$$\left(\rho_{v,\infty}-\rho_v(s)\right)=M_1(T_\infty-T(s))+M_2(\theta_\infty-\theta(s))+M_3 \tag{23}$$

where

$$M_1 = A\frac{M}{\Re}\phi \;;\; M_2 = \frac{M}{\Re}\left(\frac{P_s(s)}{T(s)}\right)^{prev}\left(\frac{\partial\phi}{\partial\theta(s)}\right)^{prev} ;M_3 = \frac{M}{\Re}\left[\begin{array}{l}\left(\frac{P_s(s)}{T(s)}\right)^{prev} R(\theta^{prev}(s)) + \\ \phi_\infty(R(T_\infty) - R(T^{prev}(s)))\end{array}\right].$$

Replacing Eq. (23) on Eq. (22), viz.:

$$D_T\left(\frac{T^0(1)-T^0(0)}{\delta x}\right) + D_\theta\left(\frac{\theta(1)-\theta(0)}{\delta x}\right) + \tag{24}$$
$$\frac{h_{m,ext}}{\rho_l}\left[M_{1,ext}Pn\left(T_{ext}-T(0)\right) + M_{2,ext}\left(\theta_{ext}-\theta(0)\right) + M_{3,ext}\right] = (\theta(0)-\theta^0(0))\frac{\Delta x}{2\Delta t}.$$

The sub-index "ext" stands for the M_1 coefficient calculated using the external conditions. However in Eq. (24), the coefficient $M_{1,ext}$ is followed by the Posnov number as it multiplies the dependent variable T. Thus, the coefficient $M_{1,ext}$ is redefined as:

$$M_{1,ext} = A_{ext}\frac{M}{R}\frac{h_{ext}}{P_n};$$

Rewriting Eq. (24) in terms of dimensionless parameters, viz.:

$$2Fo_m Pn\frac{\eta}{\Delta T_{ref}}\left(T(1)-T(0)\right) + 2Fo_m\left(\theta(1)-\theta(0)\right) + \tag{25}$$
$$\frac{2Bi_m Fo_m}{\rho_l}\left[M_{1,ext}Pn\left(T_{ext}-T(0)\right) + M_{2,ext}\left(\theta_{ext}-\theta(0)\right) + M_3\right] = (\theta(0)-\theta^0(0)),$$

which can be regrouped focusing on the MTDMA use as:

$$\left(1 + 2Fo_m + 2Bi_m Fo_m\frac{M_{2,ext}}{\rho_l}\right)\theta(0) + \left[2Bi_m Fo_m Pn\left(\frac{M_{1,ext}}{\rho_l} + \frac{\eta}{Bi_m \Delta T_{ref}}\right)\right]T(0) = \tag{26}$$
$$\left(2Fo_m\right)\theta(1) + \left(2Fo_m Pn\frac{\eta}{\Delta T_{ref}}\right)T(1) +$$
$$\frac{2Bi_m Fo_m}{\rho_l}\left[M_{1,ext}PnT_{ext} + M_{2,ext}\theta_{ext} + M_3\right] + \theta^0(0).$$

- Energy conservation Equation (x=0)

Replacing the vapor difference term (Eq. 23), the discretized energy conservation equation can be written as

$$\rho_0 c_m\frac{\Delta x}{2\Delta t}\left(T(0)-T^0(0)\right) = \frac{\lambda_{ap}}{\delta x_e}\left(T(1)-T(0)\right) + \tag{27}$$
$$+ h_{ext}(T_{ext}-T(0)) + \alpha q_r - \varepsilon R_{ol} +$$
$$Lh_{m,ext}(M_{1,ext}Pn\left(T_{ext}-T(0)\right) + M_{2,ext}\left(\theta_{ext}-\theta(0)\right) + M_{3,ext}).$$

Rewriting Eq. (27) in terms of dimensionless, we have:

$$\begin{gathered}ZM_{2,ext}\theta(0)+\left(M_{1,ext}PnZ+2Bi_TFo_T+2Fo_T+1\right)T(0)=\\2Fo_TT(1)+ZM_{2,ext}\theta_{ext}+\left(M_{1,ext}PnZ+2Bi_TFo_T\right)T_{ext}+\\ZM_{3,ext}+2Bi_TFo_T\frac{\alpha.q_r}{h_{c,ext}}+T^0(0),\end{gathered}\tag{28}$$

where: $Z = 2Bi_m Fo_m \dfrac{L}{\rho_0 . c_m .}$.

