DYNAMIC ANALYSIS OF RESPONSE VARIABILITY OF A STOCHASTIC STRUCTURE

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Abstract. *In this paper, experimental, analytical and numerical methods are used to evaluate the dynamic behavior of the reduced model of a two-storey shear building. The experimental methodology is based on sweep sine techniques and the frequency domain method (Allemang, 1988, Rades, 1988), where the modal parameters are obtained directly from the information contained on the frequency response functions. The uncertainties involved on the dynamic response are estimated primarily due to: (1) boundary conditions effects, (2) variability of the mechanical and dimensional properties of the components of the structure. A lumped system is used for the analytical formulation. Based on the Finite Element Method (Bathe, 1996), modal and harmonic analysis are performed in order to simulate the dynamic behavior of the ideal structure. The results achieved by means of the different methods were compared and it was possible to, not only, validate the numerical idealized model, but also, verify the consistency of the experimental results and quantify the uncertainties involved on the dynamic response of the structure.*

Keywords. *Modal analysis, Finite Element Method, sweep sine, frequency response function, random variable.*

1. Introduction

The maximum stresses caused by dynamic loads are an essential factor in design. Therefore, it is important that the designer is confident that the models and calculations involved in the analysis are reasonably accurate. The dynamic analysis of a structure can be performed by means of numerical, analytical and/or experimental models. However, there are always uncertainties associated to modeling that result from the numerous hypotheses involving many parameters that control the dynamic behavior of the structure such as, geometry, mechanical properties of the materials and boundary conditions. Research performed independently by Ellis (1988), Chang (1993) and Papadimitriou (1995) presents the difficulties involved in the definition of models that satisfactorily represent the structure's dynamic behavior. Such behavior, as well as the reliability associated can be very susceptible to the variations of these characteristics. Depending on the system being analyzed, even small uncertainties associated to these stochastic variables can affect in a significant way the dynamic behavior of the structure (Papadimitriou, 1995).

The main aim of this study is to evaluate the effects of the uncertainties associated to the estimates of the parameters that describe the modeling of the dynamic system and its behavior. In this sense, experimental, analytical and numerical methods were used to estimate the modal parameters of the reduced model of a two-storey shear building. The experimental methodology was based on the frequency domain analysis method described by Allemang et al (1988) and Rades (1988), where the modal parameters are achieved directly from the information contained on the frequency response functions. This parameters, such as, fundamental frequencies and modal vectors were obtained using sweep sine techniques. A two-storey benchmark steel frame was built to quantify the uncertainties involved on the dynamic response of 9 similar models due to different boundary conditions related to mounting and dismounting processes. The effects on the frequency response due to the variations of the mechanical and dimensional properties, associated to measurements of thickness, length, width, specific weight and Young Modulus of the columns that were fabricated from the same lot of material were estimated by means of sensitivity analyses. For the analytical formulation, a discrete two degree-of-freedom model was used, where the stiffness matrix was assembled based on the hypothesis that the structure is subjected to pure shear. The Finite Element method - FEM (Bathe, 1996) was used to simulate modal and harmonic analysis on a numerical model consisting of a combination of beam and shell elements. Consistency analysis parameters were used to determine the levels of correlation between the numerical and experimental results.

2. Basic Theory for Dynamic Analysis

The equation of motion that describes the dynamic behavior of an *n* degree-of-freedom linear structure subjected to external forces can be written in the form presented in Eq. (1)

$$\mathbf{M}\cdot\ddot{\mathbf{X}}+\mathbf{C}\cdot\dot{\mathbf{X}}+\mathbf{K}\cdot\mathbf{X}=\mathbf{F}(t) \tag{1}$$

where **M**, **C** e **K** represent the mass, damping and stiffness *n* x *n* matrices, respectively; and $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$, $\mathbf{X}$ e $\mathbf{F}$(t) represent the acceleration, velocity, displacement and force vectors of order *n*, respectively.

The solution of Eq. (1) consists of two distinct parts, the complementary function, which represents the homogeneous solution and the particular or transient solution. Based on the homogeneous solution, or modal solution, a system of equations allows the determination of the fundamental frequencies of vibration, called natural frequencies, as well as the characteristic deformed shapes of the structure while excited by these frequencies, denominated modes of vibration. The particular solution allows the description of the dynamic response of the structure when excited by any forcing function. Typically, such response is presented by frequency response functions – FRF.

