# Analysis of composite sleeve reinforcement systems for metallic pipelines with localized imperfections or damage

Heraldo Silva da Costa Mattos
*Laboratório de Mecânica Teórica e Aplicada, Departamento de Engenharia Mecânica, Universidade Federal Fluminense, RJ – Brazil*
Eduardo Martins Sampaio
*Instituto Politécnico, Universidade do Estado do Rio de Janeiro, RJ – Brazil*
Rodrigo Mendes Alves Cortes
*Centro de Tecnologia em Dutos - CTDUT, RJ – Brazil*

## Abstract

The present work is concerned with the analysis of composite sleeve reinforcement systems for metallic pipelines undergoing elastic or inelastic deformations with localized imperfections or damage that impair the serviceability. In these systems, a piping or vessel segment is reinforced by wrapping it with concentric coils of composite material. The main goal is to develop a methodology as simple as possible to define the necessary thickness of composite material to assure: (a) the safety of repairs under operation conditions and/or (b) the lifetime extension under operation conditions. Such methodology, although simple, is able to account for different failure mechanisms (plasticity, fatigue, fracture). A computer program, DRE20002, has been developed to automatically evaluate the minimum thickness of reinforcement to be used for different imperfections or damage that impair the serviceability of a segment of steel transmission line. It is suggested the use of the composite sleeve as repair system itself (mainly to avoid or to restrain the propagation of internal flaws) but also as a complementary procedure to enhance the reliability of weldments, eliminating the necessity of heat treatment (in the welding operation there is always a possibility of metallurgical changes in the parent metal in the vicinity of the weld). Examples concerning the use of composite reinforcement in different damage situations are presented and analyzed showing the possibilities of practical use of the proposed methodology in the design of safer and more reliable repair systems.

Keywords: composite sleeve reinforcement systems, metallic pipelines, elasto-plasticity

Mechanics of Solids in Brazil 2007, Marcílio Alves & H.S. da Costa Mattos (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-30-7

## Nomenclature

| | |
|---|---|
| $r_i$ | inner radius of the internal cylinder (pipe) |
| $r_o$ | external radius of the internal cylinder and internal radius of the composite sleeve |
| $r_e$ | external radius of the composite sleeve |
| $P_i$ | internal pressure applied to the system |
| $P_c$ | contact pressure between the pipe and the composite sleeve |
| $\sigma_r$ | radial stress component |
| $\sigma_\theta$ | tangential (hoop) stress component |
| $u_r$ | radial displacement in sleeve |
| $E_\theta$ | extensional modulus in the tangential direction |
| $E_r$ | extensional modulus in the radial direction |
| $\upsilon_{r\theta}$ | coefficient relating contraction in the circumferential direction to extension in the radial direction |
| E | Young modulus of the pipe material |
| $\sigma_y$ | yield stress of the pipe material |
| K | coefficient of plastic resistance of the pipe material |
| N | hardening exponent of the pipe material / life of the pipe without reinforcement |
| $\sigma_{Max}$ | given maximum hoop stress criterion in the pipe |
| $(P_c)_{Min}$ | limit contact pressure between the pipe and the sleeve |
| $(r_e)_{Min}$ | limit external radius of the composite sleeve |
| $\widehat{u}$ | radial displacement of the external surface of the pipe due to the internal pressure $P_{apl}$ to which the pipe was submitted when the reinforcement was applied |
| S | endurance strength related to fatigue analysis |
| $N_F$ | number of cycles until a macro-crack initiation in the pipe structure after repeated cycles of pressure |
| $N_R$ | total life of the pipe |
| $S_e$, a, b, $\eta$ | pipe parameters that depend on the environmental conditions, material, geometry and surface finish |
| $S_e$ | endurance limit of the material of the pipe |
| $S_u$ | ultimate tensile strength of the material of the pipe |
| $K_I$ | stress intensity factor |
| $a_o$ | initial length of the semi-elliptical crack |
| $a_c$ | critical length of the semi-elliptical crack in the pipe without reinforcement |
| C,m | pipe material parameters |
| $\bar{a}_c$ | critical length of the semi-elliptical crack in the pipe with reinforcement |
| $\bar{N}$ | life of the pipe with reinforcement |

Mechanics of Solids in Brazil 2007, Marcílio Alves & H.S. da Costa Mattos (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-30-7

## 1 Introduction

The present work is concerned with the analysis of composite sleeve reinforcement systems for metallic pipelines undergoing elastic or inelastic deformations. In these systems, the piping segment is reinforced by wrapping it with concentric coils of composite material. Such technique has been studied in the last 20 years and aims to provide a safe and effective alternative to more costly methods for repairing damaged pipeline.