Now, the MTDMA matrix-coefficients can be defined as

$$A(0)=\begin{bmatrix}\left(1+2Fo_m+2Bi_mFo_m\dfrac{M_{2,ext}}{\rho_l}\right) & 2Bi_mFo_mPn\left(\dfrac{M_{1,ext}}{\rho_l}+\dfrac{\eta}{Bi_m\Delta T_{ref}}\right)\\ \dfrac{2L}{C}Bi_mFo_mM_{2,ext} & \left(\dfrac{2L}{C}Bi_mFo_mM_{1,ext}Pn+2Bi_TFo_T+2Fo_T+1\right)\end{bmatrix};$$

$$B(0)=\begin{bmatrix}2Fo_m & 2Fo_mPn\dfrac{\eta}{\Delta T_{ref}}\\ 0 & 2Fo_T\end{bmatrix};C(0)=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix};\tag{29}$$

$$D(0)=\begin{bmatrix}2\dfrac{Bi_mFo_m}{\rho_l}\left(M_{1,ext}PnT_{ext}+M_{2,ext}\theta_{ext}+M_{3,ext}\right)+\theta^0(0)\\ 2Bi_TFo_TT_{ext}+2Bi_TFo_T\dfrac{\alpha.q_r}{h_{ext}}+\dfrac{2L}{C}Bi_mFo_m\left(M_{1,ext}PnT_{ext}+M_{2,ext}\theta_{ext}+M_{3,ext}\right)+T^0(0)\end{bmatrix}.$$

The discretized equations for the internal surface are similar to those of the external side, except for the absence of the short-wave related term.

7. Results and Discussions

The above equations are solved with a finite-volume approach using a fully-implicit solution scheme with coupling between the conservation equations. Using the Patankar (1980) method with uniform nodal spacing and a tridiagonal-matrix solution algorithm, a C program solves the temperature and moisture content distributions iteratively (TDMA) at each time step or simultaneously (MTDMA). The thermophysical properties were obtained from Perrin (1985).

Comparative studies between the traditional and the new methods are presented. First, it is analyzed the influence of dimensionless numbers on the stability of the traditional method and then comparisons are carried out in terms of heat flux.

7.1 - Dimensionless numbers influence on numerical method stability

Besides the Biot number for moisture diffusion (Bi_m), another important dimensionless is the mass Fourier number, $Fo_m = D_\theta \Delta t / \Delta x^2$, which expresses a mesh-size parameter that could be related to a convergence error function δ, defined as the relative difference between the wall surface moisture content values:

$$\delta = abs\left(\frac{\theta_s - \theta_s^{prev}}{\theta_s^{prev}}\right).$$

Fig. 1 illustrates the influence of the Biot and Fourier numbers on the convergence error function. It is noted, for the traditional TDMA algorithm, as the Bi_m increases the error function goes up very quickly showing a high sensitivity to this dimensionless number, which can be even higher than that caused by the mass Fourier number.

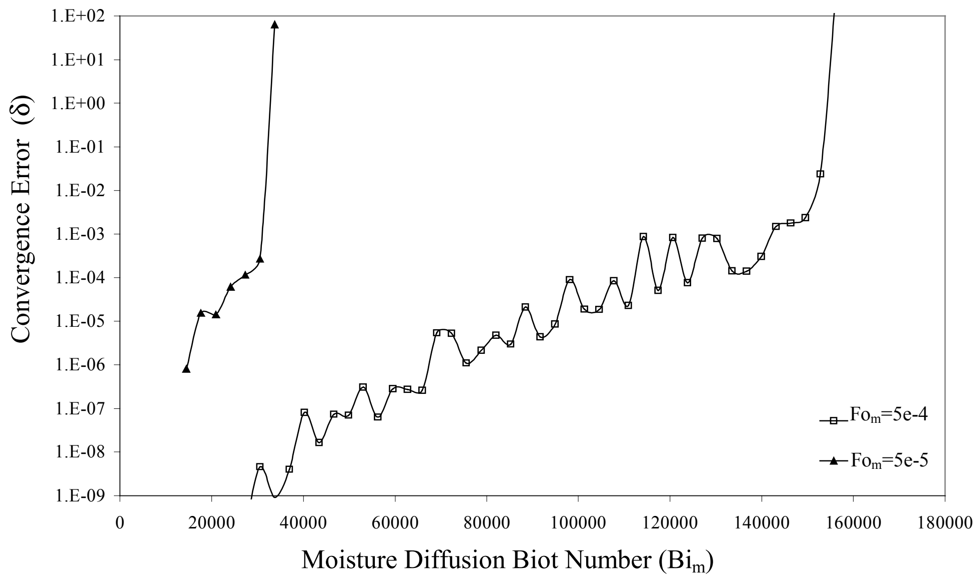

Figure 1: Convergence error as a function of Biot number for moisture Diffusion.