A modal solution of the motion equation for natural frequencies and mode shapes require a reduction of Eq. (1). In cases where there is absence of damping and forcing functions, the equation can be reduced to the form expressed in Eq. (2).

$$\mathbf{M} \cdot \ddot{\mathbf{X}} + \mathbf{K} \cdot \mathbf{X} = \mathbf{0} \tag{2}$$

To solve such equation, a hypothesis involving a harmonic solution presented in Eq. (3) has to be assumed, where ϕ represents the dislocation amplitudes and ω the system's free vibration frequencies.

$$\mathbf{X} = \phi \sin \omega t \tag{3}$$

Based on such function and on its derivates, the undamped free vibration moviment equation can be described by the form of Eq. (4), denominated eigenfunction (Oliveira, 1997), where **I** represents the identity matrix.

$$\left(\mathbf{K} \cdot \mathbf{I} - \mathbf{M} \cdot \omega^2\right) \cdot \mathbf{X} = 0 \tag{4}$$

The group of algebraic homogeneous equations composes an eigensystem presenting two possible solutions (Rao, 1995): (a) trivial solution, where the determinant of the eigenfunction assumes magnitude different of zero and there is absence of movement ($\mathbf{X}$=0); (b) Non trivial solution, where the determinant is equal to zero and there is presence of movement ($\mathbf{X}\neq 0$). The determinant of the eigenfunction assumes value equal to zero for discrete values of the eigenvalues ω_i. Such a function is solved by an eigenvector $\mathbf{X_i}$ for the respective eigenvalue and can be written in the form expressed by Eq. (5), where **A** represents the flexibility matrix.

$$\left(\frac{1}{\omega_i^2} \cdot \mathbf{I} - \mathbf{A} \cdot \mathbf{M}\right) \cdot \mathbf{X}_i = 0 \quad , \quad for\ i = 1,2,3 \ldots n \tag{5}$$

The eigenvectors and eigenvalues are related to the natural frequencies and natural modal vector, respectively and describe the free vibration behavior of the structure. For each eigenvalue ω_i , a natural frequency f_i and a modal vector $\mathbf{X}_i$ are defined. The number of eigenvalues and, therefore, the number of eigenvectors is equal to the number of degrees-of-freedom of the system (Ewins, 2000).

Based on Eq. (5) is possible to analyze the sensitivity of the frequency response. This can be done by approximating the mean value and variance of ω by means of an expansion of this function in a Taylor series about the mean values of the variables and truncating the series at the linear terms. The first-order approximate mean value, $\widetilde{\omega}$, and variance, σ_ω, of the fundamental frequencies estimator are expressed in Eq. (6) and (7), respectively (Ayyub, 1984; Harr,1987).

$$\widetilde{\omega} = \omega(\Xi) \tag{6}$$

$$\sigma_\omega^2 \cong \sum_{i=1}^{m} \sum_{j=1}^{m} \left[\left(\frac{\partial \omega(\Xi)}{\partial \Xi_i} \right) \cdot \left(\frac{\partial \omega(\Xi)}{\partial \Xi_j} \right) Cov\left(\Xi_i, \Xi_j\right) \right] \tag{7}$$

where Ξ is the vector associated to the parameters that control the motion equation and $Cov(\Xi_i,\Xi_j)$ is the covariance of Ξ_i and Ξ_j.

3. Two Degree-of-Freedom Approximate Solution

The approximate solution for the dynamic behavior of the two-storey structure, presented in Fig. (1a) was performed by considering a two degree-of-freedom model where the columns are subjected to pure shear (Beards, 1996). Moreover, as illustrated in Fig. (1b), the basic hypotheses used to describe the elastic line equation were that the columns are fixed at one end and there is absence of rotation at the other, defined by Eq. (8) and (9), respectively.

$$v = 0 \Rightarrow u = 0 \ and \ \frac{du}{dv} = 0 \tag{8}$$

$$v = h \Rightarrow \frac{du}{dv} = 0 \tag{9}$$

where v and u represent the local coordinates in the longitudinal and perpendicular directions respectively and h is the height of each storey.

Furthermore, by making use of such hypotheses associated to the elastic line equation results on the relationship between the force, F, and the displacement of the force application point, u, as shown by Eq. (10) and ilustrated in Fig. (1c)

$$F = \frac{12 \cdot E \cdot I}{h^3} \cdot u \tag{10}$$

where E represents the Young's Modulus and I the second moment of inertia of each segment.

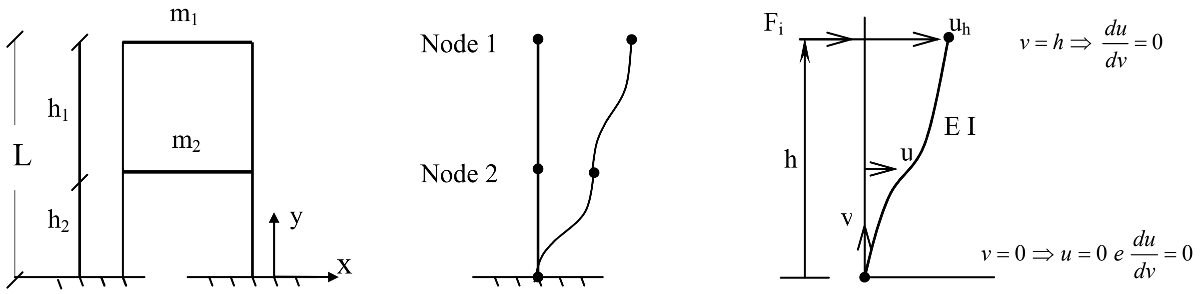


Figure 1. (a) simplified illustration of the structure; (b) two degree-of-freedom model; (c) representation of the column subjected to pure shear and the problem's boundary conditions.

