STRUCTURAL DAMPING ESTIMATION OF MECHANICAL SYSTEMS

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Abstract. *In this work the authors try to establish a procedure to damping identification of transmission line cables with interest to estimate in a simple way (proportional) the system damping matrix. The procedure is based on experimental and simulated data. The experimental data are collected through five accelerometers and the simulated data are obtained using the Finite Element Method. To validate the mathematical model it is used the reduction of the system considering the measured degrees-of-freedom. After this, the system damping matrix is estimated through different procedures.*

Keywords*. Damping, cable, transmission line, modal analysis, identification.*

1. Introduction

Several researchers have investigated the dynamical behavior of mechanical systems in the attempt to identify the physical parameters for mathematical model updating. The characterization of damping is important in making accurate predictions of both the true response and the frequency response of any device or structure dominated by energy dissipation. However, this parameter is difficult to be estimated.

Pilkey (1998) developed two procedures for computation of a damping matrix for finite element model updating. Both methods, direct and iterative, are based on partitioning of the inertia and stiffness matrices of the system, and the normalization of the eigenvectors. These methods allow a fast convergence but they need the experimental data to be quite accurate.

Adhikari (2000) presented a systematic study on analysis and identification of multiple parameter damped mechanical systems. The attention is focused on viscously and non-viscously damped multiple degree-of-freedom linear vibrating systems.

Iglesias (2000) has investigated several modal analysis extraction techniques to estimate damping ratio. The investigation focuses on the estimate of damping ratio because among the modal parameters, it is the most difficult to model.

Lamarque et. all (2000) introduced a wavelet-based formula similar to the logarithmic decrement formula to estimate damping in multi-degree-of-freedom systems from time-domain responses. Both analytical and numerical approaches are investigated.

In spite of the many techniques which have been developed, the experimental applications are restricted by well-known physical models. The study of dynamic behavior of transmission line cables needs to be refined. The inherent difficulty of this kind of system is in the joining of the vertical and lateral motions. Moreover, the mechanical properties vary with the cable type and they usually are made of aluminum and steel wires with different gauges.

Yamaguchi et. all (2001) analyze the dynamic characteristics with free and forced response of a cable system using sub structural interactions. The system consists of two identical structural sagged cables in parallel connected by another sagged cross cable. An energy-based method of damping evaluation is applied to analyze the characteristics of system modal damping. Yamaguchi and Adhikari (1995) obtain analytically the modal damping characteristics of single structural cables. An energy based representation of modal damping in structural cable is derived in the form of the product of modal strain energy ratio and loss factor. Numerical results are obtained with Finite Element Method considering a model with axial and bending deformation.

The modal analysis has been investigated by Nariboli and McConnell (1998) considering the curvature effect of the cables and non-linear equations; Gopalan et all (1987,1993) investigate the dynamic behavior of cables in indoor testing systems; Russell and Lardner (1998) investigated the dynamic behavior of elastic cables through theoretical models and experimental data in order to establish a theoretical equation for the mechanical tension in the system through the values of the natural frequencies.

In this work the authors try to establish a procedure to identify damping of transmission line cables in order to estimate in a simple (proportional) way the system damping matrix. The procedure is based on experimental and simulated data. The experimental data are collected through five accelerometers and the simulated data are obtained using the Finite Element Method. To validate the mathematical model the reduction of the system is used considering the measured degrees-of-freedom. After this, the system damping matrix is estimated through different procedures . To validate the procedure, three sample lengths and two different mechanical loads are applied. The experimental and simulated results present good approximates.

2. Mathematical model

The equations of motion of an damped system can be obtained through Finite Element Method and gives:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t) \tag{1}$$

where **M**, **C** and **K** are the mass, damper and stiffness matrices; $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are the acceleration, speed and displacement vectors and $\mathbf{f}(t)$ is the excitation vector. Usually, the values of the elements of **M** and **K** are well-known. However, the elements of **C** are difficult to obtain. To estimate the values of the elements of **C,** several procedures based on knowledge of the finite element or analytical mass and stiffness matrices and measured eigendata have been developed. To find the modal parameters several methods are used (Maia, 1998 e Iglesias, 2000): the Rational Fraction Polynomial (RFP) Method, the Prony or Complex Exponential Method (CEM), the Ibrahim Time Domain (ITD) and the Hilbert Envelope Method. All the methods are based on Frequency Response Function (FRF) between an excitation signal and the measured parameter:

$$\begin{bmatrix} X_1(\omega) \\ . \\ . \\ X_n(\omega) \end{bmatrix} = \begin{bmatrix} h_{11}(\omega) & . & . & h_{1n}(\omega) \\ . & . & . & . \\ . & . & . & . \\ h_{n1}(\omega) & . & . & h_{nn}(\omega) \end{bmatrix} \begin{bmatrix} F_1(\omega) \\ . \\ . \\ F_n(\omega) \end{bmatrix} \tag{2}$$

where: ω is the frequency; X_i is the variable value in point *i;* h_{ij} is the value of the FRF matrix element; F_i is the value of the force applied at point *i.*