Only recently the technique was recognized by the Research and Special Programs Administration (RSPA) of the US Department of Transportation (DOT) as effective to permanently restore the serviceability of the pipeline. Conventional methods of repair include one of two ways: (a) cutting out the damaged segment of pipe and replacing it by welding in a new piece of pipe or (b) covering the pipe with a metal welded sleeve over the damaged area. Different commercial repair systems based in fiber reinforced composite materials can be found: (a) dry fiberglass fabric to be wrapped with impregnation of liquid resin, (b) ready pre-cured layers ready to wrap around the pipe, (c) Flexible resin pre-impregnated bandage to be wrapped with water. No matter the application procedure, the basic idea of the reinforcement technique is to transfer the hoop stress in the pipe wall due to the internal pressure to the composite sleeve. Most of the studies about these systems are concerned with the materials (matrix, fibers, adhesive) and application procedures.

Only a few studies are concerned with the mechanical analysis of the repair system. These studies are generally restricted to particular problems (geometry, material, etc) and are very complex, requiring hours of finite element computation for each different imperfection or damage that impairs the serviceability of a segment of steel transmission line. There are a few studies trying to propose simplified models for the mechanical analysis of the repair system, but they are excessively simple, and do not account for the different kind of possible defects (wall loss, cracks, localized corrosion, dents, etc.), for the non monotone nature of the pressure history and neither for the coupling between the mechanisms involved (plasticity, fatigue, corrosion, etc.). Most of the simplified models are based on ASME B31G-1991, "Manual for Determining the Remaining Strength of Corroded Pipelines". The main goal of this paper is to propose a methodology as simple as possible to define the necessary thickness of composite material to assure: (a) the safety of repairs under operation conditions and/or (b) the lifetime extension under operation conditions for different damage situations. Such methodology is sophisticated enough to overcome the limitations of the simpler models found in the literature. A more detailed discussion can be found in [1, 2].

## 2 Basic model – pipe without localized damage

In this first step towards a simplified method of analysis of composite sleeve reinforcement systems, no localized imperfections or damage are considered. The pipe-composite sleeve system is modeled as two concentric cylinders, open at the extremities, under internal pressure – an internal thin-walled with elastic-plastic behavior and a sleeve with orthotropic elastic behavior. The internal cylinder has an inner radius $\mathrm{r}_i$ and external radius $\mathrm{r}_0$. The cylinder can be considered thin-walled if the wall thickness is less than about 1/10 of the internal radius ($(\mathrm{r}_0\text{-}\mathrm{r}_i) < \mathrm{r}_i/10$). The sleeve has an internal radius $\mathrm{r}_0$ and

Mechanics of Solids in Brazil 2007, Marcílio Alves & H.S. da Costa Mattos (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-30-7

external radius $r_e$. The system is subjected to an internal pressure $P_i$ as shown in Fig. 1. The contact pressure between the pipe and the sleeve will be noted $P_c$. Assuming that the radial displacement in the contact surface is the same for both cylinders, it is possible to obtain analytical expressions for the stress, strain and displacement fields. With this expressions, it can be obtained the minimum composite sleeve thickness in order to verify a given safety criterion. Different failure mechanisms (plasticity, fatigue, burst rupture) can be considered.

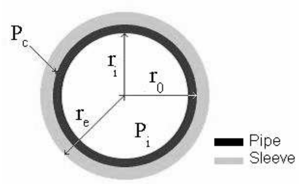


Figure 1: Pipe and sleeve with internal pressure

Generally unidirectional glass reinforced epoxy is used for the sleeve (epoxy resin is the matrix and the reinforcement is glass fiber). Neglecting a decrease in time of the polymer composite strenght due to the environment, analitycal expressions for the radial stress component $\sigma_r$, the tangential (hoop) stress component $\sigma_\theta$ and the radial displacement $u_r$ in sleeve are obtained in [1, 2] and are given by

$$\sigma_\mathrm{r} = B\left(1-\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\right)r^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)} + C\left(1+\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\right)r^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)} \tag{1}$$

$$\sigma_\theta = -B\left(1-\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\right)\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}r^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)} + C\left(1+\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\right)\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}r^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)} \tag{2}$$

$$\mathrm{u_r} = -\mathrm{B}\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\mathrm{r}^{-\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}}\left(\frac{1}{\mathrm{E}_\theta}+\frac{\nu_{\mathrm{r}\theta}}{\mathrm{E_r}}\right) + \mathrm{C}\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\mathrm{r}^{\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}}\left(\frac{1}{\mathrm{E}_\theta}-\frac{\nu_{\mathrm{r}\theta}}{\mathrm{E_r}}\right) \tag{3}$$