The flexibility influence coefficient a_{ij} is defined as the displacement occurred in a position i due to a unitary force applied to a position j. In order to determine the flexibility matrix, expressed symbolically in Eq. (11), the superposition principle was applied, associated to the equation of the elastic line (Thomson, 1978, Steidel, 1986).

$$\mathbf{A} = \mathbf{K}^{-1} = \begin{bmatrix} h_2^{\,3} + h_1^{\,3} & h_2^{\,3} \\ h_2^{\,3} & h_2^{\,3} \end{bmatrix} \cdot \frac{1}{24\,E\,I} \tag{11}$$

where h_1 and h_2 represent the distance between storeys and the distance between the lower storey and the base of the structure, respectively.

The system's mass matrix is determined by Eq. (12) and was calculated based on the storeys weights and the equivalent mass due to the inertia of the columns (Rao, 1995).

$$\mathbf{M} = \begin{bmatrix} m_1 + m_{1\ equiv} & 0 \\ 0 & m_2 + m_{2\ equiv} \end{bmatrix} \tag{12}$$

where m_1 and m_2 represent the mass of the upper and lower storey, respectively.

By calculating these two matrixes it is possible to solve the eigensystem given by Eq. (5).

3. Numerical Model - FEM

The Finite Element Method is based on the Rayleigh-Ritz theory and considers the partition of the integration dominium in a finite number of regions denominated finite elements that together form a mesh of elements containing a certain number of degrees of freedom (Bathe, 1996). For the modeling of the columns and the storeys, beam and shell elements were used, respectively. The beam elements present two nodes and three degrees of freedom per node: translation in the x and y directions and rotation in z. The quadrilateral shell elements present four nodes and six degrees-of-freedom per node: translation and rotation in all three directions. The boundary condition was established by a restriction of all degrees-of-freedom of the nodes located at the base of the columns, simulating a full-constrained support situation. The subspace method was used in order to determine the modal parameters with precision and speed (Bathe, 1996). Mesh convergence analysis were performed, allowing the achievement of a model conciliating computational effort and results precision. Such convergence study was prepared for the first fifteen natural vibration modes. However, the results presented are related only to the first two fundamental modes. In order to guarantee the compatibility between the order of the eigenvectors numerically and experimentally obtained, the dimensions concerning the beam elements were set in a way that presented nodes corresponding to the experimental analyzed positions, avoiding the necessity of utilizing numerical techniques for reduction or expansion of modes, as described by Guyan (1965), Ewins et al (1991) and, Kim et al (1984). Once the model was specified and determined the two first natural frequencies, harmonic analyses were performed. A unitary forcing function and normal to the structure, applied to a similar position to the one caused by the shaker in the physical model, was used in order to match up numerical and experimental results. The full method, where Eq. (1) is directly solved, was used to determine the FRF (Bathe, 1996). A 0 to 30 Hz frequency range was initially used with the aim of obtaining the FRF of the structure and to observe the amplitude peaks related to different positions and due to the natural frequencies previously determined by the modal analysis. Latter, near the frequencies of interest, more specific analysis were done, allowing the definition of the modal vectors associated to the first two natural vibration modes. In all of the above situations 100 equal segments divided the frequency ranges. The finite element model, the types of finite elements used, as well as the displacement-analyzed positions are illustrated in Fig. (2).