The solution of the equation of motion (1) is given by:

$$\mathbf{x}(t) = e^{-\lambda_i t}(A_i \cos(\omega_{di}\, t) + B_i \sin(\omega_{di}\, t)) \tag{3}$$

where: $\omega_{di} = \xi_i \omega_{ni}$ being ξ_i the damping ratio and ω_{ni} the natural frequency, A_i and B_i are constants that depend of the initial conditions and ω_{di} is the damped frequency. These are the parameters obtained through the modal analysis.

An iterative procedure described in Friswell et all (1995) can be used to adjust a reduced system of equations. The procedure is based on the partitioned equation of motion:

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{sm} & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_m \\ \ddot{\mathbf{x}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_m \\ \mathbf{x}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_m \\ 0 \end{Bmatrix} \tag{4}$$

where: $\mathbf{M}$ is the inertia matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{x}$ is the displacement vector, $\mathbf{f}$ is the excitation vector, *m* are the measured nodal points and *s* are the non-measured nodal points . The reduced system is:

$$\mathbf{M}_r \ddot{\mathbf{x}}_m + \mathbf{K}_r \mathbf{x}_m = \mathbf{f}_m \tag{5}$$

where : $\mathbf{M}_r$ and $\mathbf{K}_r$ are the reduced matrices. In this study the first five mode shapes obtained through five accelerometers were considered (**r** = 5).

With the five estimated values of the damping ratio (ξ_i) and the natural frequencies (ω_i) of the selected modes shape, it is possible to obtain a function of the form (Adhikari, 2000):

$$\xi(\omega) = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 \tag{6}$$

The modal damping ratios in terms of the discrete natural frequencies can be obtained by

$$2\xi_i\omega_i = 2\omega_i(a_0 + a_1\omega_i + a_2\omega_i^2 + a_3\omega_i^3 + a_4\omega_i^4) \tag{7}$$

The equation (7) is a function of ω_i and simply replace ω_i^2 by $\mathbf{M}_r^{-1}\mathbf{K}_r$ and any constant terms by constant times **I** (identity matrix) to obtain the damping matrix:

$$\mathbf{C} = 2\mathbf{M}_r\sqrt{\mathbf{M}_r^{-1}\mathbf{K}_r}\left[a_0\mathbf{I} + a_1\left(\mathbf{M}_r^{-1}\mathbf{K}_r\right)^{1/2} + a_2\left(\mathbf{M}_r^{-1}\mathbf{K}_r\right)^{2/2} + a_3\left(\mathbf{M}_r^{-1}\mathbf{K}_r\right)^{3/2} + a_4\left(\mathbf{M}_r^{-1}\mathbf{K}_r\right)^{4/2}\right] \tag{8}$$

Another procedure to find the damping matrix C is described in Adhikari (2000). Suppose λ_j and $\mathbf{z}_j$ are the measured complex natural frequency and complex mode shape:

$$\lambda_j \approx \pm\omega_j + iC'_{jj}/2 \tag{9}$$

$$\hat{\mathbf{z}}_j = \mathbf{u}_j + i\mathbf{v}_j \tag{10}$$

The imaginary part of $\hat{\mathbf{z}}_j$ can be expanded as linear combination of $\hat{\mathbf{u}}_j$:

$$\hat{\mathbf{v}}_j = \sum_{k=1}^{m} B_{kj}\hat{\mathbf{u}}_k \tag{11}$$

where

$$B_{kj} = \frac{\hat{\omega}_j C'_{kj}}{\hat{\omega}_j^2 - \hat{\omega}_k^2} \tag{12}$$