With

$$\begin{aligned} B &= \frac{-P_C r_e^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)}}{r_0^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)} r_e^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)} - r_0^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)} r_e^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)}}; \\ C &= \frac{-P_C r_e^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)}}{r_0^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)} r_e^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)} - r_0^{-\left(\sqrt{\frac{E_\theta}{E_r}}+1\right)} r_e^{\left(\sqrt{\frac{E_\theta}{E_r}}-1\right)}} \end{aligned} \tag{4}$$

where $P_c$ is the contact pressure between the sleeve and pipe, $E_\theta$ the extensional modulus in the tangential direction and $E_r$ the extensional modulus in the radial direction and $\nu_{r\theta}$ the coefficient relating contraction in the circumferential direction to extension in the radial direction. The stress components and the radial displacement in the sleeve are functions of the contact pressure $\mathrm{P}_c$ which is not known “a priori”. If the wall of the pipe is thin, it can be shown that the stress component $\sigma_\theta$ and the radial displacement $\mathrm{u}_r$ for the pipe are approximated by the following expressions [2]

$$\sigma_\theta = \frac{P_i r_i - P_C r_0}{r_0 - r_i} \tag{5}$$

$$u_r = r\left[\frac{\sigma_\theta}{E} + \left\langle \frac{\sigma_\theta - \sigma_y}{K} \right\rangle^{\frac{1}{N}}\right] \tag{6}$$

where E is the Young modulus of the pipe material, $\sigma_y$ the yield stress, K and N are material parameters that characterize the plastic behavior of the material. K is the coefficient of plastic resistance and N is the hardening exponent. The angular brackets have the following meaning: $\langle x\rangle = \max\{0,x\}$. The term $\sigma_\theta/E$ corresponds to the elastic deformation and the term $\langle(\sigma_\theta - \sigma_y)/K\rangle^{1/N}$ to the plastic deformation in the pipe.

Generally, from the practical point of view, it is important to define the sleeve thickness in order to assure a given maximum hoop stress criterion in the pipe

$$\sigma_\theta < \sigma_{\mathrm{Max}} \text{ in the pipe} \tag{7}$$

In this case, the limit contact pressure $(P_C)_{\mathrm{Min}}$ is obtained from the condition $\sigma_\theta = \sigma_{\mathrm{Max}}$

$$\sigma_\theta = \frac{\mathrm{P_i r_i} - (\mathrm{P_C})_{Min}\mathrm{r_0}}{\mathrm{r_0 - r_i}} = \sigma_{\mathrm{Max}} \Rightarrow (\mathrm{P_C})_{Min} = \frac{\mathrm{P_i r_i} - \sigma_{\mathrm{Max}}(\mathrm{r_0 - r_i})}{\mathrm{r_0}} \tag{8}$$

And the limit external radius $(\mathrm{r}_e)_{Min}$ may be obtained from the condition that the radial displacement of the contact surface between pipe and sleeve must be the same

$$[\mathrm{u_r}(\mathrm{r}=\mathrm{r_0})]_{\mathrm{pipe}} - \widehat{u} = [\mathrm{u_r}(\mathrm{r}=\mathrm{r_0})]_{\mathrm{sleeve}} \tag{9}$$

where $\widehat{u}$ is the radial displacement of the external surface of the pipe due to the internal pressure $\mathrm{P}_{apl}$ the pipe was submitted to when the reinforcement was applied

$$\widehat{u} = r_0\left[\frac{\hat{\sigma}_\theta}{E} + \left\langle \frac{\hat{\sigma}_\theta - \sigma_{\mathrm{y}}}{K} \right\rangle^{\frac{1}{N}}\right] \text{ with } \hat{\sigma}_\theta = \frac{P_{apl} r_i}{r_0 - r_i} \tag{10}$$

Or, in other words, the limit external radius $(\mathrm{r}_e)_{Min}$ is the root of the function $\Phi$, given by

$$\Phi(\mathrm{r_e}) = \overbrace{\mathrm{r_0}\left[\frac{\sigma_{\mathrm{Max}}}{\mathrm{E}} + \left\langle \frac{\sigma_{\mathrm{Max}} - \sigma_{\mathrm{y}}}{\mathrm{K}} \right\rangle^{\frac{1}{\mathrm{N}}}\right]}^{[\mathrm{u_r(r=r_0)}]_{\mathrm{pipe}}} - \widehat{\mathrm{u}} - \overbrace{\left[-\mathrm{B}(\mathrm{r_e})\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\mathrm{r_0}^{-\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}}\left(\frac{1}{\mathrm{E}_\theta} + \frac{\nu_{\mathrm{r}\theta}}{\mathrm{E_r}}\right) + \mathrm{C}(\mathrm{r_e})\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}\mathrm{r_0}^{\sqrt{\frac{\mathrm{E}_\theta}{\mathrm{E_r}}}}\left(\frac{1}{\mathrm{E}_\theta} - \frac{\nu_{\mathrm{r}\theta}}{\mathrm{E_r}}\right)\right]}^{[\mathrm{u_r(r=r_0)}]_{\mathrm{sleeve}}} = 0 \tag{11}$$

It is important to remark that the sleeve composite is supposed to be stronger than the pipe itself and it will not fail in operation.


Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-30-7

The internal pressure $P_{apl}$ in the pipe when the sleeve is applied is one of the most important variables in the application of reinforcement systems. The wrong choice of this pressure may result in ineffective reinforcements as it can be seen in Fig. 2 ($P_i$ = 5 MPa, $r_i$ = 240 mm, $r_e$= 250 mm, $\sigma_y$= 133 MPa, E = 200000 MPa, K = 435 MPa, N = 0.22, $E_\theta$=34400 MPa, $E_r$=9600 MPa, $\nu_{r\theta}$= 0.3 and $\sigma_{\text{Max}}$= 146.3 MPa). If $P_{apl}$is closer to $P_i$, the reinforcement must be very thick and will only share hoop stresses with the sleeve when a pressure surge above the value $P_i$ occurs. Most of commercial repair systems recognize that reducing pressure during repair is a good practice but this pressure reduction is not quantified and is not a mandatory requirement.

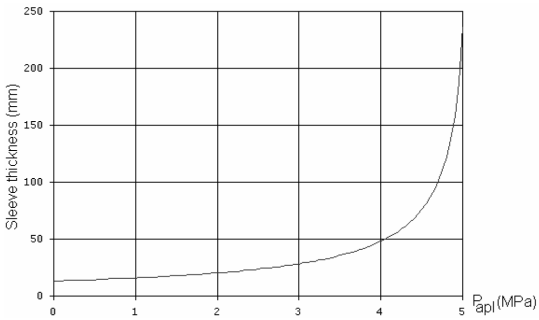


Figure 2: Influence of the internal pressure $P_{apl}$ in the sleeve thickness to assure a maximum hoop stress $\sigma_{\text{Max}}$= 146.3 MPa in the pipe. $P_i$ = 5MPa.

Figure 3 shows the influence of $\sigma_{\text{Max}}$ in the limit sleeve thickness for $P_{apl}$= 5 MPa and $P_0$= 7.2 MPa.

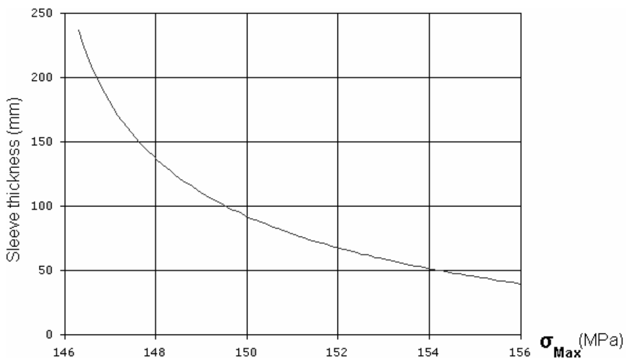


Figure 3: Influence of $\sigma_{\text{Max}}$in the limit sleeve thickness for $P_{apl}$= 5 MPa and $P_0$= 7.2 MPa.

## 3 Possible choices of $\sigma_{\text{Max}}$

The choice of $\sigma_{\text{Max}}$ is very important in order to define the role of the sleeve reinforcement. The goal of the present section is to present different possible choices of $\sigma_{\text{Max}}$ depending on the integrity criteria the pipe must verify in operation.

### 3.1 Von Mises criterion

This is the most obvious criterion. The pipe won't be submitted to permanent deformation provided the hoop stress is smaller than the yield stress

$$\sigma_\theta < \sigma_y \Rightarrow \sigma_{Max} = \sigma_y \tag{12}$$

### 3.2 High cycle fatigue criterion

In this paper we will only consider high cycle fatigue due to repeated cycles of pressure as shown if Fig. 4. Variable amplitude pressure histories and cumulative damage rules won't be discussed.

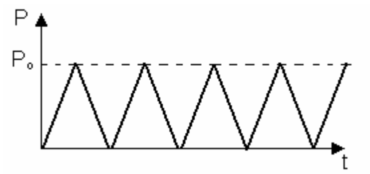


Figure 4: Repeated cycles of pressure

Experimentally, the tipical behavior of the curve S-$N_F$ shown in Fig. 5 is observed. Where S, called the endurance strenght, is defined in (13) and $N_F$is the number of cycles until a macrocrack initiation. Generally, for a metallic structure, a macro-crack appears in the last 10% to 20% of the total life $N_R$. Since the error in life previsions is very big, due to modeling limitations and the large dispersion of experimental results, it is reasonable to consider $N_R \approx N_F$.

$$S = \frac{\sigma_\theta^{\text{a}}}{1 - \left(\frac{\sigma_\theta^{\text{m}}}{\text{Su}}\right)} \quad \text{with} \quad \begin{array}{l} \sigma_\theta^{\text{a}} = (_{\max t}\,\sigma_\theta + {}_{\min t}\,\sigma_\theta)/2 = 1/2\sigma_\theta(P = P_o) \\ \sigma_\theta^{\text{m}} = (_{\max t}\,\sigma_\theta + {}_{\min t}\,\sigma_\theta)/2 = 1/2\sigma_\theta(P = P_o) \end{array} \tag{13}$$

The value of the parameters $S_e$, a, b, and $\eta$ depend on the environmental conditions, material, geometry and surface finish. In general $0.7 \leq \eta \leq 0.9$, a $\leq 3$ and b $\geq 6$. In the lack of specific experimental results it is suggested to consider a = 3, b = 6 and $\eta = 0.7$. $S_e$ is the endurance limit. In the lack of specific experimental results it is suggested to consider the following conservative value:

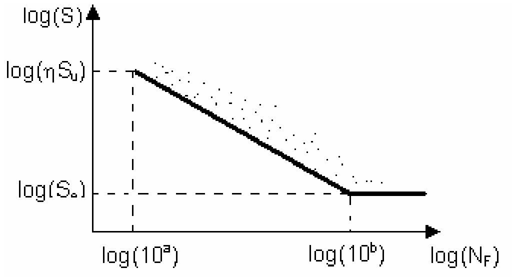


Figure 5: Typical S-N$_F$ curve.