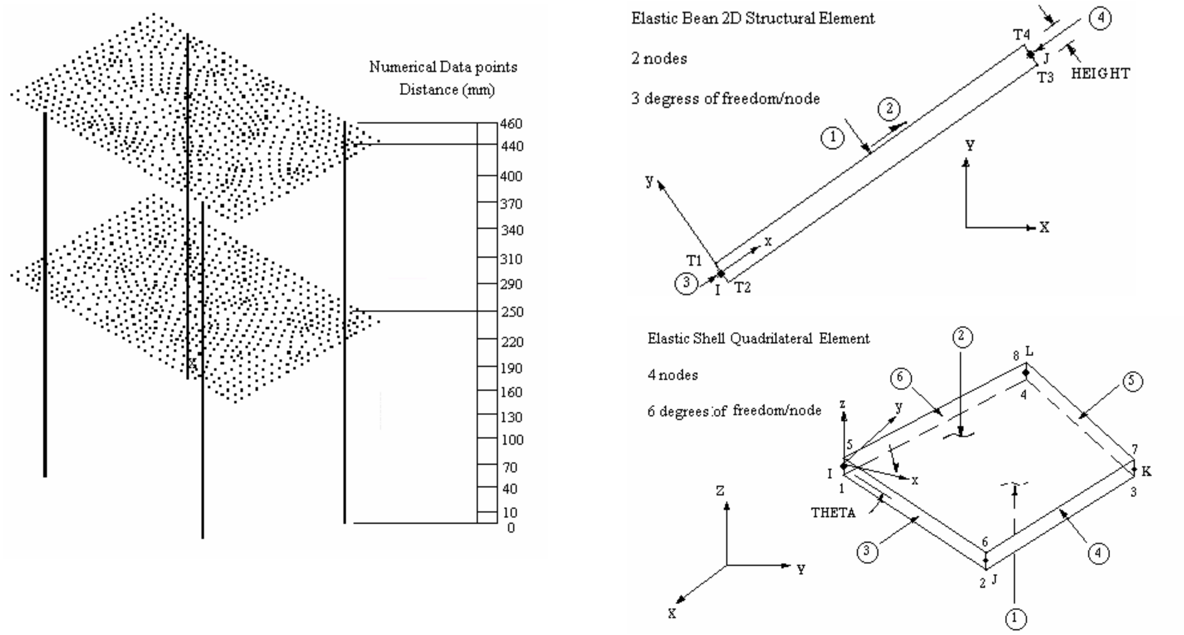


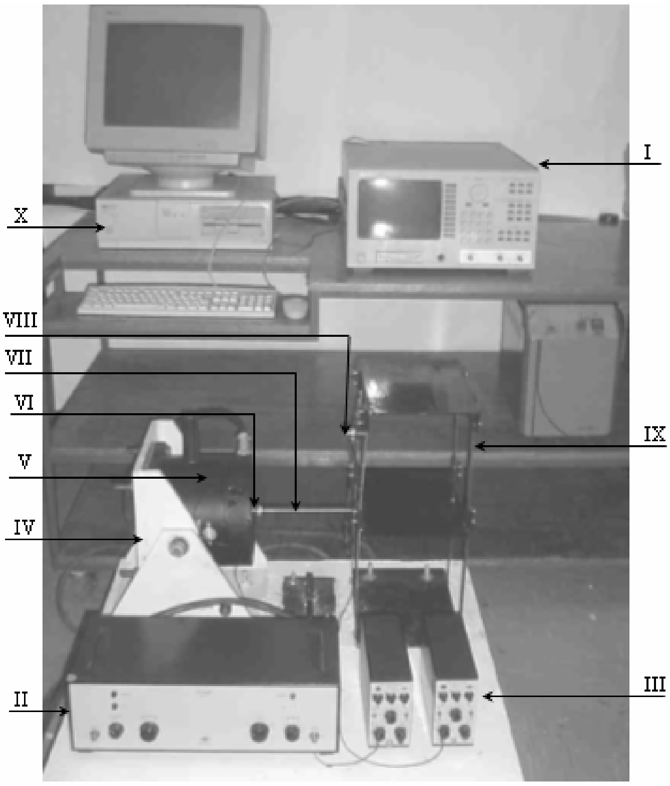
Figure 2.Finite element model, finite elements used and displacement analyzed positions.

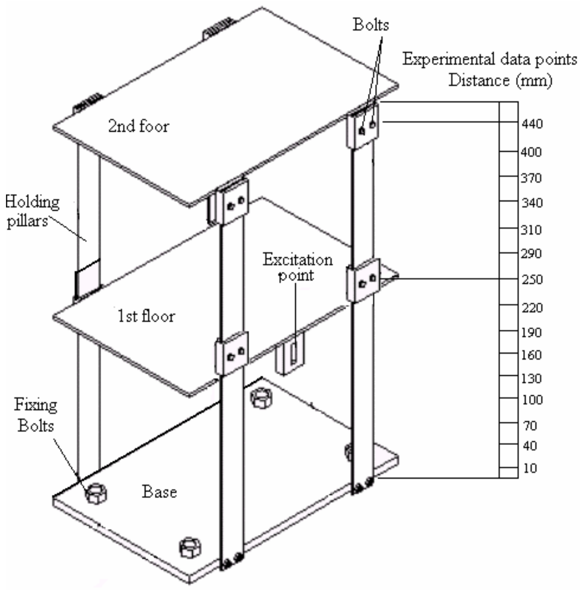
4. Experimental Model

Experimental tests of harmonic analyses were performed to determine the modal parameters of the model. The formulation and appliance of experimental modal analysis were based on sweep sine techniques and the frequency domain method. In that sense, the experimental benchmark steel frame illustrated in Fig. (3) was built. The total constrained support situation was approximated by means of a group of fixing bolts that fixed the reduced model to an inertial base represented by a 350 kg block of concrete. The four columns were assembled from a 1.2 mm steel plate and cut to nominal dimensions of 470 x 30 mm, presenting mass of approximately 0.132 kg. The upper and lower storeys were assembled from a 3 mm steel plate and cut to nominal dimensions of 200 x 300 mm, presenting mass of approximately 1784 e 1926 g, respectively. This difference is primarily due to the presence of a small steel plate, welded to the lower storey, possessing a notch with the intention of fixing the stinger for the horizontal excitation by the shaker. The storeys are fixed to the columns by means of a group of bolts that permit variations on its positioning. In the analysis, the upper and lower storeys were positioned at heights of 460 and 235 mm. The structural response was evaluated with a B&K Type 4366 piezoelectric axial accelerometer, wax fixed and positioned in 15 points on the columns. The signal acquisition procedures were performed with a HP35665A two-channel spectral analyzer that computed the FRF. Wide range (1 a 20 Hz) sweep sines were accomplished in order to identify the natural frequencies relative to the first two natural modes of vibration. Based on these initial analyses, specific sweeps were performed near the frequencies of interest. For determining the modal parameters, single-mode parameter estimation techniques were used (HP Application Note, 1986; Mesquita, 1996). The modal vectors associated to the first two natural frequencies were achieved based on the FRF obtained from the fifteen points of the columns. In order to obtain statistical consistency and evaluate the variability of the dynamic characteristics, the above procedure was repeated for 9 nominally similar models. Each one of these models was mounted with a particular group of four columns, randomly chosen from the same lot of material. Tab. (1) presents the mean and variation coefficient of the dimensions of the components. As a result of this experimentation, 9 similar mode shapes and 135 natural frequencies were obtained for each studied mode. Independently, the boundary condition effects on the dynamic response of the structure were examined, where the mounting and dismounting procedure was performed six times for the same model and evaluated the dispersion of the natural frequency measurements.