The estimated element of the modal damping matrix is:

$$C'_{kj} = \frac{\left(\hat{\omega}_j^2 - \hat{\omega}_k^2\right)B_{kj}}{\hat{\omega}_j}, k \neq j \tag{13}$$

and

$$C'_{jj} = 2\Im(\hat{\lambda}_j) \tag{14}$$

where: $\hat{\lambda}_j$ is the complex natural frequency, $\hat{\omega}_j$ is the undamped natural frequency (real part of $\hat{\lambda}_j$), $\Im(\hat{\lambda}_j)$ is the imaginary part of $\hat{\lambda}_j$, $\mathbf{B}$ is an auxiliary matrix. The damping matrix is:

$$\mathbf{C} = \left[(\hat{\mathbf{U}}^T\mathbf{U})^{-1}\hat{\mathbf{U}}^T\right]^T \mathbf{C}'\left[(\hat{\mathbf{U}}^T\mathbf{U})^{-1}\hat{\mathbf{U}}^T\right] \tag{15}$$

where $\mathbf{U}$ is the real part of the complex mode shape matrix.

Two procedures to matrix damping identification are described by Pilkey (1998), iterative and direct damping identification. The iterative method begins with the initial choice of damping matrix $\mathbf{C}_0$ and the eigenvectors normalization as:

$$\boldsymbol{\varphi}_i^T(2\mathbf{M}_r\lambda_i + \mathbf{C}_{m-1})\boldsymbol{\varphi}_i = 1 \tag{16}$$

where m is the variable increment, $\boldsymbol{\varphi}_i$ is the eigenvector and λ_i is the eigenvalue. The damping matrix is:

$$\mathbf{C}_m = -\mathbf{M}_r(\boldsymbol{\Phi}\boldsymbol{\Lambda}^2\boldsymbol{\Phi}^T + \overline{\boldsymbol{\Phi}\boldsymbol{\Lambda}^2\boldsymbol{\Phi}^*})\mathbf{M}_r \tag{17}$$

where the overbar represents the complex conjugate and $*$ represents the complex conjugate transpose; $\mathbf{C}_m$ is the damping matrix after m iterations, $\boldsymbol{\Phi}$ is a matrix of eigenvectors and $\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues.

Finally, a structural damping matrix was adjusted for the following form: $\mathbf{C} = \alpha\mathbf{M}_r + \beta\mathbf{K}_r$, where α and β are parameters to be adjusted. In this case the damping matrix reference is the damping matrix estimated through one of the previous methods.

3. Results

The cable used was the Ibis type in which the parameters are: specific mass 0,8127 [kg/m]; rigidity flexural (EI) 11,07 Nm^2 .

For the modal identification, five accelerometers were placed in the cable, in the positions L/2, 3L/8, L/4, L/8 and L/16. The excitation of the system was applied through an impact hammer. In the numeric solution 10 finite elements were used and the eigenvalues and eigenvectors were obtained through the state matrix and by using the software MATLAB. The sample lengths for the mechanical load of 10700 N are: 13,385, 32,3 and 65,355m for the load of 15860 N are: 13.395m, 32,322m and 65,378m.

Fig. 1 shows the experimental and estimated FRF curves of the first five mode shapes. In this case the mechanical load is 10700 N and the sample length is 32,300m. The RFP method was used to estimate the damping ratio and mode shapes. The results obtained with the other methods were similar. The estimated curve presents very accurate behavior.

