AIRPLANE PARAMETER IDENTIFICATION USING FREQUENCY RESPONSE ERROR METHOD

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Abstract. *The search for a mathematical model that well represents an aircraft dynamics is a challenge that requires a lot of work and is excessively time consuming. A large number of tools have been developed to help the engineer through the process. The output error is extensively applied, but, besides being very effective, this method is not stochastic and does not take into account the process noise. One way to overcome this disadvantage is to use the frequency response error method, in the frequency domain. The use of the coherence function and the possibility to separate uncorrelated inputs and known correlated inputs effects makes possible to have good frequency responses curves even is a noisy environment. A cost function is stated as a sum of the differences between the magnitude and between the phase of model and flight test identified frequency response curves. A minimization process furnishes the parameters that best fit the test data through a given model. In this paper an EMBRAER regional jet longitudinal dynamic model is identified using frequency response error method. Input data were obtained concatenating mutisteps "3-2-1-1" maneuvers to improve the averaging process. The assumption of a liner model proved to be very reasonable, since model and flight-test data presented good agreement.*

Keywords. *Parameter Identification, Frequency Response, Airplane, Longitudinal Dynamic*

1. Introduction

Aircraft dynamic model identification is still a challenge even tough it is not a new problem and engineering and computational resources have continuously been improved. If fact its applications have also become more stringent in terms of accuracy and precision. Typical applications for aircraft dynamic models are related to Full Flight Simulator, a training tool applied to pilots and aircraft development, and control laws development. The last one has been demanding more efforts by aircraft manufactures recently, mainly due to fly by wire controls. An aircraft mathematic model has to by very faithful to its real behavior to be accepted by the certification authorities and, nowadays, it is a multidisciplinary task incorporating aerodynamics, engines, actuators, systems and sometimes even the pilot. Embraer has been putting a great effort in this area to use modern techniques and extract good results for its purpose.

The aerodynamic part of the problem involves a previous established mathematical model that characterizes well the dynamic under interest and the related parameters. A good procedure to extract those parameters from flight test data has to take into account a carefully analysis of the problem, considering the flight test instrumentation, the maneuvers applied to excite the aircraft, the methodology to accomplish the flight test campaign and the mathematical procedures to extract all the information from flight data.

In this paper a frequency domain technique is applied to identify the parameters related to a previous established linear dynamic model of the longitudinal dynamics of an Embraer typical regional jet. Although time domain methods have become popular in parameter identification applications, frequency domain methods have some advantages over time domain methods that make them quite attractive.

Besides a good insight into the system, one can select a frequency range of interest and pre-filter the data before the identification process. After that, a long period of flight-test data can be stored in a fixed reduced set of transformed data pair.

Also some complex mathematical tasks, like differentiation, integration or convolution become easier algebraic operation, like division or multiplication. In a space state form, numerical integration methods are avoided, simplifying the equation set and its solution.

Some time domain methods have the characteristic to consider the stochastic nature of the process in the algorithms, but frequency responses methods have some properties that handle with this allowing managing with process noise, measurement noise and even partial correlated known inputs.

Nomenclature

A = state matrix	**g** = measurement function	γ^2_{xy} = coherence function
$\mathbf{a_z}$ = load factor (g)	g = grav. acceleration (9.80665 m/s^2)	**I** = identity matrix
B = input matrix	**Gxy** = cross spectrum matrix	J = cost function
f = model function	**H** = Frequency response function	N = total number of samples

q = pitch rate (rad/s)
t = time, s
Δt = *sample time, s*
u(t) = control vector
V, V_0 = aircarft speed (m/s).
x(t) = state vector
y(t) = model output vector
z(t) = in-flight measurement vector
ww(t) = state noise vector
w = frequency
v(i) = measurement noise vector

W = coherence weighting matrix
T = period (seconds)
CR = Cramer-Rao lower bound

Greek Symbols

δ_{ij} = Kronecker delta
θ = vector of unknowns
θ = Pitch attitude angle (rad)
α = angle of attack (rad)
δ_e = elevator angle (rad)

Superscripts

T = transpose
-1 = matrix inverse
^ = estimated
~ = Fourier transformed
* = complex conjugate transpose