Table 1. Dimensional parameters of the columns.

Column	Width		Thickness		Length		Mass	
	Mean (mm)	V. C. (%)	Mean (mm)	V. C. (%)	Mean (mm)	V. C. (%)	Mean (g)	V. C. (%)
1	30.112	1.135	1.207	2.621	475.7	0.105	133.089	1.143
2	30.094	1.285	1.201	1.882	475.4	0.111	133.356	0.916
3	29.784	2.115	1.206	1.906	475.7	0.105	132.056	1.907
4	29.384	1.615	1.208	2.221	475.4	0.111	132.001	1.415
All	29.844	1.814	1.205	2.094	475.6	0.106	132.625	1.409





I. Spectral analyzer;
II. Amplifier;
III. Charge amplifier;
IV. Shaker support;
V. Shaker;
VI. Stinger;
VII. Accelerometer;
VIII. Load cell;
IX. Reduced model;
X. Control unit.

Figure 3. Experimental apparatus, two-storey benchmark steel frame and experimental measurement positions.

5. Analysis Methodology

Graphical and statistical methods were used in order to validate the results. The graphical methods were used to compare the natural frequencies and the mode shapes of interest. In this methodology, a diagram correlating the numerical and experimental results, as well as a regression analysis is used to determine the confidence limits of the scattered points, as well as the best-fit straight line that passes through the origin.

Besides the basic statistics of the results found, comparison parameters, such as, the MAC - “Modal Assurance Criterion” and the MSF - “Modal Scale Factor” were used to compare and to test the consistency and correlation of the numerical and experimental modal vectors (HP Application Note, 1986; Ewins, 1984). The MSF represents the slope of the best straight line through the points in the scatter diagram, being numerically expressed as Eq. (13).

$$MSF(x,p) = \frac{\sum_{i=1}^{N} (\phi_x)_i (\phi_p)_i}{\sum_{i=1}^{N} (\phi_p)_i (\phi_p)_i} \qquad (13)$$

where ϕ_x and ϕ_p represent, respectively the experimentally measured mode shape and the numerically calculated one.

The MAC, also called MSCC - “Mode Shape Correlation Coefficient”, provides a measure of the least squares deviation of the points from the straight-line correlation, and is determined by Eq. (14).

$$MAC_{(p,x)} = \frac{\left| \sum_{i=1}^{N} (\phi_x)_i (\phi_p)_i \right|^2}{\left(\sum_{i=1}^{N} (\phi_x)_i (\phi_x)_i \right) \left(\sum_{i=1}^{N} (\phi_p)_i (\phi_p)_i \right)} \tag{14}$$

With the aim of evaluating the variations of the dynamic parameters due to stochastic variations of the inherent characteristics of the model and the ones involved on the measurement procedures, variance analysis technique was used (Murteira, 1990), as well as sensitivity analyses by means of Eq. (6) e (7) (Ayyub, 1984; Harr, 1987).

6. Results and Discussions

Based on the analytical model, values of 4.69 and 17.77 Hz were predicted for the first two natural frequencies. Such values were extremely important as an initial estimation of the results expected for the numerical and experimental analysis.

The convergence analysis for the results calculated with the finite element method is presented in Fig. (4) where it is possible to verify that the relative deviation of the values estimated with the use of a particular mesh when compared to the results obtained for the most refined one is always inferior to 1%, fact that allows the use of any particular mesh level of refinement evaluated. However, in order to permit certain compatibility between the order of the eigenvectors numerically and experimentally obtained, the most refined mesh, presenting 25896 equations, was used. In this sense, the predicted values for the first and second natural frequencies obtained with the use of such method and mesh refinement level were, respectively, 4.01 and 16.05 Hz, results approximately equal to those obtained by the simplified two degree-of-freedom model.

