

ERRORS IN TRANSMISSION LOSS PREDICTION - THE BISPECTRUM AND KURTOSIS APPROACHES

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Abstract. *The measurements of the acoustic performance of automotive mufflers are influenced by the wave propagation with reflection and absorption. This way, the measured signal can have linear and nonlinear interactions of the wave components. The bispectrum, which is the measure of the phase relationship between three spectral components, has been shown to be a useful tool in the study of linear and nonlinear wave interactions. The bicoherence spectrum may be used to discriminate between nonlinearly coupled waves and spontaneously excited waves. At the same time, the kurtosis parameter is used as an indicative of the signals of microphones. The performances of two different physical models (bipartite chamber and a chamber with concentric perforated tube) are analyzed considering the transmission loss (TL). The experimental data are obtained through a two microphone method and the numerical values through the Finite Element Method.*

Keywords. *Muffler, transmission loss, bispectrum, kurtosis, automotive.*

1. Introduction

Structural defects in rotating machinery components can be detected by monitoring vibration and/or sound emissions. The bispectrum, a third-order statistic and kurtosis, a fourth order moment, helps to identify faults in mechanical components (Parker Jr. et all, 2000; Fackrell et all, 1995a-1995b and McCormick, 1998). The bispectrum technique relates one set of mixing waves through the spectral coupling. The kurtosis gives an indication of the proportion of samples that deviate from the mean by a small value compared to those, which deviate by a large value.

Seybert and Ross (1977) introduced a experimental technique to determine the acoustic properties in tubes considering the effect of the mean flow. The technique is based on measurement of sound pressure by two microphones placed before the muffler (chamber). The theory and treatment of the signals are developed considering the pressures incident and reflected by the silencer that characterize the coupling (mixing) of waves.

Abom and Bodén (1986,1988) analyze the errors derived from the use of the technique of two microphones and present an appropriate frequency range for the measurements.

Munjal (2001) analyzes the errors committed by the technique of two microphones considering the hypothetical case of anechoic source (for an arbitrary load). The method of one microphone can be used for any load.

Dalmont (2001a, 2001b) evaluates several measurement methods of sound pressure and the inherent errors due to the use of these methods. Linearity and stability are the basic hypotheses for a good measurement. It is possible to detect nonlinearities through measurement in several load levels. The stability can be analyzed by repeating the calibration in different times and comparing the results. The main calibration errors are due to: fluctuations in the acoustic excitation signal, temperature changes, incorrect measurement of the speed of sound, irregularities in the geometry, vibrating modes, etc.

The theory of high order modes in circular tubes is reviewed and applied to expansion chambers (Erikssom, 1980) and the cutoff frequencies for these modes are presented.

In this work, the kurtosis and bispectrum, are used as parameters of evaluation of the quality of the signals obtained by four microphones for experimental calculation of transmission loss for two different muffler models.

2. Definitions

A quadratic nonlinearity will relate three wave components in such a way that:

$$X_m = \sum_{m=k+l} A_{k,l} X_k X_l + \varepsilon \tag{1}$$

where X_k and X_l denote the complex Fourier spectral components at ω_k and ω_l, with phase θ_k and θ_l respectively. $A_{k,l}$ denotes the coupling coefficient and is dependent on the properties of the nonlinearity system. The term ε

denotes any errors associated with this model. In this system X_k and X_l will interact to create a third component X_m, where $\omega_m = \omega_k \pm \omega_l$ and $\theta_m = \theta_k \pm \theta_l$.

The bispectrum is defined as:

$$b(k,l) = \left\langle X_k X_l X_m^* \right\rangle \tag{2}$$

where X_m^* denotes the complex conjugate of X_m. It can be clearly seen how this takes into account the mixing between two frequencies. If ω_k, ω_l and ω_{k+l} are independent, each will have an independent random phase (relative to each other).