S$_e$ = 0.25 S$_u$(S$_u$ is the ultimate tensile strength. In a brief and simplified way, from the curve in Fig. 3, it is possible to derive [1, 2] the following fatigue criteria

$$\text{"infinite" life } \Rightarrow \mathrm{S} < Se \tag{14}$$

$$\text{life of } \mathrm{N_F} \text{ cycles } (10^{\mathrm{a}} \leq \mathrm{N_F} \leq 10^{b}) \Rightarrow \mathrm{S} < Sn$$

$$\text{with } Sn = 10^{\beta}(\mathrm{N_F})^{\alpha} \text{ and } \alpha = \frac{-1}{(b-a)}\log\left(\frac{\eta Su}{S_e}\right), \beta = \log\left[\left(\frac{(\eta S_u)^2}{S_e}\right)^{\frac{a}{b-a}}\right] \tag{15}$$

Hence, from the definition of S and Eq. (14), (15), it is possible to verify that

$$\text{"infinite" life } \Rightarrow \sigma_{\mathrm{Max}} = 2\left[\frac{SeSu}{Se+Su}\right] \tag{16}$$

$$\text{life of at least } \mathrm{N_F} \text{ cycles } (10^{\mathrm{a}} \leq \mathrm{N_F} \leq 10^{b}) \Rightarrow \sigma_{\mathrm{Max}} = 2\left[\frac{SnSu}{Sn+Su}\right] \tag{17}$$

## 4 Accounting for a localized damage

The expressions presented up to now are valid only if there are no localized imperfections or damage in the pipe section. In this section, a simple procedure to account for a localized damage is proposed. The basic Idea is to suppose the hoop stress far from a localized imperfection is approximately the same than the verified in an undamaged cylinder under pressure. Since the internal radius is much bigger than the thickness (the wall thickness is less than $1/10$ of the internal radius, at least), the stress analysis near the damaged region can be performed in a conservative way by considering an infinite plate under traction with an equivalent imperfection (external or internal crack, trough-the-thickness crack, notch, etc.), as shown schematically in Fig. 6. Hence, the same equations proposed in the last section are used and the problem is reduced to an adequate choice of $\sigma_{\mathrm{Max}}$.

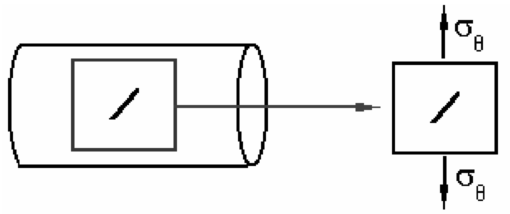


Figure 6: Approximate problem.

### 4.1 External or internal cracks

Internal or external cracks in thin-walled pipes may be approximate in a conservative way by a semi-elliptical crack in an infinite plate as shown in Fig. 6. The stress intensity factor $K_I$,in this case is given by the following relation

$$K_I = \sqrt{\pi a}(M/\varphi) \tag{18}$$

where $\sigma = \sigma_\theta$, for an external crack and $\sigma = (\sigma_\theta + P_i)$, for an internal crack. $\sigma_\theta$ is the hoop stress defined in (5). $\varphi$ is given by

$$\varphi \approx \frac{3\pi}{8} + \frac{\pi}{8}\left(\frac{a}{c}\right)^2 \tag{19}$$

and M is a function of the geometry

$$\begin{aligned} M &= M_s M_e \\ M_s &= 1.12 + (1 - 0.75 \text{ a/c}) \\ M_e &= \sqrt{(2\text{e}/\pi\text{a}) \tan(\pi\text{a}/2\text{e})} \end{aligned} \tag{20}$$

The formula for $M_s$ was proposed by Maddox [3] to correct the basic solution of Irwin considering a semi-infinite medium. In general it is adopted the following value $M_s$=1.12. The formula for $M_e$ is a correction factor for finite thickness and was proposed by Paris and Sih [4]. Alternative expressions for M can be found, for instance, in [5].