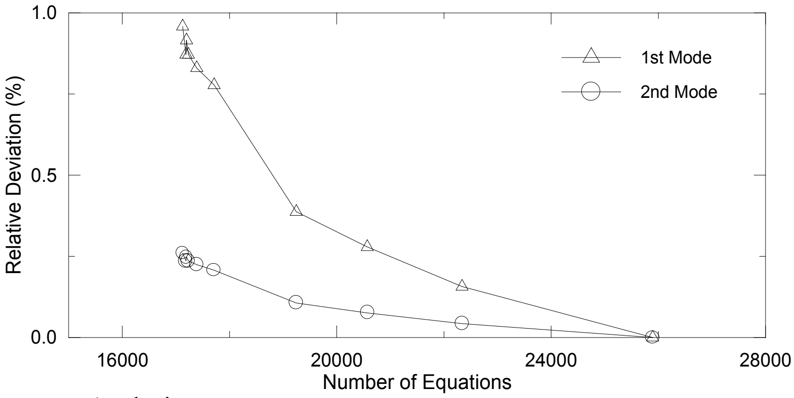


Figure 4. Mesh Convergence Analysis.

The mean values and variation coefficient of the natural frequencies of the nine models experimentally evaluated are presented in Tab. (2)

Table 2. Statistics of the natural frequencies experimentally obtained for the 9 similar structures.

Frequency		Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
1st	Mean (Hz)	4.27	3.84	5.09	3.70	4.07	3.97	3.83	3.78	4.04
	V. C. (%)	0.00	0.00	0.00	0.00	0.22	0.87	0.62	0.33	0.51
2nd	Mean (Hz)	14.54	15.29	15.46	14.54	15.43	15.05	15.25	14.91	15.52
	V. C. (%)	0.00	0.15	0.13	0.13	0.14	0.17	0.69	0.16	0.14

Analyzing the results presented in Tab. (2) it is possible to verify the existence of approximated 10 and 2% variations for the first and second natural frequencies, respectively. However, such fact is not observed on the measurements taken for the same model, where maximum variations of approximately 0.9 % were verified, indicating the presence of an unimportant experimental error and that the variations associated to the boundary conditions and random variations of the parameters that control the structure's dynamic behavior, such as mass and stiffness of the columns, affect in a significant way its response. Such statements are corroborated by the variance analysis results presented in Tab. (3) and (4), where the hypothesis that the variations between treatments are equal to the ones observed within the treatment is true, since F assumes a higher value than F_{critic}.

Table 3. Variance analysis for the 1st natural frequency measurements.

Source of Variation	*SQ*	*Gl*	*MQ*	*F*	*P-value*	*F critic*
Between treatments	21.217	8	2.652	9885.4	2.3E-172	2.013
Within treatments	0.034	126	0.0002			
Total	21.251	134				

Table 4. Variance analysis for the 2nd natural frequency measurements.

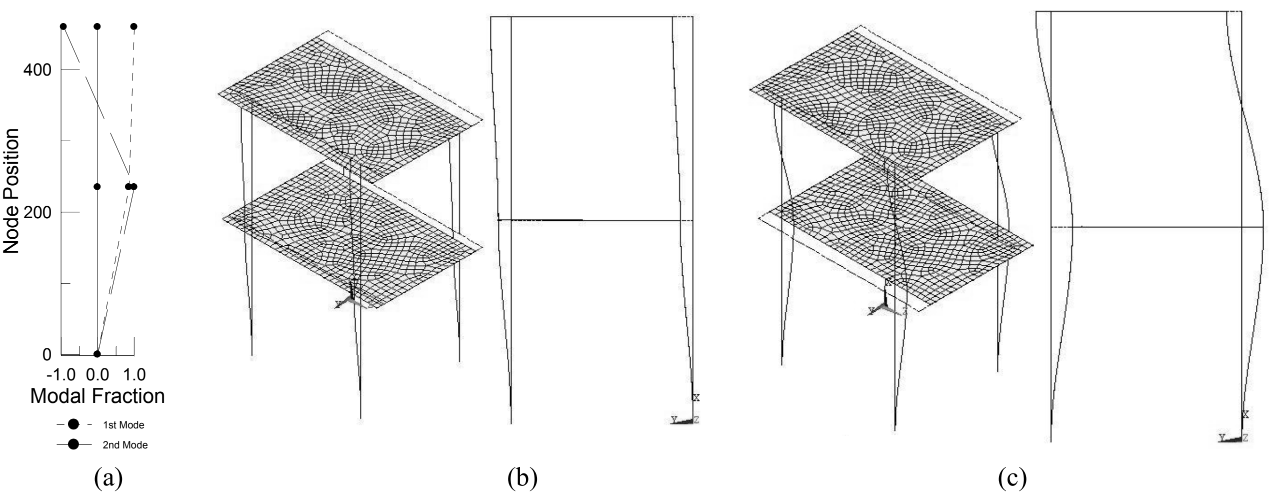
Source of Variation	*SQ*	*Gl*	*MQ*	*F*	*P-value*	*F critic*
Between treatments	11.887	8	1.486	913.3	1.2E-107	2.013
Within treatments	0.205	126	0.0016			
Total	12.092	134				

The statistics related to the measurements of the fundamental frequencies obtained by means of the boundary condition analysis is shown in Tab. (5). Based on such results it is possible to verify that the uncertainties associated to the response are approximately constant.

