THE CURZON-AHLBORN EFFICIENCY OF COMBINED CYCLES

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Abstract. *The publication of the paper of Curzon and Ahlborn in 1975 on the thermal efficiency of an endothermic Carnot engine set a revolution on the evaluation methodology of the thermal efficiency of thermal engines and created the roots for an inovation on the second law analysis of thermodynamic processes. This new analytical procedure, the Finite Time Exergy Analysis, emerged as a major break through in Thermodynamics.*
The present work is based on the adaptation of the Curzon-Ahlborn efficiency to the evaluation of combined cycles. Combined cycles in series, with and without intermediate heat losses, will be analysed from a theoretical point of view. Obtained results and conclusions following this new approach, will be compared with classical evaluation methods.

Keywords. *Curzon-Ahlborn efficiency, combined cycles, finite time exergy.*

1. Introduction

The conciliation of the optimum thermal efficiency working conditions with the natural limitations imposed by the reversible heat transfer constraints that are one of the main basis of the Carnot cycle concept lead to the development by Novikov (Bejan, 1994) and lately by Curzon and Ahlborn (1975) of the simple and effective concept of the internally reversible and externally irreversible "Carnot" engine, known afterwards as the endoreversible Carnot engine. Based on this simple idea a new methodology for thermodynamic analysis was developed, initially turning its attention to thermal engines (Leff, 1987; Bejan, 1994; Bejan, 1996). For example, Bejan (1996) analyses models for power plants that generate minimum entropy while operating at maximum power output. This endorreversible engine analysis was further enhanced by the development of new techniques for the evaluation of thermodynamic processes, the endoreversible thermodynamics (Hoffman, et al., 1997), as a non-equilibrium approach in viewing a thermodynamic system as a network of endoreversible systems, exchanging energy in an irreversible fashion. This concept was widened by Sieniutycz and co-workers studies (Sieniutycz, 1998; Sieniutycz and Von Spakovsky, 1998; Sieniutycz 1999; Sieniutycz, 2001; Sieniutycz and Kubiak, 2002). More recent studies covered refrigeration cycles, Agnew et al. (2002), as well as heat pump cycles, Kaushik et. al (2002). Another result of the development of the new concepts around endoreversible thermodynamics was either the refinement of new thermodynamic diagrams already proposed by several authors, or even the elaboration of new approaches (Bucher, 1986; Yan and Chen, 1990; Yan and Chen, 1992; Bucher, 1993; Wallinford, 1999; Branco et al, 2002 and Branco et al., 2002a), just to refer a few ones.

2. Series combined cycle without intermediate heat losses

Considering the series combined cycle where all the rejected heat from the top or high temperature cycle (H) is used by the bottom or low temperature cycle (L) (Horlock, 1992) it would be convenient to find out what is the overall thermal efficiency of the combined cycle when each component cycle is operating at maximum work output conditions.

Both engines of Fig. (1) are endorreversible and the main objective is to find out what is the combined cycle thermal efficiency corresponding to a work maximization situation for each one of them.

According to the scheme, the top cycle receives a heat amount of Q_H from the hight temperature heat reservoir, produces an amount of work W_H and rejects the heat Q_R towards the bottom cycle. As far as this second cycle is concerned, while receiving this last amount of heat, it produces an amount of work W_L rejecting heat Q_L to the external low temperature heat reservoir. As the main objective of this cycle is to maximize the work production, according to the Curzon-Ahlborn analysis as well as basic common sense, finite temperature differences must occur during the heat exchanges among engines and heat reservoirs, as well as between the two engines. For the heat exchange with the high temperature reservoir there is a finite temperature difference $(T_H - T_{HC})$, for the heat exchange between both cycles, the upper cycle is at T_{RH} whereas the bottom cycle is at T_{EL}. However, from the theoretical point of view a hipothetical heat reservoir, at temperature T_{int}, is assumed to exist between these two cycles. For the final heat rejection from the bottom cycle towards the external low temperature reservoir there is a temperature diference of $(T_{LC} - T_L)$. On Fig. (1) there is also a *T-s* diagram with the relative positions of all the involved heat reservoirs.