2. Theoretical Formulation

Parameter identification in the frequency domain has basically the same formulation as related methods in the time domain. General nonlinear dynamic equations are stated in state space format as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f(\mathbf{x}(t),\mathbf{u}(t),\theta) + ww(t) \\ \mathbf{x}(0) &= x_0 \\ \mathbf{y}(t) &= g(\mathbf{x}(t),\mathbf{u}(t),\theta) \\ \mathbf{z}(i) &= \mathbf{y}(i) + v(i) \end{aligned} \tag{1}$$

All states are function of the states itself, the input vector and the parameters under investigation. Those values are corrupted by a process noise *ww(t)* that represents unmeasured excitations that can not be modeled deterministically, like aerodynamic turbulences in aeronautical applications. In stochastic time domain methods the process noise is considered white and Gaussian, but in frequency response methods an uncorrelated noise is enough, no matter its statistical distribution.

In fact another source of error is implicit in the formulation. It would be extremely grateful if the mathematical model could correspond to the actual aircraft behavior. It is just an acceptable approximation, but accurate enough for its purpose. The model contains, though, a modeling error component, generally lumped with measurement noise, which is not stochastic. This violates the white Gaussian measurement noise assumption for time domain methods and, probably, the uncorrelated assumption for frequency response method. The problem can be solved anyway, but care should be taken while analyzing the results and its accuracy, especially in flight test applications.

An aircraft dynamic model can be linearized without loosing accuracy, even in typical nonlinear applications, like rotary wings (Tischler, 1987). In those problems frequency domain formulation returns the best linear model for the dynamic under analysis. In fact, aircraft linear dynamic models are largely used in control laws projects and in flight simulator math models (Hui et. all, 2001).

It is possible to write Eq. (1) in a linearized form as follows:

$$\begin{aligned} \dot{x}(t) &= A.x(t) + B.u(t) + ww(t) \\ x(0) &= x_0 \\ y(t) &= C.x(t) + D.u(t) \\ z(t) &= y(t) + v(t) \end{aligned} \tag{2}$$

Matrix A, B, C and D now contains all the relevant parameter necessary to characterize the aircraft dynamic under interest. A cost function can be defined as a mean to search for the best parameter values through a convenient optimization method.

For frequency domain methods all data is transformed into a complex pair corresponding to magnitude and phase of the vector. A Continuous Fourier Transform applied to Eq. (2) results in:

$$\begin{aligned} j.w.\tilde{x} &= A.\tilde{x} + B.\tilde{u} \\ \tilde{y} &= C.\tilde{x} + D.\tilde{u} \\ \tilde{z} &= \tilde{y} \end{aligned} \tag{3}$$

The measurement over input vector is isolated in the complex frequency response form:

$$\frac{\tilde{y}}{\tilde{u}} = \left[\mathbf{C}.\left[jw - \mathbf{A}\right]^{-1}.\mathbf{B} + \mathbf{D}\right] \tag{4}$$

Experimental frequency responses and related coherence functions are extracted from flight test data through spectral functions. Mean and tendencies are removed from flight data prior to spectral calculations. Multiple time slices related to the same aircraft configuration and flight condition can be concatenated to improve high and low frequency contents of the FRF´s. Even after drift and DC level removal, windowing is quite necessary to prevent leakages in the spectral estimates. Large window preserves low frequency contents in the sign while small windows allows best hi frequency quality through averaging process. A compromise should exist regarding the expected bandwidth of interest in the signal to a proper window size.

A set of experimental input, output spectrum and cross-spectrum, frequency response and coherence is calculated in a traditional way:

$$\begin{aligned}
Gxx(f) &= \frac{2}{TU} X(f).X(f)^* \\
Gxy(f) &= \frac{2}{TU} X(f).Y(f)^* \\
Gxx(f) &= \frac{2}{TU} X(f).X(f)^* \\
H(f) &= \frac{Gxx(f)}{Gxx(f)} \\
\gamma^2{}_{xy}(f) &= \frac{\|Gxx(f)\|^2}{Gxx(f).Gyy(f)}
\end{aligned} \tag{5}$$

Now is possible to formulate a cost function to be minimized, where the residual is the difference between experimental and model estimated FRF´s. A weighting function based on the residual covariance, in a maximum likelihood sense, or on the coherence function could be applied. The cost function assumes the form:

$$J(\Theta) = \sum_{1}^{n} \varepsilon(w_n, \Theta).W.\varepsilon(w_n, \Theta) \tag{6}$$

A coherence cost function weighting is preferable because it represents the fraction of the power that is linearly related to the input power. This removes nonlinearities, secondary input effects and measurement noise.