Table 5. Mean values and variation coefficient obtained by mounting and dismounting 6 times the same model.

1st Natural Frequency		2nd Natural Frequency	
Mean (Hz)	V. C. (%)	Mean (Hz)	V. C. (%)
3.87	3.31	15.01	3.08

The influence of the stochastic variations on the frequency response of the model, concerning the thickness, width, Young's Modulus of the columns, as well as the mass and displacement of the storeys, were evaluated by means of a sensitivity analysis and it was possible to verify that the thickness of the columns and the displacement of the storeys represent the two parameters that influence the most on the dynamic response, with values of approximately 30 % each on the global uncertainties associated to the measurements. Based on the dimensional results presented in Tab. (1) it is viable to verify that, only due to dimensional variations, the experimental results should present uncertainties identical for both modes analyzed and with variations of approximately 4.4 %. Comparing the magnitude order of these variations with the ones estimated accounting the connection effects, it can be verified that the combination of these elucidate the global dispersion of the frequency data measured for the first natural frequency of vibration. On the other hand, the same is not valid to explain the lower deviation obtained for the second natural frequency. The analytical and numerical results for the two fundamental mode shapes are presented in Fig. (5). Based on the numerical results of the deformed shapes, presented in Fig. (5a) and (5b) it is possible to verify the veracity of the basic hypotheses used to assemble the analytical model of the columns: fixed at one end and null rotation at the other.



(a) (b) (c)

Figure 5. Vibration Modes: (a) analytical model, (b) 1st mode - FEM, (c) 2nd mode – FEM.

By means of sweep sine techniques, the experimental mode shapes of the 9 similar models were estimated. A graphical comparison of the experimentally obtained results for the first and second vibration modes is presented in Fig. (6) and (7), respectively. These figures illustrate not only the dispersion of the results, but also a comparison of the mean experimental behavior and the numerical mode shape. A more obvious dispersion of the experimental data related to the first natural mode shape can be observed, when compared to the second one. However, a more coherent analysis of such results is done via scatter diagram analyses and the calculation of the MAC and MSF parameters.

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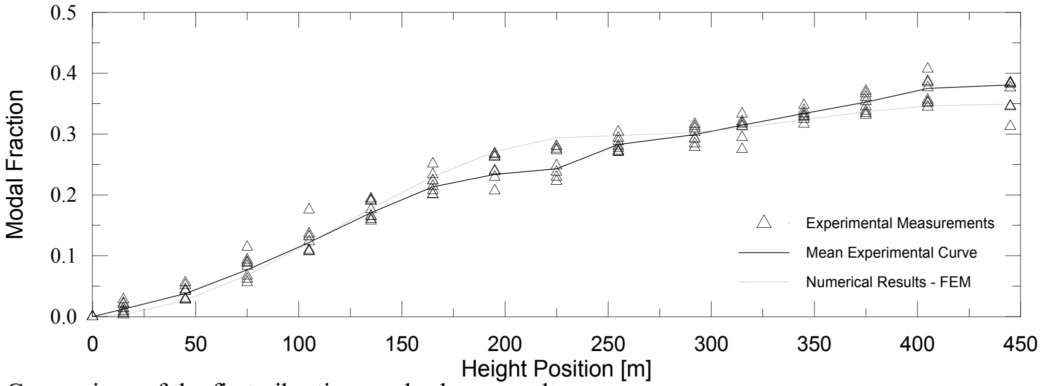

Figure 6. Comparison of the first vibration mode shape results.

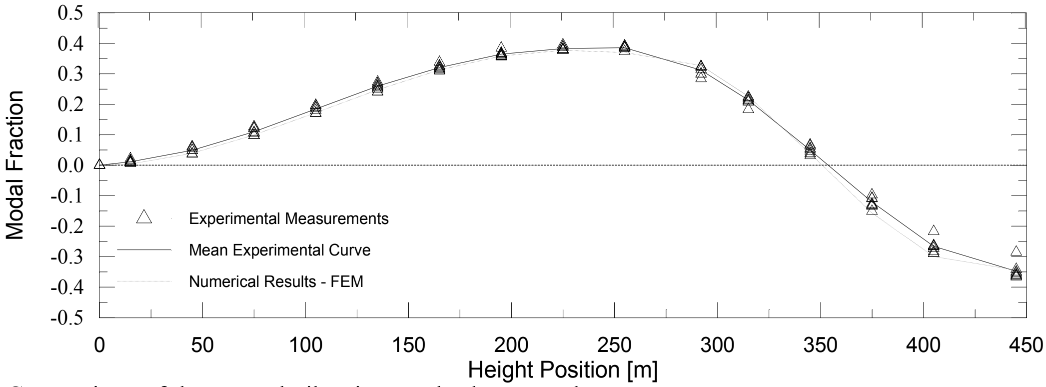

Figure 7. Comparison of the second vibration mode shape results.