The probability distribution of a random variable X is defined as:

$$F(x) = P(X < x) \tag{3}$$

The probability density function (p.d.f.) is the derivative of this:

$$f(x) = \frac{dF(x)}{dx} \tag{4}$$

The expectation operation, which gives the expected value of a function $g(x)$, is defined as:

$$E\{g(x)\} = \int_{-\infty}^{\infty} g(x)f(x)dx \tag{5}$$

In most cases, the probability density function can be decomposed into its constituent moments or cumulants. If a change in condition causes a change in the p.d.f of the signal then the moments and cumulants may also change.

The moments of the signal are defined as:

$$m_n = E\{x^n\} \tag{6}$$

where $E\{.\}$ can be estimated.

The most common simple statistical feature used in signal monitoring is the mean square value (second-order moment) of the signal:

$$m_2 = \frac{1}{N}\sum_{i=1}^{N} x(n)^2 \tag{7}$$

A second common statistical feature used is the kurtosis which gives an indication of the proportion of samples which deviate from the mean by a small value compared to those which deviate by a large number. The fourth order moment can be normalized by the second order moment squared:

$$\gamma_4 = \frac{m_4}{m_2^2} \tag{8}$$

The zero mean Gaussian distributed variable has a kurtosis of 3.

The experimental investigation of the transmission loss of automotive mufflers involves the sound generation (signal source) and the measurement of the sound pressure along the inlet and outlet ducts. The acoustic waves in the inlet and outlet ducts contain incoming waves as well as reflected waves.

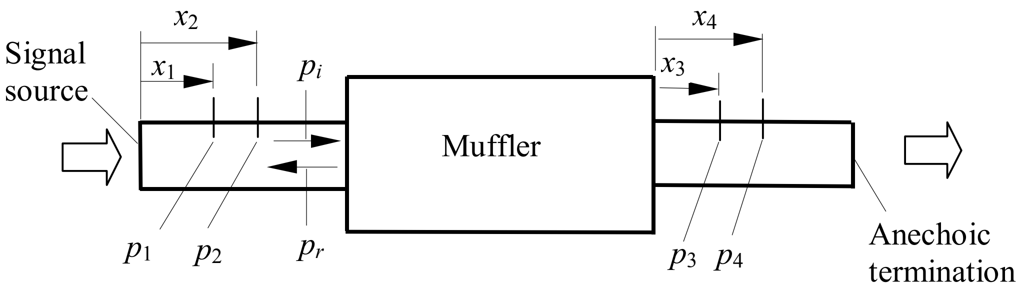

Figure 1 – Muffler system.

The pressures p_1 and p_2 at two points can be written as:

$$p_1 = p_i e^{-kx_1} + p_r e^{ikx_1} \tag{9}$$

$$p_2 = p_i e^{-kx_2} + p_r e^{ikx_2} \tag{10}$$

where p_i represents the incident wave and p_r represents the reflected wave.

Solving equations (9) and (10) for p_i gives:

$$p_i = \frac{1}{2i\sin[k(x_2 - x_1)]}(p_1 e^{ikx_2} - p_2 e^{ikx_1}) \tag{11}$$

The same procedure can be used to determine the transmitted wave p_t in the outlet duct. The *TL* of the muffler can be evaluated by:

$$TL = 20\log_{10}\left(\left|\frac{p_i}{p_t}\right|\right) + 10\log_{10}\left(\frac{S_1}{S_2}\right) \tag{12}$$

where S_1 and S_2 are the cross-sectional area of the inlet and outlet ducts.

It is very common to have a mathematical evaluation of the transmission loss of automotive vehicular mufflers with the ***Four-Pole Parameters Method*** (Igarashi and Toyama, 1958; Igarashi and Arai, 1960). In this method, the sound pressure and the velocity in the inlet of the muffler, Fig.2, can be related to the sound pressure and the velocity in the outlet with the use of four parameters in the following form (Wu, Zhang and Cheng, 1998):

$$\begin{bmatrix} p_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} p_2 \\ -u_2 \end{bmatrix} \tag{13}$$

where the pairs (p_1, u_1) and (p_2, u_2) represent sound pressure and velocity in the inlet and outlet, respectively. **A**,**B**,**C** and **D** are the four parameters that name the method and its numerical determination is made as follows.