The frequency response residual is a complex number but, for the cost function formulation, it is represented as a magnitude residual and phase residual. A typical weighting constant is used to compensate those different units. This formulation is extremely convenient because the results can be represented in a Bode plot, where it is possible to visualize all the cost functions components in a post-processing.

An optimization algorithm is applied to search the parameters that minimize the cost function. Once the cost function was formulated using real numbers, magnitude and phase, any common optimization algorithm used in time domain methods is usable.

An initial estimation is necessary and a wind tunnel result probably is enough for aerodynamic estimation. In its absence, a direct search method should be applied prior to a Newton based methods. The latter are more sensible to a rough initial guesses because the minimum proximity is implicit, and it has a possibility not to converge. Other more robust methods like the Levenberg Marquardt can be used as an intermediate search method. If there is no wind tunnel data available to use as reference, care should be taken not to and up having only a good multiple curve fitting but also having a good estimate of the parameters values.

2. Aircraft Dynamic Model

A fixed wing aircraft dynamic model can be split in a longitudinal and a latero-directional uncoupled equations. The linear model is a linearization around to the trim condition – equilibrium – and the states become a perturbated state around the trim value. This approach proved to be well consistent with an Embraer regional jet dynamic analysis, as shown by Mendonça, C. B. et Hoff, J. C. (2003).

In this work the main objective is to investigate the longitudinal dynamic of a typical Embraer regional aircraft. The purpose is to identify a four-order model as stated in the next matrix equation

$$\mathbf{A} = \begin{bmatrix} X_V & X_\alpha & 0 & -g\cos\theta_e \\ Z_V & Z_\alpha & Z_q & \frac{-g}{V_0}\sin\theta_e \\ M_V & M_\alpha & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} \quad \mathbf{u} = [\delta_e] \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-V}{g}Z_V & \frac{-V}{g}Z_\alpha & 0 & 0 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{-V}{g}Z_{\delta_e} \end{bmatrix} \tag{7}$$

The formulation above consider two typical longitudinal modes presented by a common airplane: i) the “short period”, a second order dynamic with higher frequency and high damping coefficient; and ii) the “phugoid”, or “long period”, also a second order dynamic but with lower frequency and lightly damped. For the “short period” a period of 5 seconds and a damping ratio of 0.7 are a good baseline, while for the “phugoid” a period from 20 to 40 seconds and a damping ratio close to zero are expected.

3. Data Analysis

Input data is a primary concern to excite properly the model. The identification process is basically extracting from the flight test data the model characteristics and adjust the related parameters. If only pour information is available regarding any model characteristic, certainly the result will be also pour and inaccurate. Parameter insensitiveness to flight test data is quite common. In this case the test point should be redesigned and repeated or the parameter discarded from the model structure. A Cramer-Rao lower bound limit can be used to identify the insensitivity problem. This number is computed taken the square root of the diagonal terms of Fischer information matrix inverse, as shown in Eq. (8).

$$CR = \sqrt{diag\left[\sum_{i=1}^{N} \frac{\partial y}{\partial \theta} W \frac{\partial y}{\partial \theta}\right]^{-1}} \quad (8)$$

Despite being a “lower bound” Eq.(8) provides a good indication of data information contents. Some heuristic corrections are proposed in the literature to match real parameter scatter, Tischler (1987) propose a factor of “2” for frequency response methods, and that is the value used is this work. Tischler (1987) also considers a Cramer–Rao value inferior of 20 % of the parameter as satisfactory. Morelli (1997) proposed a method to post-process parameter identification results and consider colored residuals information to recalculate a “corrected” Cramer-Rao value. The method was presented considering output error time domain applications, but its results can also be extended to other methods.