The scatter diagrams for the mode shapes of interest are presented in Fig. (8). Analyzing these diagrams it is possible to compare the numerical and experimental modal vectors, verify the positioning of most of the results within the 95% confidence limits, as well as confirm the presence of a more obvious dispersion of the data related to the first mode of vibration when compared to the second mode. There is significant similarity between the slope of the best-fit straight line and the ideal value of 45°, ratifying the excellent correlation level of the numerical and experimental results.

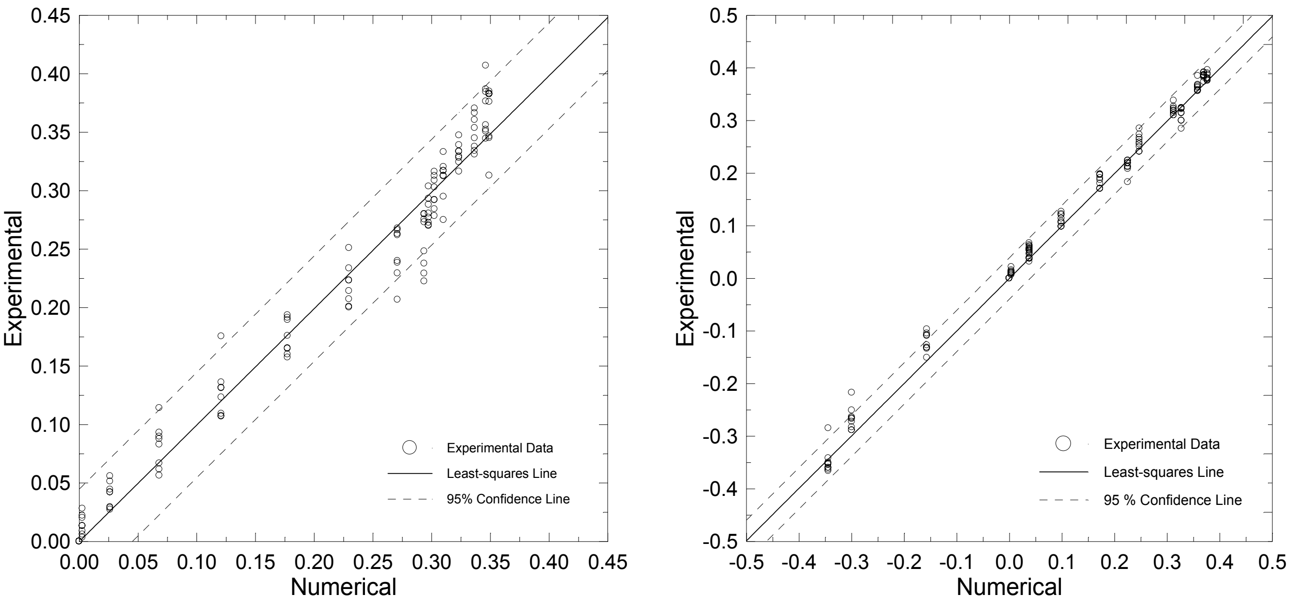

Figure 8. Scatter diagrams of (a) 1st vibration modal vector (a) 2nd vibration modal vector.

The values presented in Tab. (6) and (7) were calculated using the MAC and MSF, respectively for the comparison of the modal vectors. Analyzing the MSF values it is possible to quantify and confirm the fine level of similarity between the slope of the best fit line of the scatter diagram and the ideal 45° slope line for most of the models. This consistency analysis is completed with the MAC values, determining that the least squares deviations of the points from the straight line correlation are close to the unity, indicating a high correlation level of the obtained results.

Table 6: MSF values for the first and second modal vectors.

Mode	MSF								
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
1st	0.927	0.987	0.911	0.902	0.985	0.852	0.896	0.994	0.687
2nd	0.972	0.980	0.980	0.976	0.943	0.964	0.962	0.997	0.984

Table 7: MAC values for the first and second modal vectors.

Mode	MAC								
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
1st	0.997	0.998	0.995	0.987	0.989	0.986	0.986	0.996	0.926
2nd	0.995	0.999	0.999	0.998	0.981	0.992	0.994	0.995	0.992

7. Conclusions

In this research, theoretical, numerical and experimental estimations of the dynamic response of a two-storey shear building were intended. In that sense, 9 similar models were built for the experimental analysis. The analytical solution was based on a two degree-of-freedom model where the stiffness matrix was assembled based on the hypothesis that the structure is subjected to pure shear. A numerical solution, based on the finite element method, was calculated by means of a beam and shell elements combination model. Based on the results it was possible to verify that the numerical and experimental previsions of the first two modes of vibration frequencies were similar, presenting deviations of approximately 5%. However, this can not be computed for the analytical results, where deviations of approximately 15% were observed, serving only as an initial comparative value. The quantification of the uncertainties involved in the dynamic response of the structure was made evident. A combination of uncertainties estimated analytically by the sensitivity analysis, of approximately 4.4 %, together with the ones estimated for the boundary condition effects, of approximately 3 %, allowed the explanation of the global dispersion observed for the principal mode of vibration. The MAC and MSF values demonstrated the consistency and concurrence of the numerical and experimental results.

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