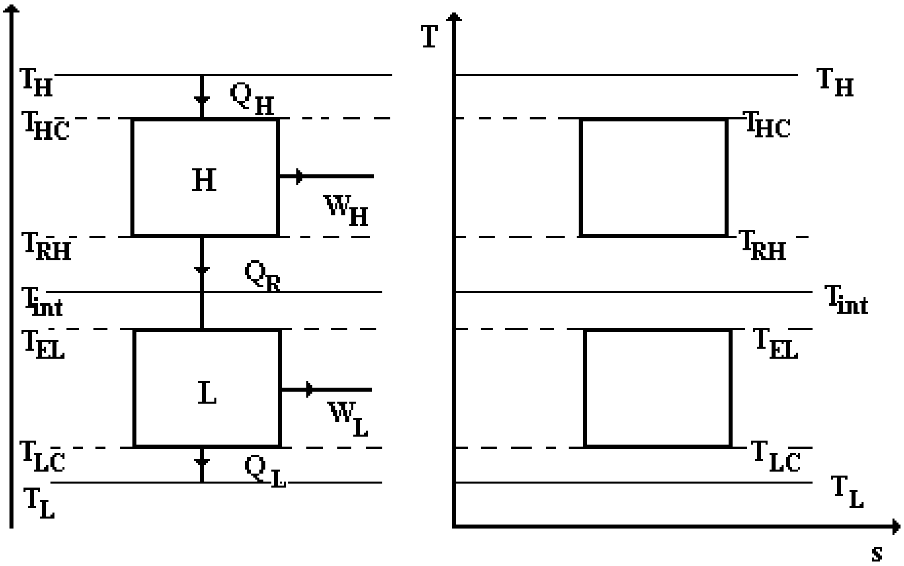

Figure 1. Curzon-Ahlborn combined cycles.

Applying the first law of thermodynamics to the top and bottom cycles as well as the definitions of the corresponding thermal efficiencies,

$$\eta_H = \frac{W_H}{Q_H} \tag{1}$$

and

$$\eta_L = \frac{W_L}{Q_R} \tag{2}$$

the deduction of the global thermal efficiency for the series combined cycle gives (Horlock, 1992),

$$\eta_{CC} = \eta_H + \eta_L - \eta_H \eta_L \tag{3}$$

As previously referred the objective of the present analysis is to study the possibility of maximizing the output work from each one of the two cycles composing the combined cycle. The original analysis for an single endothermic Carnot cycle working between two heat reservoirs (at T_H and T_L respectively), was carried out by Curzon and Ahlborn (1975), who showed that the cycle thermal efficiency for a work output maximization situation is given by $\eta_{C-A} = 1 - \sqrt{T_L / T_H}$.

In the adopted symbology when an asterisk is used it means that the energies are for the situation of maximization of the work output. So for the top cycle at the work maximization conditions,

$$\frac{W_H^*}{Q_H^*} = 1 - \sqrt{\frac{T_{\text{int}}}{T_H}} = \eta_{C-A,H} \tag{4}$$

while for the bottom cycle,

$$\frac{W_L^*}{Q_R^*} = 1 - \sqrt{\frac{T_L}{T_{\text{int}}}} = \eta_{C-A,L} \tag{5}$$

As, $Q_{RH}^* = Q_{EL}^*$ then,

$$W_H^* = \eta_{C-A,H} Q_H^* \tag{6}$$

and

$$W_L^* = \eta_{C-A,L}\left(1 - \eta_{C-A,H}\right)Q_H^* \tag{7}$$

Defining now the combined cycle efficiency at maximum work output as,

$$\eta_{CC}^* = \frac{W_H^* + W_L^*}{Q_H^*} \tag{8}$$

the conclusion is that,

$$\eta_{CC}^* = \eta_{C-A,H} + \eta_{C-A,L} - \eta_{C-A,H}\eta_{C-A,L} \tag{9}$$

which is an expression equal to that corresponding to the conventional thermal efficiency for the series combined cycle.