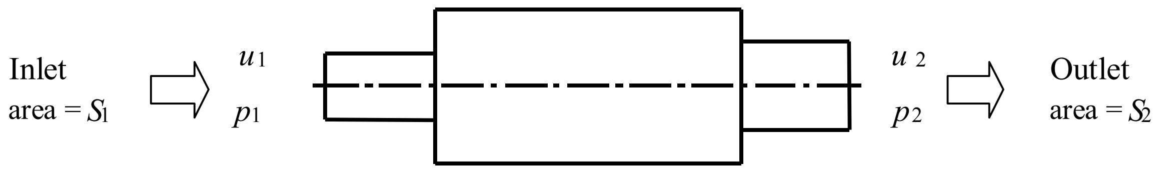

Figure 2- References for expansion chamber.

Parameter **A** is calculated by solving Helmholtz's equation with the boundary condition given by $u_2 = 0$ and $u_1 = 1$ (Fig. 3a). Using the hypothesis that the excitation frequency is below the first cutoff frequency for the inlet duct, only plane sound pressure waves propagate in the input side. In this situation all nodes of the finite element mesh in the input area have the same sound pressure value. The same behavior is repeated for outlet area if the excitation frequency is also below the first cutoff frequency of the outlet duct.

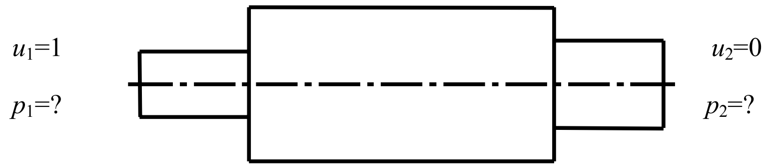

Figure 3a-Boundary Conditions to evaluate **A** and **C**.

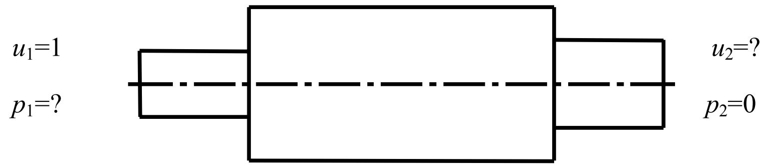

Figure 3b-Boundary Conditions to evaluate **B** and **D**.

After solving Helmholtz's equation with these boundary conditions the numerical value for parameter **A** is easily calculated with the values of sound pressure in the inlet and outlet by using the following equation:

$$\mathbf{A} = \left.\frac{p_1}{p_2}\right|_{u_2=0,u_1=1} \tag{14}$$

Also, with this same solution it is possible to find the numerical value for the parameter **C** that is defined as:

$$\mathbf{C} = \left.\frac{u_1}{p_2}\right|_{u_2=0,\ u_1=1} \tag{15}$$

The other two parameters, **B** and **D**, are calculated in a similar way, by solving *again* Helmholtz's equation with boundary conditions given by $p_2 = 0$ and $u_1 = 1$ (Fig. 3b). The numerical evaluation of these parameters is performed by using the values of the input and output:

$$\mathbf{B} = \left.\frac{p_1}{-u_2}\right|_{p_2=0,\ u_1=1} \tag{16}$$

$$\mathbf{D} = \left.\frac{u_1}{u_2}\right|_{p_2=0,\ u_1=1} \tag{17}$$

The transmission loss of automotive mufflers can be obtained (Igarashi and Toyama, 1958 and Igarashi and Arai, 1960) as:

$$TL = 20\log\left\{\frac{1}{2}\left|\mathbf{A} + \mathbf{B}\frac{1}{\rho_o c} + \mathbf{C}\rho_o c + \mathbf{D}\right|\right\} + 10\log\left(\frac{S_1}{S_2}\right) \tag{18}$$

The physical mean properties are the density ρ_o and the speed of sound c. The Improved Four-Pole Parameters Method was used in this work. This method was first proposed by Kim and Soedel (1989a, 1989b,1990) and it was used by Wu, Zhang and Cheng (1998) for the calculation of the transmission loss of acoustic mufflers with the Boundary Element Method (BEM).

3. Experimental results

The Figure 4 shows one physical model of chamber with concentric perforated tube.