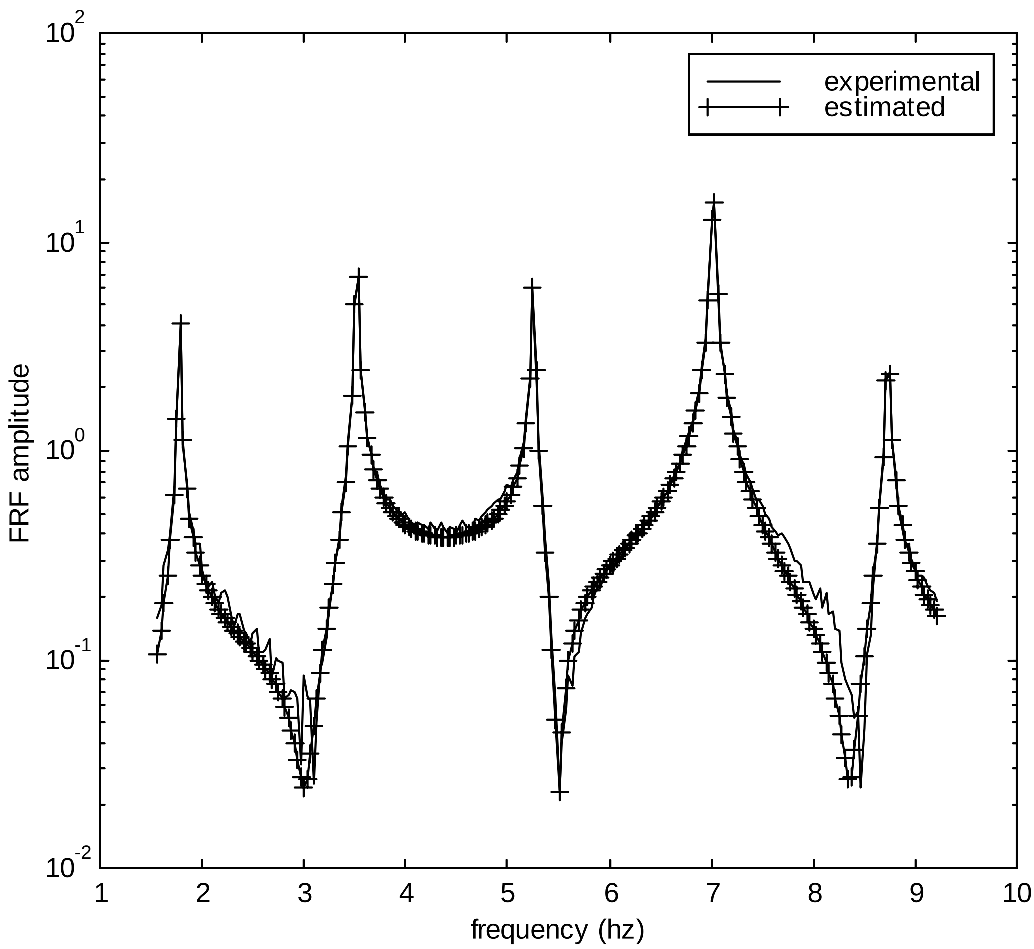

Figure 1 - FRF plots of the experimental and RFP curve fit

Figure 2 shows the curves of damping ratio of the first five mode shapes obtained for all sample lengths and mechanical load of 10700 N. Figure 3 shows the same curves for load of 15860 N. It can be noticed that the damping ratio increases with the increase of the length of the sample and decreases with the increase of the mechanical load.

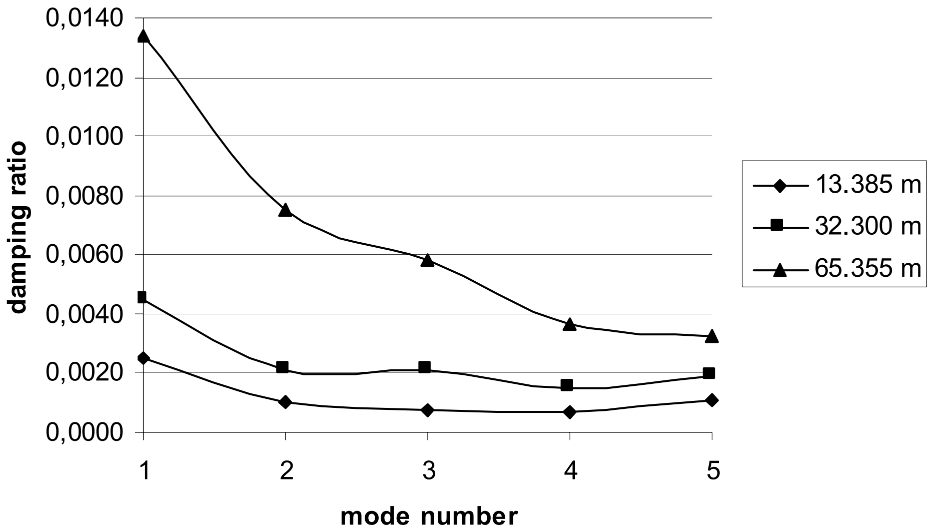

Figure 2 - Damping ratio (load of 10700 N).