A “3-2-1-1” elevator deflection is a common input used for longitudinal dynamic identification purpose. Its frequency contents and energy transmitted to aircraft has higher quality than pulse or doublets inputs. The input is scaled in amplitude not to violate the linear dynamic behavior considered in the model structure. Its duration is determined based on the dominant mode to be excited. The “2” is scaled to match de frequency under interest and the “3” and “1” are scaled proportionally.

A Hanning window was applied to the time history to remove spectra leakage. Windows of 6, 12 and 20 seconds were applied to accomplish the “short period” and the “phugoid” modes. A small window improves higher frequency contents removing scatter through averaging. A larger window allows low frequency identification, because at least one complete period should be included in the time history. A single frequency response mixing all three windows was generated using a CIFER® routine, in such a way that low and high frequency contents were preserved.

Four test points were generated after proper aircraft stabilization in the correct configuration, speed and altitude. All of them were concatenated, after mean and trend removal, as a single source for the analysis.

For aircraft identification purpose a bandwidth from 0 to 2 Hz is quite applicable. Pilots hardly excite the aircraft with frequency higher than that. A 20 Hz sampling rate was adopted and considered adequate to prevent “aliasing” and not to increase computer demand unnecessarily.

4. Results

One of the records used in the identification process is depicted in Fig. (1) to Fig.(5). Figure (1) represents a typical pilot produced “3-2-1-1”elevator input, while Fig.(2) to Fig.(5) are the related aircraft responses.

Table (1) shows all model parameters identified followed by its related corrected Cramer-Rao value. It is noticeable that in general accurate values were obtained, with most of the Cramer-Rao less than 20 %, as desired. The others with higher scatter are the ones related to the “phugoid” mode and indicate less related information contents in the data for that dynamic.

Time slices should have longer record to carry more low frequency information. A twenty-second window was not long enough to allow accurate low frequency mode estimates also. It was expected higher scatter, as a consequence of reduced number of windows, to parameters related to low frequency modes.

A correct approach should be to drop from the model all parameters with high Cramer-Rao, one by one, and run the model again. It was done with the X_v, because of excessively high covariance indicating practically no information in the data to estimate it. This approach was not adopted for others parameters, just to have an initial guess of its values. To allow better results, estimating low frequency parameters, other flight test points with proper time slice record should be analyzed.

DERIVATIVE	VALUE IDENTIFIED	CRAMER-RAO (%)
Z_α	-0.5839	8.014
Z_v	-7.491e-4	128.8
Z_q	0.9200	2.080
$Z_{\delta e}$	0.1512	5.700
M_α	-0.8056	11.62
M_q	-0.9224	8.930
M_v	1.423e-3	81.80
$M_{\delta e}$	-1.216	4.287
X_α	11.17	19.43
X_v	-	-
$X_{\delta e}$	-4.934	29.79

Table 1. Longitudinal dimensional aerodynamic estimates.

In the "phugoid" mode the kinetic and potential energy of the aircraft is interchanged. This effect is observed through speed and altitude behavior. While speed increases altitude decreases, and vice-versa. The speed behavior of one of the records is shown in Fig.(2), and the speed FRF matching in Fig.(8). The coherence function of Fig.(8) gives consistency to the scatter of low frequency parameters. Its value ranged from 0.2 to 0.5, witch is considerably low when compared to other coherence estimates. The coherence function is certainly an important tool for results analysis in frequency response parameter identification methods.

Figure (3) to Fig.(5) compare flight test data and model results. Although parameter identification is not a curve fit itself, those figures depicts a model result compared to one realization of a stochastic event. Even being one of the records used in the matching, what is not indicated to model check, all time histories look very similar to model results.

It is clear that the model fits better in the "short period" dynamic. After around 16 seconds, when the "phugoid" become a dominant mode, the model is less representative, as justified above.