Replacing now the Curzon-Ahlborn efficiencies for both the component cycles in the above expression,

$$\eta_{CC}^* = \left(1 - \sqrt{\frac{T_{\text{int}}}{T_H}}\right) + \left(1 - \sqrt{\frac{T_L}{T_{\text{int}}}}\right)\sqrt{\frac{T_{\text{int}}}{T_H}} \tag{10}$$

the final result after a proper simplification is,

$$\eta_{C-A,CC} = \eta_{CC}^* = 1 - \sqrt{\frac{T_L}{T_H}} \tag{11}$$

The Curzon-Ahlborn efficiency for the series combined cycle will be then independent of the reference intermediate temperature for the heat exchange between both engines. This result makes sense as the Carnot efficiency for the case of series combined Carnot cycles would be,

$$\eta_{C,CC} = 1 - \frac{T_L}{T_H} \tag{12}$$

There is only a dependence upon the high and low heat source temperatures between which the series combined cycle is working. Once again, similarly to the Carnot cycle thermal efficiency, the Curzon-Ahlborn is also a reference or standard against which all series combined cycle efficiencies may be compared. There is however an efficiency reduction when moving from the combination of the series Carnot combined cycle towards the maximization of the work output,

$$\Delta\eta_{CC} = \eta_{C,CC} - \eta_{C-A,CC} = \sqrt{\frac{T_L}{T_H}} - \frac{T_L}{T_H} \tag{13}$$

and introducing the definition of the temperature ratio $\tau = {T_L}/{T_H}$, Leff (1987),

$$\Delta\eta_{CC} = \sqrt{\tau} - \tau \tag{14}$$

This efficiency reduction is shown by the upper line in Fig. (3), as a function of the heat sources temperature ratio. It is clearly that the work maximization for a series combined cycle leads also to a thermal efficiency decrease compared to the ideal Carnot situation.

Through the calculation of $\frac{\partial\,\Delta\eta_{CCS}}{\partial\,\tau} = 0$ it is found that when $\tau = 0.25$ the thermal efficiency penalization is maximized and reaches the value of 25 %, as it is evident from the analysis of same curve on Fig. (3).

3. Series combined cycles with intermediate heat losses

A closer situation to the daily practice is when there are heat losses during the heat exchange between the two cycles in series. Such situation will penalize the overall cycle behaviour. Schematically, Fig. (2) is quite clear.

Again and according to the definition of thermal efficiency for the combined cycle,

$$\eta_{CCSL} = \frac{W_H + W_L}{Q_H} \tag{15}$$

while for each of the component cycles,

$$\eta_H = \frac{W_H}{Q_H} \qquad \text{and} \qquad \eta_L = \frac{W_L}{Q_{EL}} \tag{16}$$

Through the application of the first law of thermodynamics to the top cycle, and to the heat transfer between both cycles, and defining the parameter,

$$\nu_{UN} = \frac{Q_{UN}}{Q_H} \tag{17}$$

the deduction of the thermal efficiency for the series combined cycle with intermediate heat losses gives (Horlock, 1992),

$$\eta_{CCSL} = \eta_H + \eta_L - \eta_H \eta_L - \nu_{UN} \eta_L \tag{18}$$

This expression can be compared with the corresponding equation for the thermal efficiency of the series combined cycle without losses during the intermediate heat exchange, and it is immediately realized that the efficiency penalty of intermediate heat losses upon the combined cycle overall thermal efficiency is the last term on the left hand side of Eq. (18) $\nu_{UN}\eta_L$. In other words, when there are not heat losses, $\nu_{UN} = 0$ and Eq. (18) $\rightarrow$ Eq. (3).

It would again be convenient to find out the thermal efficiency of the combined cycle for a output work maximization and taking into account this intermediate heat loss, Fig. (2). Remembering that for maximum output work the asterisk superscript is used in all involved energy forms.

For the top cycle it can be written that,

$$W_H^* = \left(1 - \sqrt{\frac{T_{int}}{T_H}}\right) Q_H^* \tag{19}$$

and for the bottom cycle,

$$W_L^* = \left(1 - \sqrt{\frac{T_L}{T_{int}}}\right) Q_{EL}^* \tag{20}$$

where,

$$Q_{RH}^* = Q_{UN}^* + Q_{EL}^* \tag{21}$$

then,

$$W_L^* = \eta_{C-A,L}\left[\left(1 - \eta_{C-A,H}\right) - \nu_{UN}\right] Q_H^* \tag{22}$$

But as,

$$\eta_{CCSL}^* = \frac{W_H^* + W_L^*}{Q_H^*} \tag{23}$$

$$\eta_{CCSL}^* = \eta_{C-A,H} + \eta_{C-A,L}\left[\left(1 - \eta_{C-A,H}\right) - \nu_{UN}\right] \tag{24}$$

which is an expression quite similar to the previous one for the general situation.