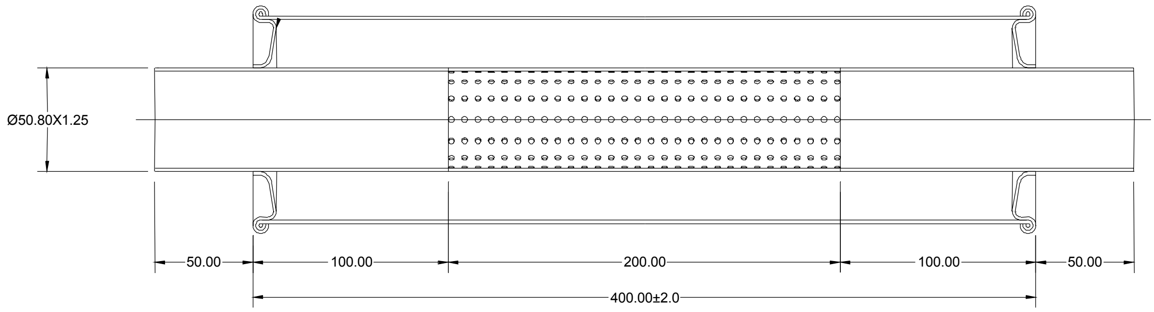

Figure 4 - Chamber with concentric perforated tube. (dimensions in mm)