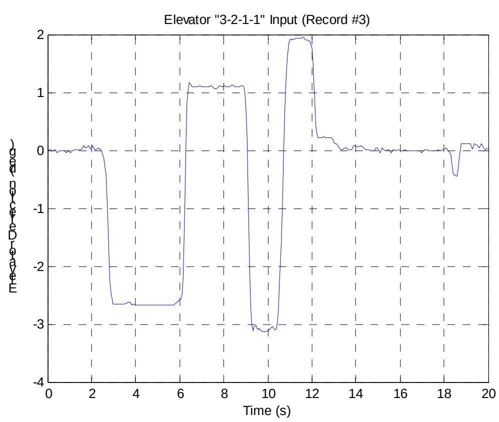


Figure (1) – Elevator input.

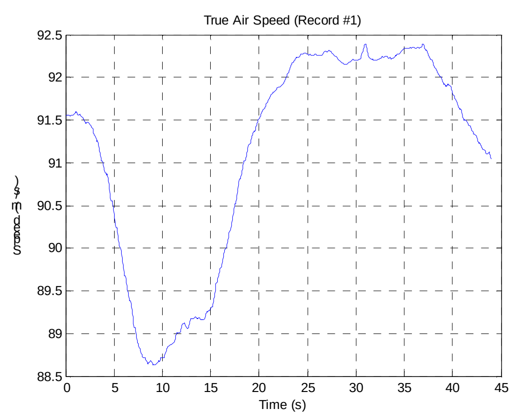
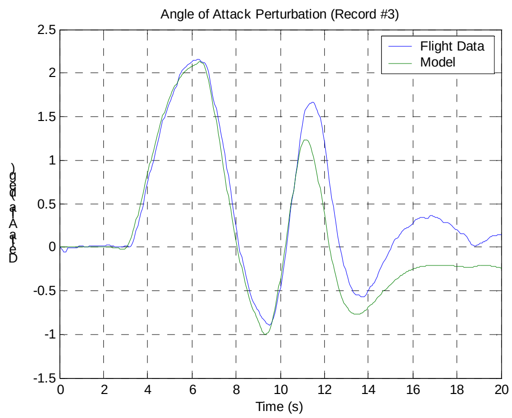


Figure (2) – Aircraft speed response.



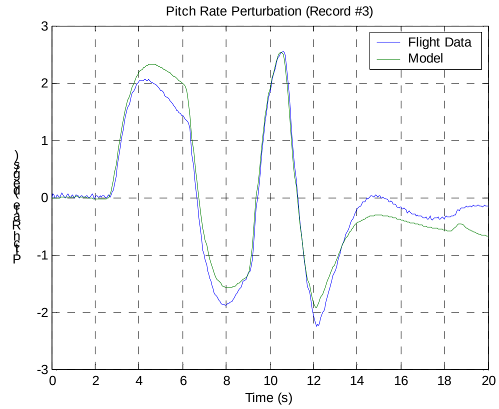


Figure (3) – Model and aircraft angle of attack responses.

Figure (4) – Model and aircraft pitch rate responses.

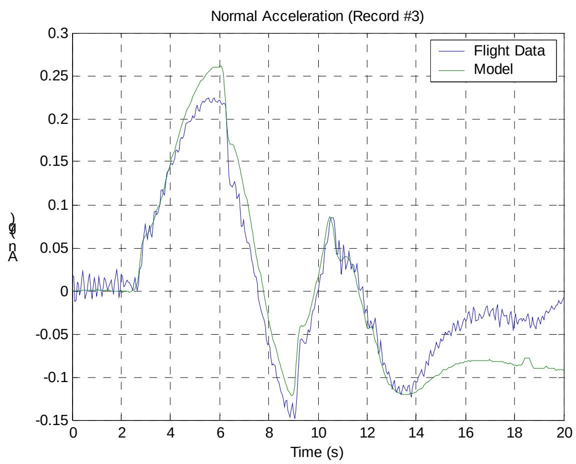
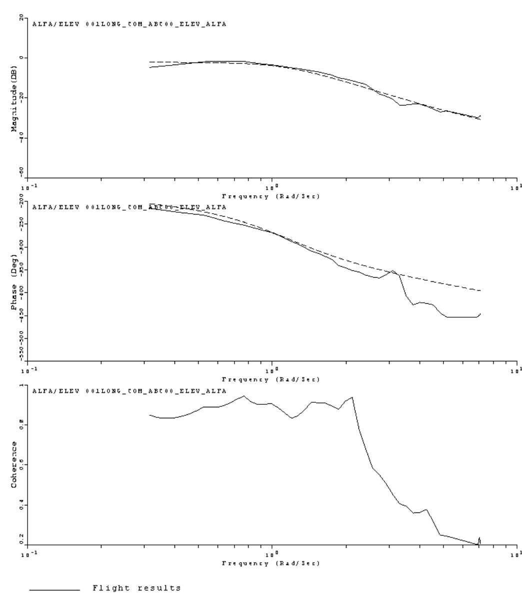