If the pipe is submitted to repeated cycles of pressure the crack may propagate until a brutal rupture. In this paper we will only consider cycles of pressure as shown if Fig. 2. Variable amplitude pressure histories and cumulative damage rules won't be discussed. Supposing the validity of the well known Paris law

$$\frac{da}{dN} = C(\Delta K)^m \Rightarrow N = \frac{1}{C}\int_{a_o}^{a_c} \frac{1}{(\Delta K)^m} da, \text{ with } K_{Ic} = \sqrt{\pi a_c}(M(a_c)/\varphi(a_c)) \tag{21}$$

(where $a_o$ is the initial lenght of the crack, $a_c$ is the critical lenght correponding to the value $\Delta K = K_{IC}$ and C , m are material parameters), it is possible to prove [2] that the critical length $\bar{a}_c$ of the

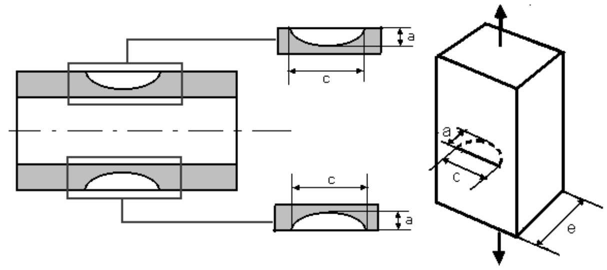


Figure 7: Approximation of an external or internal crack in a cylinder.

pipe with reinforcement will be, at least $\lambda$ times bigger than the critical value $a_c$ if the same equations proposed in section 2 were used with

$$\begin{aligned} \sigma_{Max} &= \left( \frac{K_{IC}\ \varphi(\lambda a_c)}{M(\lambda a_c)\ \sqrt{\pi \lambda a_c}} \right) \text{ for external cracks} \\ \sigma_{Max} &= \left( \frac{K_{IC}\ \varphi(\lambda a_c)}{M(\lambda a_c)\ \sqrt{\pi \lambda a_c}} + P_0 \right) \text{ for internal cracks} \end{aligned} \tag{22}$$

It is also possible to prove [2] that the life $\bar{N}$ of the pipe with reinforcement will be, at least $\lambda$ times bigger than the life N ($\bar{N} = \lambda$ N) if the same equations proposed in section 2 were used with

$$\begin{aligned} \sigma_{Max} &= (1/\lambda)^{(1/m)} \left( \frac{P_i r_i}{r_0 - r_i} \right) \text{ for external cracks} \\ \sigma_{Max} &= (1/\lambda)^{(1/m)} \left( \frac{P_i r_i}{r_0 - r_i} + P_i \right) \text{ for internal cracks} \end{aligned} \tag{23}$$

### 4.2 Trough-the-thickness crack

Trough-the-thickness cracks in thin-walled pipes may be approximate in a conservative way by a crack of the same length in an infinite plate under traction as shown in Fig. 6. Generally pipes conveying fluids are projected to have not a brutal rupture if the length of this kind of crack is smaller than a given limit ("leak before break" criteria). In general, the maximum size of trough-the-thickness crack a pipe must present without a brutal rupture must be bigger than the smaller detectable size. In this case, after a leak detection there will be time to repair (permanently or not) the pipe in order to assure its serviceability. The composite sleeve reinforcement system must be designed, in this case, to maximize the interval between inspections and to assure a given residual life. The stress intensity factors are given by

$$K_I = \sigma_\theta \sqrt{\pi a} \cos^2 \alpha \qquad ; \qquad K_{II} = \sigma_\theta \sqrt{\pi a} \cos \alpha sen\alpha; \tag{24}$$

Using the maximum normal stress criterion it is possible to show that the direction of crack propagation will not change provided $\alpha = 0$ (only mode I). Since this kind of situation ($\alpha = 0$) is the most severe from the safety point of view, it is suggested to only consider an "equivalent crack" with the same length of the real crack but with $\alpha = 0$ in the design of the adequate thickness of the reinforcement sleeve. Exactly like in the case of elliptical cracks, if the pipe is submitted to repeated cycles of pressure and Eq. (21) hold, it is possible to prove [2] that the critical length $\bar{a}_c$ of the pipe with reinforcement will be, at least $\lambda$ times bigger than the critical value $a_c$ if the same equations proposed in section 2 were used with

$$\sigma_{Max} = \left(\frac{K_{IC}}{\sqrt{\pi \lambda a_c}}\right) \text{ for trough-the-thickness cracks} \tag{25}$$

It is also possible to prove [2] that the life $\bar{N}$ of the pipe with reinforcement will be, at least $\lambda$ times bigger than the life N ($\bar{N} = \lambda$ N) if the same equations proposed in section 2 were used with

$$\sigma_{Max} = (1/\lambda)^{(1/m)} \left(\frac{P_i r_i}{r_0 - r_i}\right) \text{ for trough-the-thickness cracks} \tag{26}$$