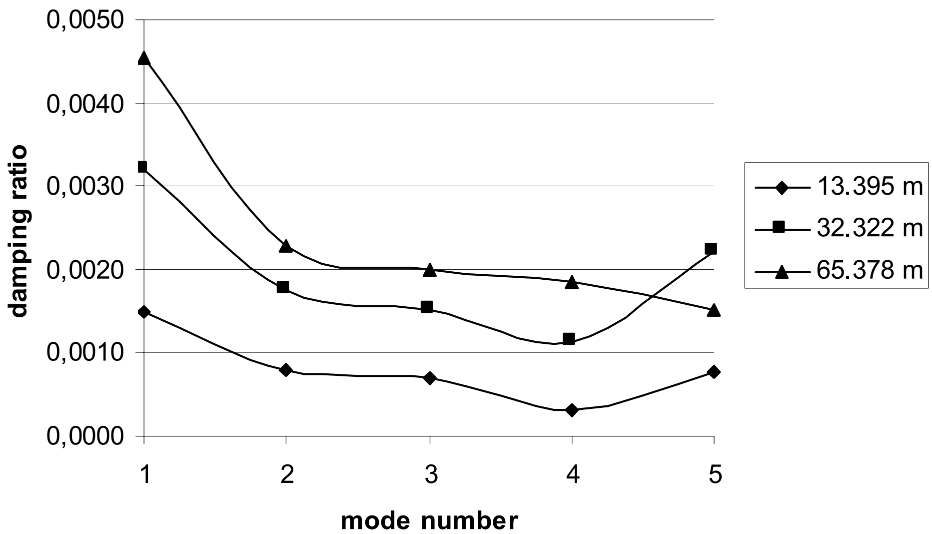

Figure 3 - Damping ratio (load of 15860 N).

In the attempt to work with reduced models, the partitioned equation (5) is used. The original system with 10 finite elements has order of 29 x 29. As five accelerometers were used in the positions mentioned previously, the reduced system has order of 5 x 5. To create reduced mass and stiffness matrices, the iterated improved reduced system (iterated IRS) method was used. The eigenvalues of the original system (undamped system), reduced system (undamped), experimental and estimated through a fourth order function (6) are shown in Tab. 1. The results for the theoretical and reduced models present great agreement. The real part of the eigenvalues introduce small differences, that were expected since the interpolation function uses theoretical values of mass and stiffness matrices.

Table 1 - Computed and experimental eigenvalues.

			Data	
mode shape	Original	reduced system	Experimental	function
1	1.7761i	1.7761i	-0.0499+1.7755i	-0.0499+1.7761i
2	3.5523i	3.5524i	-0.0466+3.5181i	-0.0470+3.5524i
3	5.3286i	5.3291i	-0.0698+5.2558i	-0.0708+5.3291i
4	7.1051i	7.1071i	-0.0651+7.0178i	-0.0660+7.1071i
5	8.8818i	8.8835i	-0.1024+8.7348i	-0.1042+8.8835i

Adjusting the damping matrix through other two procedures described in the equations (9) to (17), the convergence was noticed for the same values. The errors were negligible between methods.

Finally, it was adjusted a damping matrix of the following form $C = \alpha M + \beta K$, where α and β are constants. The Figures 4 and 5 show the results obtained for the two mechanical loads and three sample lengths. Figure 4 shows the values of the constant α and Fig. 5 the values of constant β. The curves present an inflection for samples greater than 32 meters and mechanical load of 10700 N. For the load of 15860N this behavior is not noticed.

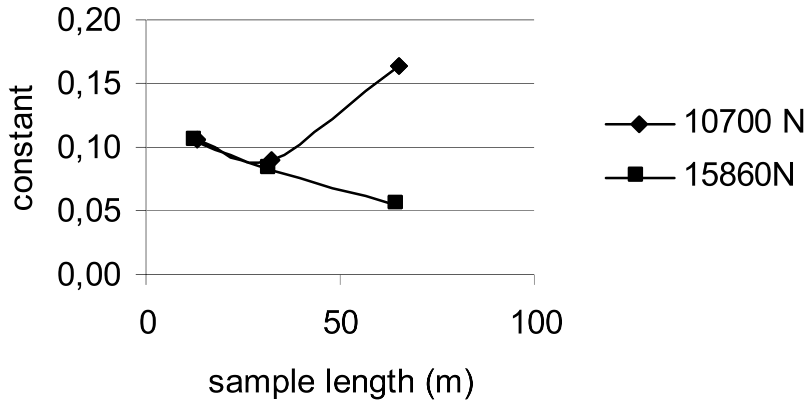

Figure 4 - Values of α constant.