Figure (5) – Model and aircraft acceleration responses.



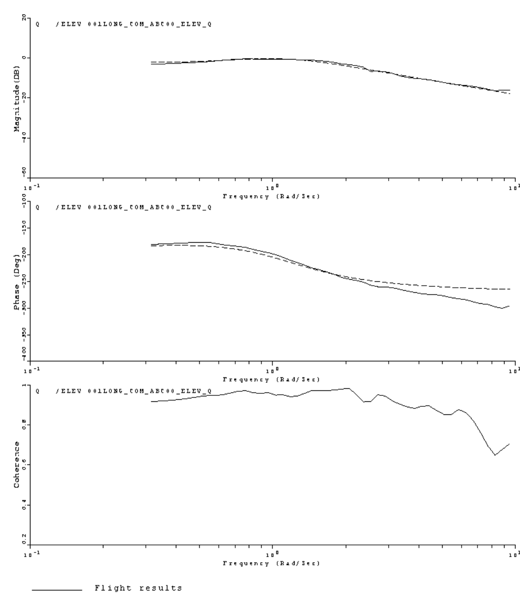


Figure (6) – Angle of attack FRF matching.

Figure (7) – Pitch rate FRF matching.

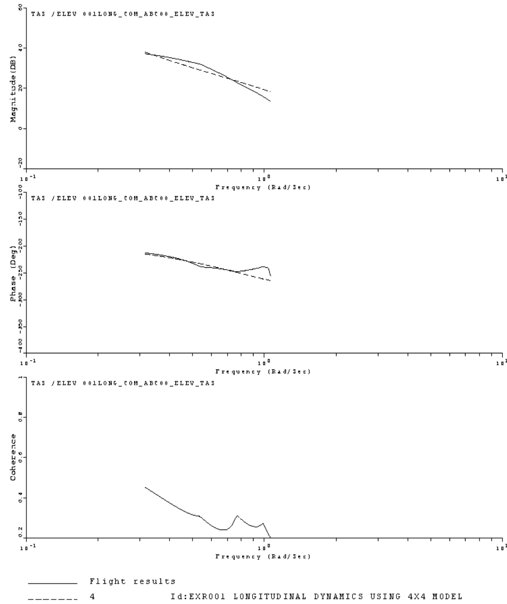


Figure (8) – Speed FRF matching.

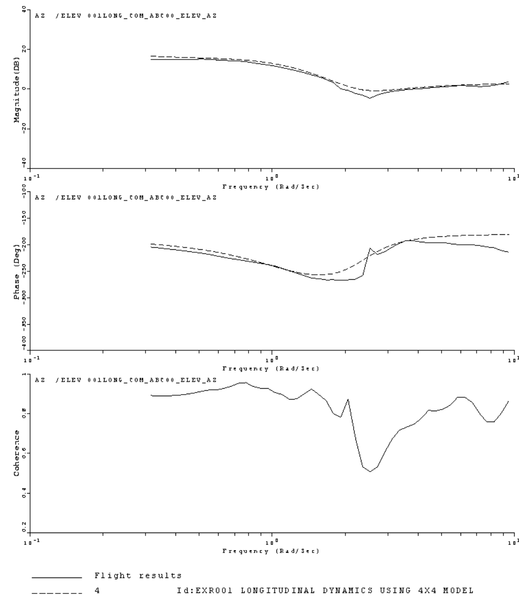


Figure (9) – Acceleration FRF matching.

5. Acknowledgement

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