REMARK: The study presented in this section is valid if the radius $\rho$ of the plastic zone at the tip of the crack is smaller than 20% of the length of the crack. An extension of the analysis to account for a bigger plastic zone is suggested in [1] but will not be discussed on this paper. The following conservative approximate expression may be used

$$\rho = \frac{K_I^2}{2\pi\, \sigma_y^2} (1 - 2\nu)^2 < 0.2a \tag{27}$$

### 4.3 Corrosion defects

The most widely used criteria for assessment of corrosion defects under internal pressure loading is a family of criteria described in [6] as the effective area methods. These include the ASME B31G criterion, the RSTENG 0.85 criterion (also Known as the modified B31G criterion). These criteria were developed in the beginning of the late 1960s and early 1970s to evaluate the serviceability of corroded gas transmission lines. The basic empirical assumption is that the strength loss due to corrosion is proportional to the amount of metal loss measured axially along the pipe, as illustrated in Fig. 8. The main ideas are very similar to those presented for cracks in the present paper. Other approaches can be considered (damage mechanics, approximation of the corrosion defect by a notch and use elasto-plasticity to determine the stresses at the root of the notch, assume that corrosion defects are blunt and hence they all fail by plastic collapse, etc. ) but will not be discussed in the present paper.

The resulting metal loss is treated as a part-through defect in the pipe. The effective area methods assume that the maximum depth profile lies in one plane along the axis of the pipe. To accommodate the irregular nature of most corrosion defects, a profile of the defect is measured and the deepest

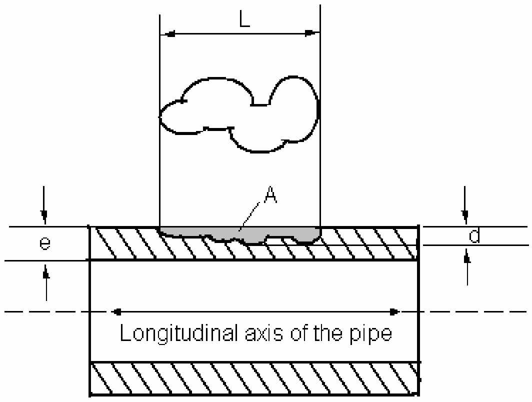


Figure 8: Metal loss in the pipe

points are translated to a single axial plane for analysis, as illustrated in Fig. 8. These criteria may be expressed in the following form

$$\sigma_\theta < \bar{\sigma}\left[\frac{1 - A/Ao}{1-(A/Ao)(M^T)^{-1}}\right] \Rightarrow \sigma_{\mathrm{Max}} = \bar{\sigma}\left[\frac{1 - A/Ao}{1-(A/Ao)(M^T)^{-1}}\right] \tag{28}$$

A is the area of defect in the longitudinal plane through the wall thickness, Ao = Le is the original cross-sectional area, $M^T$ is the Folias factor for a through-wall defect, $\bar{\sigma}$ is the "flow stress", which is a computed parameter that is between the material's yield stress and ultimate strength.

The B31G criterion may be expressed in the following form

$$\sigma_\theta < 1.1\sigma_{\mathrm{y}}\left[\frac{1-(2/3)(d/e)}{1-(2/3)(d/e)(M_1^T)^{-1}}\right];$$

$$M_1^T = \sqrt{1+0.8\left(\frac{L}{\sqrt{2r_i e}}\right)^2} \Rightarrow \sigma_{\mathrm{Max}} = 1.1\sigma_{\mathrm{y}}\left[\frac{1-(2/3)(d/e)}{1-(2/3)(d/e)(M_1^T)^{-1}}\right] \tag{29}$$

In this case $\bar{\sigma}$ =: 1.1 $\sigma_y$; A= (2/3)Le. The B31G criterion uses a simplified, two term form of the Folias bulging factor that is applicable to $\left(\frac{L}{\sqrt{2r_i\ e}}\right)^2 \leq 20.0$ and d/e $< 0.8$.

The modified B31G criterion may be expressed in the following form

$$\sigma_\theta < \bar{\sigma}\left[\frac{1-0.85(d/e)}{1-0.85(d/e)(M_2^T)^{-1}}\right] \Rightarrow \sigma_{\mathrm{Max}} = \bar{\sigma}\left[\frac{1-0.85(d/e)}{1-0.85(d/e)(M_2^T)^{-1}}\right] \tag{30}$$

with

$$\bar{\sigma} = \sigma_{\mathrm{y}} + 10\mathrm{ksi}(68.94\mathrm{MPa}) \tag{31}$$

and

$$M_2^T = \left\{ \begin{array}{c} \sqrt{1+0.06275\left(\frac{L}{\sqrt{2r_i\ e}}\right)^2 - 0.003375\left(\frac{L}{\sqrt{2r_i\ e}}\right)^4}\ ;\ \text{for}\ \left(\frac{L}{\sqrt{2r_i\ e}}\right)^2\ \leq\ 50 \\ 0.032\left(\frac{L}{\sqrt{2r_i\ e}}\right)^2 + 3.3\ ;\ \text{for}\ \left(\frac{L}{\sqrt{2r_i\ e}}\right)^2\ <\ 50 \end{array} \right\} \tag{32}$$