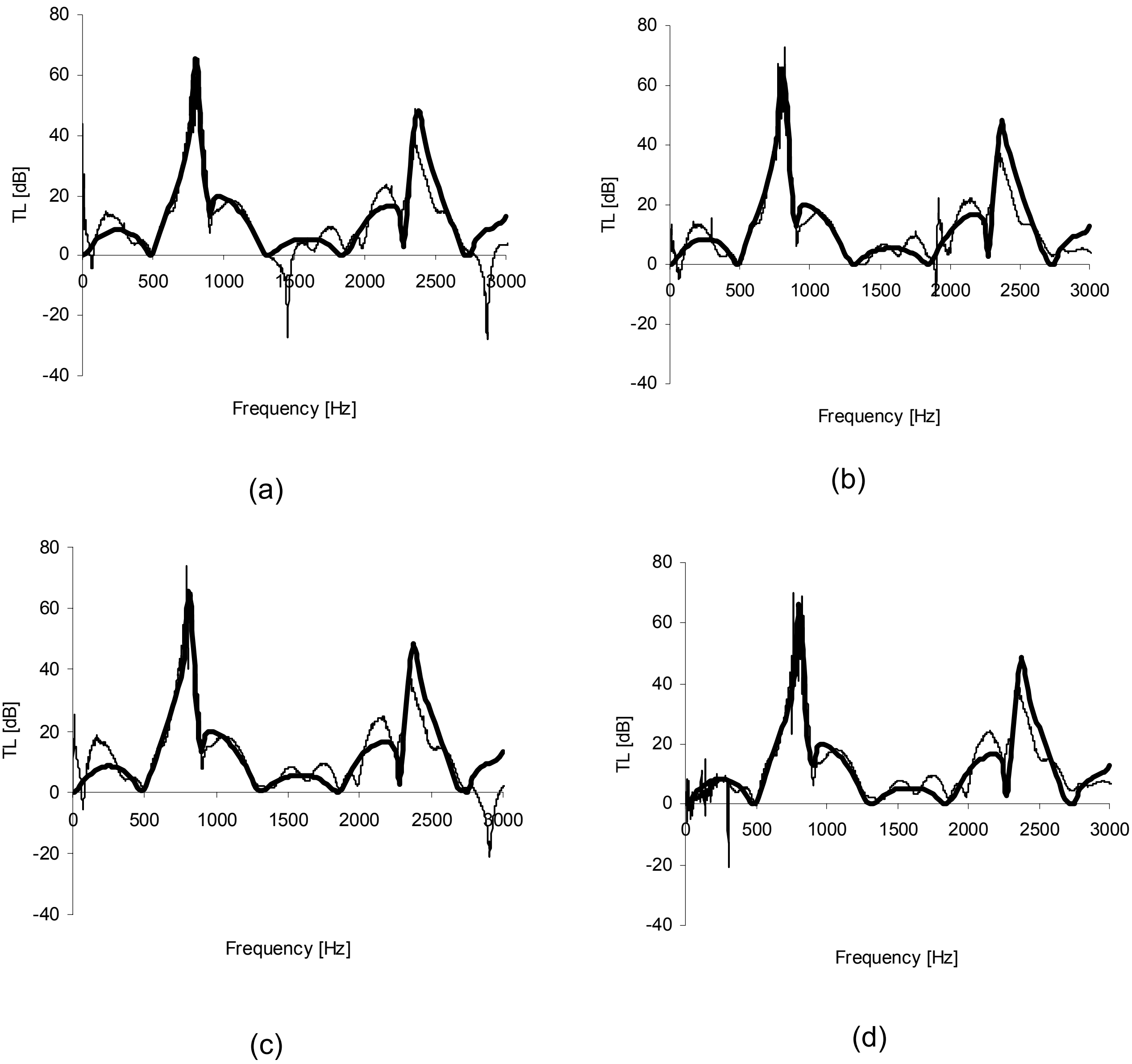

Figure 5 – Transmission loss for a chamber with concentric perforated tube. Distance between microphones: (a) d = 120 mm, (b) d = 90 mm, (c) d = 60 mm and (d) d= 30mm. (▬ FEM; — experimental)

The experimental data are obtained by using four microphones (microphones 1 and 2 in the inlet tube and microphones 3 and 4 in the outlet tube).

The experimental and numerical values of the transmission loss are shown in Figure 5. The experimental data were obtained for four different distances between microphones: d = 30, 60, 90 and 120 mm. The random excitation signal was used. It can be noticed that these chamber model presents some oscillations between numeric and experimental results. The differences are greater for the lower and high frequencies range. This fact was already expected and is directly related to limitations due to distance between microphones (Abom and Bodén, 1986-1988) and cutoff frequencies (Erikssom, 1980). The results present higher differences to distance between microphones of 120 and 60 mm.

To investigate the possible error source in the measurements, the Figure 6 show the kurtosis values for the signals of the four microphones for all distances between microphones. Figure 6 shows that the kurtosis value of microphone 4 (near the anechoic termination) presents more elevated values. This fact characterizes a larger dropping of the spectral components.

It can be noticed in Figure 7 that the bispectrum of the signal of microphone 4 has a distribution of concentrated peaks on lower frequency range. Another two frequency ranges (near to 1500 and 3000 Hz) also present higher peaks concentration. In these three frequency ranges the experimental values of the transmission loss are bad, Figure 5a and 5c. For this case the kurtosis values are higher (near to 3) (Fig. 6).

Other results for the physical model described in Fig. 4 and other different physical model can be found in Barbieri et all (2003).

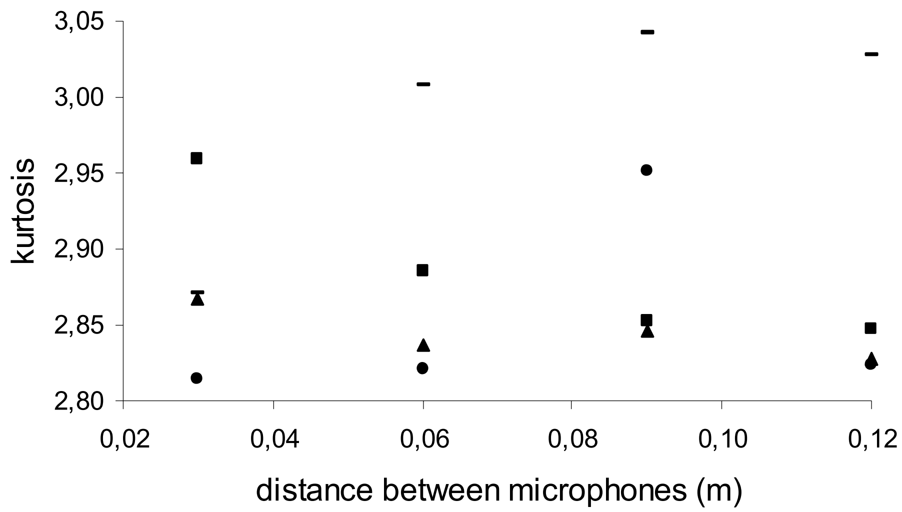

Figure 6 - Values of the kurtosis for different distances between microphones. Chamber with concentric perforated tube. (• microphone 1; ▲ microphone 2; ■ microphone 3; ▬ microphone 4)