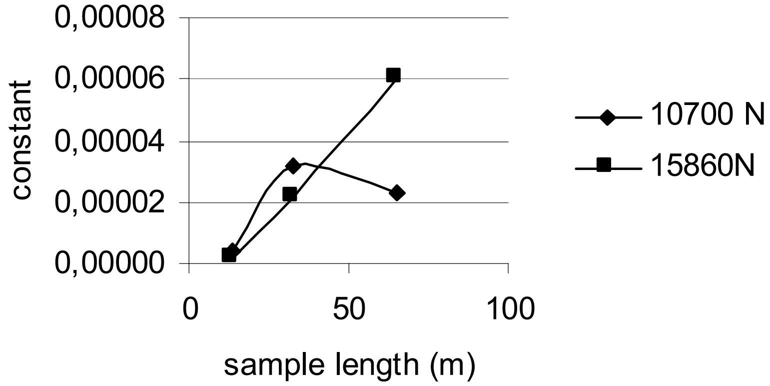

Figure 5 – Values of β constant.

To estimate these constants a gradient search routine was applied. The matrix reference was the matrix found with a fourth order function, Eq. (8). The errors due to this last approach are shown in Fig. 6. The errors of the 25 matrix elements are represented in these figure. The maximum percentile error introduced with this simplification of the damping matrix is around 5,5 %.

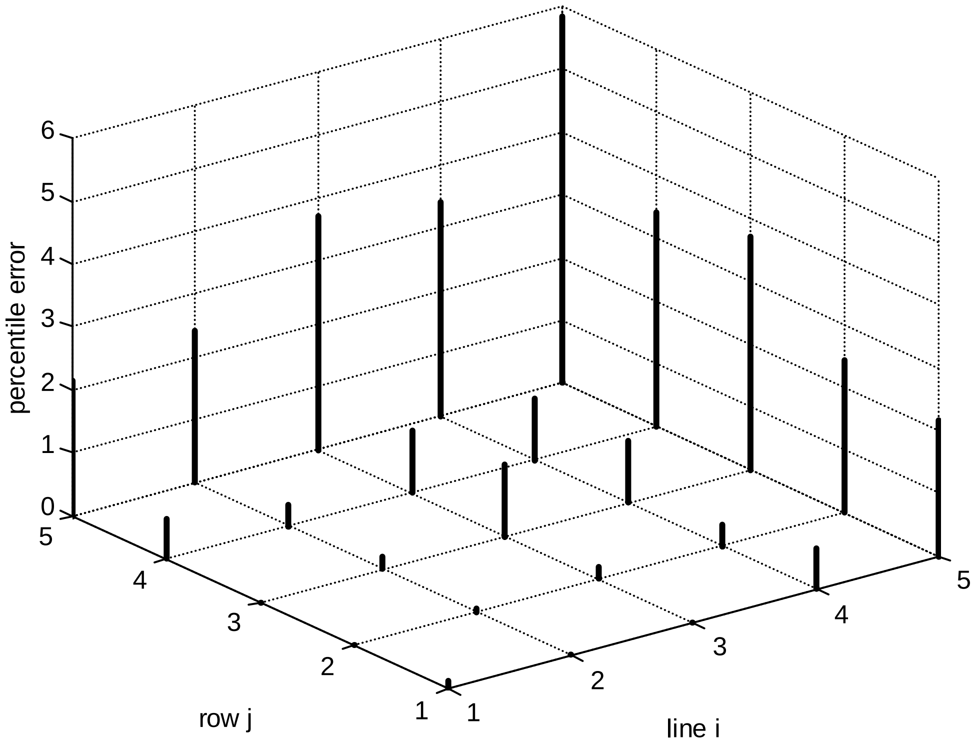

Figure 6. Percentile error of $\mathbf{C}_{ij}$ (mechanical load 10700 N, sample length 13,385m)

A more complete study about dynamical analysis of transmission line cables can be found in Barbieri et all (2003a, 2003b).

4. Conclusions

Several techniques were used to find the dynamical behavior of transmission line cables. The main goal is to establish the damping matrix of the system. The basic procedure involves the mathematical modeling of the system (Finite Element Method), modal analysis (eigendata), a iterative technique to reduce the system order and the estimate of the system damping matrix.

To obtain the experimental data, five accelerometers were used. The system was excited though an impact hammer. The function FRF was estimated by using different methods of modal analysis. The experimental and estimated results presented the same behavior for all methods.

To estimate the damping matrix of the system, three procedures were used: interpolation of a fourth order function, Eq. (8), using an auxiliary modal damping matrix, Eq. (9-15), and using the iterative procedure, Eq. (16-17). All the results converged for the same damping matrix. The real part of the eigenvalues presented small differences due to the fact that theoretical natural frequencies were used.

Finally, a proportional damping matrix was adjusted. This fact introduced small errors in the components of the damping matrix. Thus, this last damping matrix serves as an initial reference for the simulation of the system dynamical behavior.

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