The mass conservation equation at the boundaries could be also written as:

$$\left(\frac{1}{2Fo_m}+1\right)\theta(s)=\theta(s+1)+\frac{\theta^{Ant}(s)}{2Fo_m}+Bi_m\frac{(\rho_{v,\infty}-\rho_v(s))}{\rho_l}-\frac{D_T}{D_\theta}\left(T^{Prev}(s)-T^{Prev}(1)\right) \quad (30)$$

The system of equations is stable when the Eq.-(30) left-hand term is much higher than the source terms (from the 2nd to the 4th right-hand terms). The last right-hand term is normally lower than the 2nd and 3rd ones. As the second right-hand term is comparable to the first parcel at the right hand side of the equation above, it is believed that a good stability criterion, dependent on the boundary condition, but independent on the Fourier number, could be proposed as:

$$Bi_m\frac{(\rho_{v,\infty}-\rho_v(s))}{\rho_l}<<1\,, \quad (31)$$

which can be satisfied for materials with high moisture diffusivity or for very low permeance surfaces, under any boundary condition value. However, as it can be seen in Fig. 1, the higher the Fo_m number the lower the Bi_m number to avoid numerical divergence.

7.2 - Constant Coefficient Analysis - Dimensionless numbers effects on Heat Flux

The Biot number, for either mass or energy conservation boundary condition equation, expresses the relative contribution of thermal hygric resistances within the solid porous media matrix. For instance, when the mass Biot number (Bi_m) is high, the convective moisture transport within the air adjacent to the solid surface is much greater than the diffusive transport of moisture through the porous medium. Therefore, a high-Bi_m wall has an elevated hydraulic resistance. Analogously, heat flux analyses can be made in terms of Bi_T.

The Fourier numbers for heat and mass are dimensionless forms for the independent variable time. It is noticed that the ratio between Fo_m and Fo_T gives the Luikov number, i.e., the ratio between the development rapidness of moisture content and temperature profiles. For building material, except wood similar materials, the Luikov number is very low (Lu < 0.01), which makes possible to decouple the two fields (moisture content and temperature) so that the temperature profile is rapidly established independently on θ Variations. However, for the boundary control volumes, the coupling between T and θ is very important due to water vapor exchanged between the air and the surfaces.

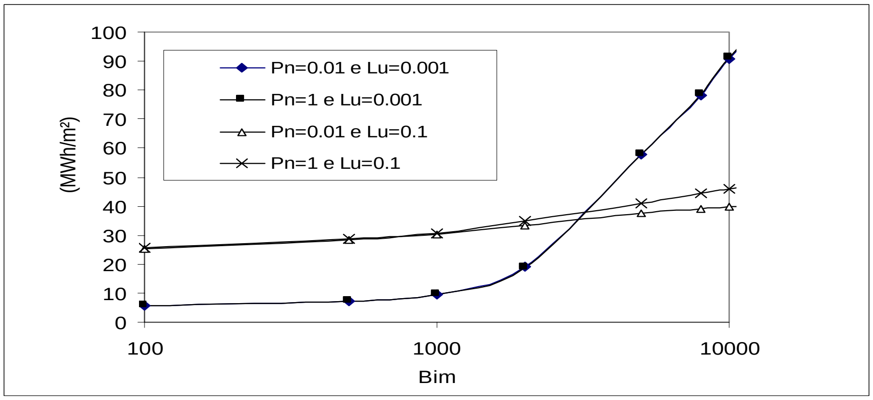

Figure 2: Bi_m , Lu and Pn Effects on YHF for $Bi_T = 0.01$ and $Fo_T = 1$.

Figures 2 and 3 show the yearly-integrated conduction heat flux (YHF) variation with Lu, Pn, Bi_m and Bi_T by using the MTDMA. The YHF quantity is an important parameter used in building material simulation codes.