Replacing $\eta_{C-A,H}$ and $\eta_{C-A,L}$ by their definitions

$$\eta_{C-A,H} = 1 - \sqrt{\frac{T_{\text{int}}}{T_H}} \text{ and } \eta_{C-A,L} = 1 - \sqrt{\frac{T_L}{T_{\text{int}}}} \tag{25}$$

the result is,

$$\eta_{CCSL}{}^{*} = 1 - \sqrt{\frac{T_L}{T_H}} - \nu_{UN}\left(1 - \sqrt{\frac{T_L}{T_{\text{int}}}}\right) \tag{26}$$

and considering the Curzon-Ahlborn efficiency for the series combined cycle without intermediate heat losses

$$\eta_{CCSL}{}^{*} = \eta_{C-A,CCS} - \nu_{UN}\left(1 - \sqrt{\frac{T_L}{T_{\text{int}}}}\right) \tag{27}$$

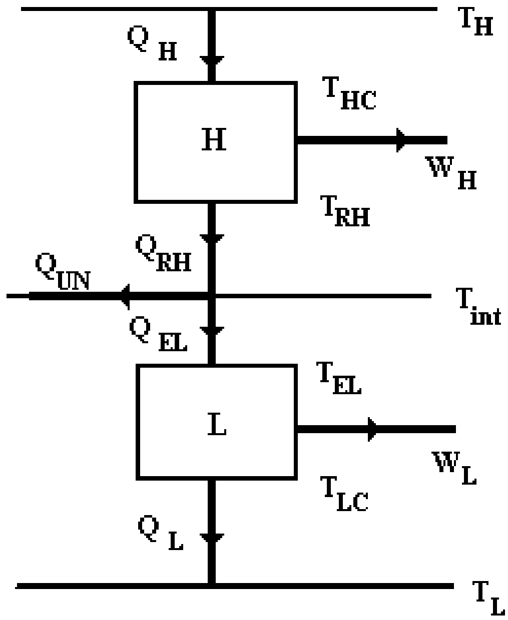

Figure 2. Series combined cycle with intermediate heat losses. Maximization of work output.

In this case it is clear that there is a global thermal efficiency penalization dependent upon the heat losss ratio ν_{UN}, the intermediate temperature T_{int} and the temperature of the cold reservoir T_L and that there is not a simple overall thermal Curzon-Ahlborn efficiency equation as found in the previously analysed situation.

Considering now the possibility of using a series combined cycle with intermediate heat losses but using Carnot cycles,

$$\eta_{C,CCSL} = \eta_{C,H} + \eta_{C,L}\left[\left(1 - \eta_{C,H}\right) - \nu_{UN}\right] \tag{28}$$

where,

$$\eta_{C,H} = 1 - \frac{T_{\text{int}}}{T_H} \text{ and } \eta_{C,L} = 1 - \frac{T_L}{T_{\text{int}}} \tag{29}$$

and evaluating the thermal efficiency reduction when going from a combination of Carnot cycles towards the combination of Curzon-Ahlborn cycles,

$$\Delta\eta_{CCSL} = \sqrt{\tau} - \tau + \nu_{UN}\left(\tau_{\text{int}} - \sqrt{\tau_{\text{int}}}\right) \tag{30}$$

where besides the previously defined temperature ratio τ a second temperature ratio τ_{int} is used,