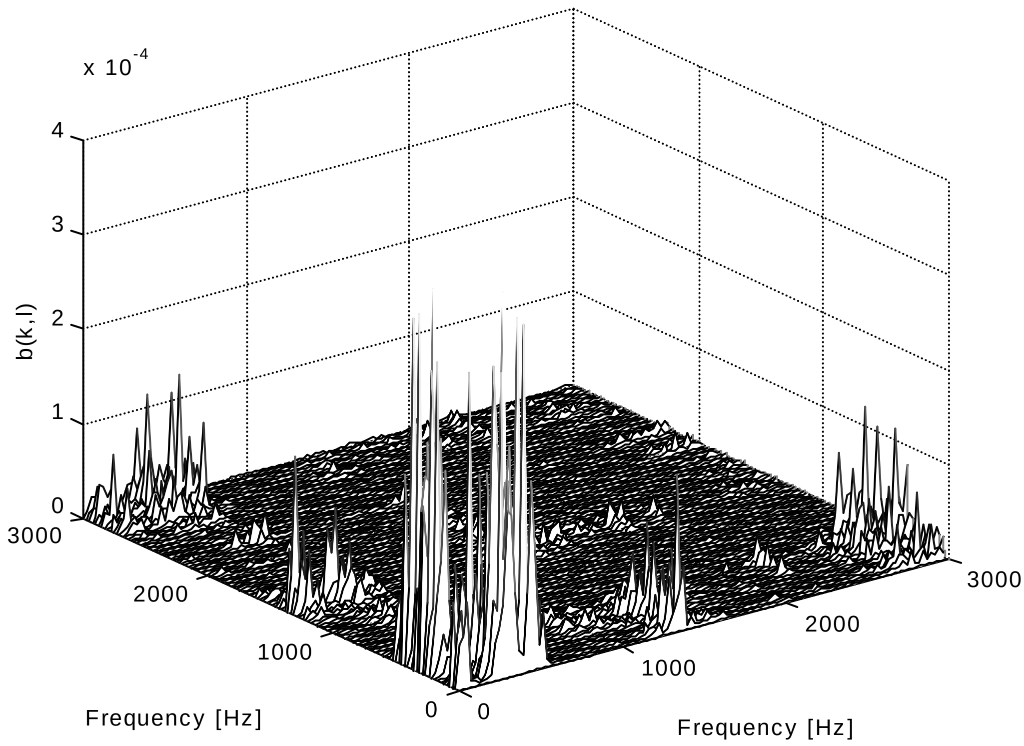

Figure 7 - Bispectrum of signal of microphone 4 (d = 120 mm)

4. Conclusions

In this work the signals of sound pressure are analyzed and used for the evaluation of the transmission loss in one acoustic muffler. The bispectrum and kurtosis techniques were applied to the signals. The results reveal a possible indicative of experimental errors. The main source of coupling wave is associated to the signals near the anechoic termination.

Comparing TL's values (Fig. 5) with the values of bispectrum (Fig. 7), it can be noticed that there are deviations between mathematical and experimental results, in the regions where the bispectrum presents great coupling waves. The errors are more significant for lower and high frequency ranges.

Comparing the results obtained by bispectrum and by kurtosis a great correlation between these analyses can be noticed. The highest values in the kurtosis (above or near to 3) are related to a larger distribution of peaks in the bispectrum (higher coupling wave ranges) where there are more errors between mathematical and experimental results. The lower values in the kurtosis (minor than 3) reveals concentrated regions of coupling waves in the bispectrum where the errors are more concentrated.

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