It is noticed, from Figures 2 and 3, the lower Luikov number the higher the inward conduction heat flux. This can be explained by the fact that high Luikov number walls are well insulated, *i.e.*, they normally have a low thermal diffusivity. On the other hand, the presence of moisture, under a certain range of mass Biot number, can cool down walls due to evaporation loads, which can reduce thermal loads when applied to buildings integrated to HVAC systems. Fig. 2 presents a low influence of Posnov number. This number becomes more important as the Bi_m and Lu numbers increase. For very low Lu numbers, the temperature field gets much more dependent on boundary conditions than the previous temperature value, i.e., the energy conservation equation transient term becomes smaller.

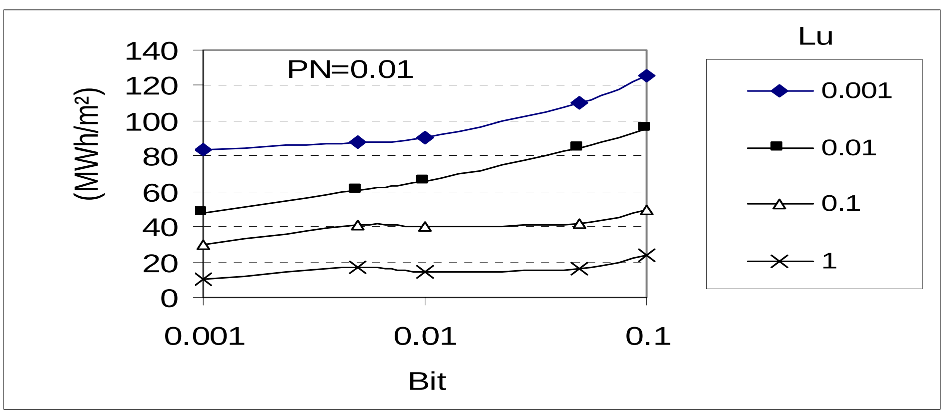

Figure 3: Bi_T and Lu Effects on YHF for Bi_m= 10000, $Fo_T = 1$ and Pn = 0.01.

Fig. 3 shows a lower sensitivity to Bi_T for high Lu numbers, which may be explained by the implicit thermal diffusivity in the Luikov number.

For the boundary condition nodes, we have noticed the predominancy of a new dimensionless group given by the product of Fourier and Biot numbers – Eqs. (26 – 28) – and that depends no longer on transport diffusive coefficients, but on air-side convection coefficients, which implies that stability may be related much more with psychrometrics variations of the free air flow adjacent to the wall than variations present within the wall. However, an exception should be made to low Biot number walls.

The parametric analysis presented in the section 7.2 should be carefully examined as it is useful on the comprehension of physics behind the strongly-coupled heat and moisture transfer phenomenon and helpful on the results interpretation. However, most of the analysis was carried out by using a constant coefficient model and, by the fact, when a parameter is varied and the remaining are fixed, a parameter interrelation is hidden and it may bring some mistakes. For example, a Bi_m variation implicitly modifies also the Lu number as it contains D_θ as a common term.

8. Conclusions

It is noted, for traditional algorithms such as TDMA, the risk of divergence goes up very quickly as the moisture Biot number increases, showing a high sensitivity to this dimensionless parameter, which can be even higher than the one caused by the mass Fourier number.

The MTDMA (MultiTriDiagonal-Matrix Algorithm) was utilized to solve the heat and mass transfer governing equations in porous media. This method avoids numerical instabilities (Mendes and Philippi, 2003) by solving simultaneously the governing equations, allowing the use of high time steps which are very important for long-term simulation of combined heat and mass transfer such as in building materials and drying processes, with transport coefficients highly moisture-content-dependent.

We have noticed of a new dimensionless group given by the product of Fourier and Biot numbers, which represents the difficulty of introducing energy and moisture into the porous structure when the simulation time step is high, *i.e.*, time step has to be reduced when Biot number is high in order to reach numerical convergence. On the other hand, this effect that causes numerical divergence can be greatly reduced when a robust algorithm such as MTDMA is used and the boundary condition terms associated to de vapor concentration difference are linearized in terms of temperature and moisture content differences.

The Biot number for moisture diffusion has also shown that it is possible to speed up simulations for high Bi_m Number walls so that further research work is recommended to be carried out on that direction in order to improve computer run time of simulation codes such as the Domus program.

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