$$\tau_{\text{int}} = \frac{T_L}{T_{\text{int}}} \tag{31}$$

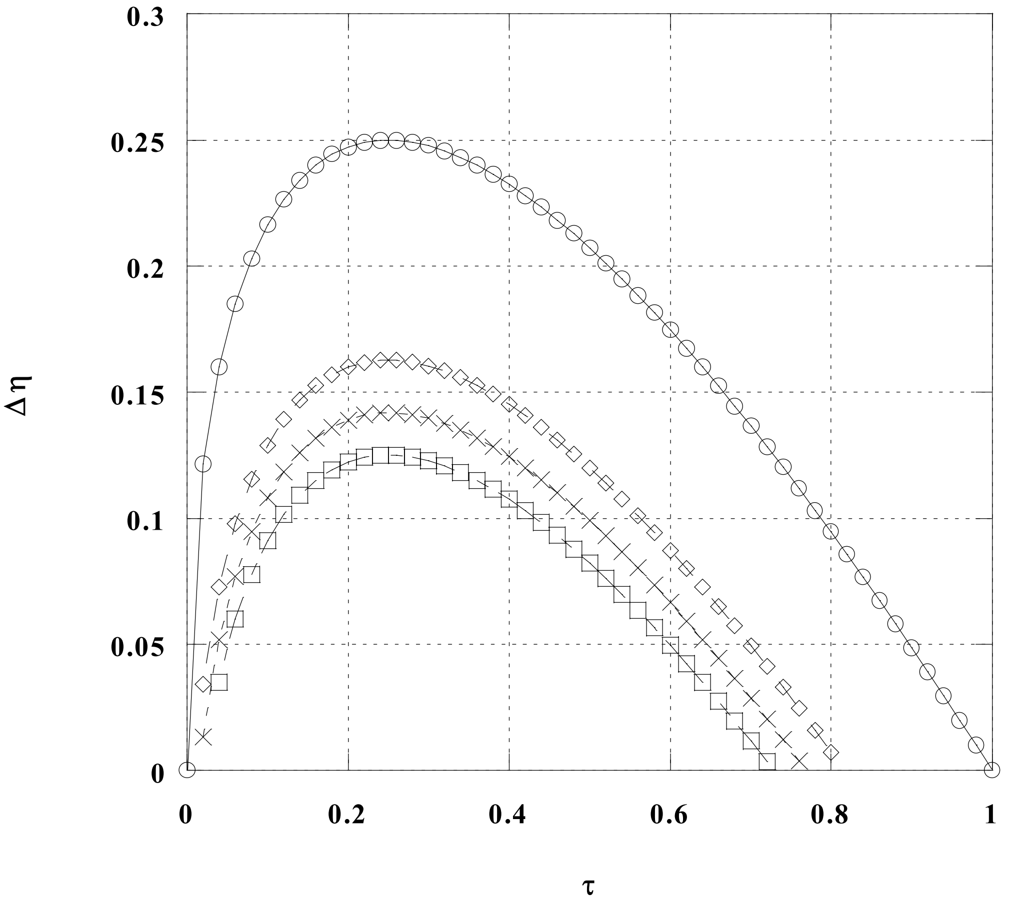

Figure 3. Reduction of the series combined cycle thermal efficiency for the maximization of work output. Symbol O refers to Eq. (14), Other symbols refer to Eq. (30): □ is for $\tau_{\text{int}} = 0.6$, × for $\tau_{\text{int}} = 0.1$ and □ for $\tau_{\text{int}} = 0.25$.

Looking for the maximization of the thermal efficency degradation through the calculation of $\frac{\partial\ \Delta\eta_{CCSL}}{\partial\ \tau} = 0$ and $\frac{\partial\ \Delta\eta_{CCSL}}{\partial\ \tau_{\text{int}}} = 0$ the result is, $\tau = \tau_{\text{int}} = 0.25$, see Fig. (3), whereas if it is done $\frac{\partial\ \Delta\eta_{CCSL}}{\partial\ \nu_{UN}} = 0$ the result is that $\tau_{\text{int}} = 1 \Leftrightarrow T_{\text{int}} = T_L$, i. e.,

$$\Delta\eta_{CCS} = \eta_{CCSL} \tag{32}$$

On Fig. (3) besides the plot of Eq. (14) , Eq. (30) is also shown but for three different values of the temperature ratio τ_{int} . For all situations the maximization of efficiency degradation happens for $\tau = 0.25$. The worst case takes place, as said before, when $\tau = \tau_{\text{int}} = 0.25$.

4. Conclusions

Through a simple approach it was shown how the Curzon-Ahlborn efficiency concept can be applied to combined cycles and what is the thermal efficiency penalization that is obtained when the maximization of the combined cycle work output is looked for in detriment of the theoretical combination of Carnot cycles.

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