For long areas of corrosion, a rectangular shape is assumed. The criterion is to consider that, due to the corrosion process the internal radius is given by ($r_i$ –d), hence

$$\sigma_{\text{Max}} = \bar{\sigma}\left[\frac{e-d}{e}\right] \tag{33}$$

In order to avoid fatigue failure, the following expressions, based on the ideas presented in section 2 and used to obtain Eq. (16) and (17) can be adopted:

$$\text{“infinite” life} \Rightarrow \sigma_{\text{Max}} = \min\left\{2\left[\frac{SeSu}{Se+Su}\right]\left[\frac{e-d}{e}\right];\ \bar{\sigma}\left[\frac{e-d}{e}\right]\right\} \tag{34}$$

$$\text{life of at least } N_F \text{ cycles } (10^a \leq N_F \leq 10^b) \Rightarrow \sigma_{\text{Max}} = \min\left\{2\left[\frac{SnSu}{Sn+Su}\right]\left[\frac{e-d}{e}\right];\bar{\sigma}\left[\frac{e-d}{e}\right]\right\} \tag{35}$$

## 5 Examples

### 5.1 Pipe with internal flaw

The inspection of a pipeline segment using an ultra-sonic pig detected an internal crack of 3 mm depth and 10 mm width that can be approximated by a semi-elliptic geometry (with a= 3mm and c = 10 mm). The pipe, submitted to an internal pressure $P_i$= 3 MPahas an internal radius $r_i$ = 342,9 mm, a wall thickness e = 6,4 mm and the following material properties: $\sigma_y$ = 300 MPa , E = 200 GPa, $S_u$ = 500 MPa, K = 435 MPa, N = 0.22, $K_{IC}$ = 30 MPa $\sqrt{m}$, m = 3.25 and C = 5.60X$10^{-12}$ . The sleeve has the following properties: $E_\theta$=34400 MPa, $E_r$=9600 MPa, $\nu_{r\theta}$= 0.3. The necessary reinforcement thickness $e_r$ to duplicate the number of cycles to the crack pop through the wall is obtained by taking $\sigma_{max} = (1/2)^{1/m}$ ($P_i$ $\frac{r_i}{e}$+ $P_i$) = 134,7 MPa. Hence, $e_r$ > 22mm if $P_{apl}$= ½ $P_i$; $e_r$ > 12mm if $P_{apl}$ = ¼ $P_i$; $e_r$ > 8.5 mm if $P_{apl}$= 0.

### 5.2 Pipe with uniform wall loss due to corrosion

The inspection using a pig detected a internal uniform wall loss of 30% due to corrosion in the pipe segment. The pipe is the same of the previous example. The necessary reinforcement thickness $e_r$ to assure a life $N_F$ of 50000 cycles before fatigue failure is obtained by taking using the value of $\sigma_{max}$ defined by expression (35). Hence, $\sigma_{max}$= 206.8 MPa and $e_r$ > 7.5mm if $P_{apl}$= ½ $P_i$; $e_r$ > 5mm if $P_{apl}$ = ¼ $P_i$; $e_r$ > 3.5 mm if $P_{apl}$= 0.

### 5.3 Pipe with a localized corrosion defect

A corrosion defect with maximum depth d = 2.24 mm and length L = 374.77 mm was found in a pipe submitted to an internal pressure $P_i$= 3 MPawith an internal radius $r_i$ = 685.8mm, a wall thickness e = 6,4 mm and the following material properties: $\sigma_y = 300$ MPa , E = 200 GPa. In this case, d/e = 0.35 and $\left(\frac{L}{\sqrt{2r_i\ e}}\right)^2 = 16$ , $\sigma_{max}$= 270.6 MPa . The minimum necessary reinforcement thickness $e_r$ to assure that it will not fail with an internal pressure $P_i$=3 MPa is $e_r > 20$mm if $P_{apl} = 1/2\ P_i$; $e_r >$ 11.5mm if $P_{apl} = 1/4\ P_i$; $e_r > 8.0$ mm if $P_{apl}$= 0.

## 6 Concluding remarks

This paper presents a summary of a simplified methodology proposed as an auxiliary tool in the design of polymer composite reinforcement systems. This methodology can be helpful in the definition of the pressure of application and sleeve thickness necessary to assure the design of safer and more reliable repair systems. Although the presentation in this paper in this restricted to open ended cylinders, expressions to determinate the adequate thickness of the sleeve for closed-ended pipes are easily obtained. The extension of the methodology to account for other theories for corrosion in pipes is simple. Generally these theories assume that, since corrosion defects are blunt, they all fail by plastic collapse. The extension of the methodology to account for localized damage is also simple (gouges, dents, etc.) and is not presented due to the limited space. For instance, all criteria presented in [6] and [7] can be taken into account in the proposed methodology. It is important to remark that, for external damage, the defect area must be filled with a high compressive strength filler material before the application of the sleeve. Comparison between the previsions of the proposed methodology and more complex finite element simulations can be found in [1]. The validation of this simplified methodology still requires an extensive program of experimental investigation.

#### References

[1] Côrtes, R., *Modeling of polymer composite sleeve reinforcement systems for metallic pipelines*. Master's thesis, UFF, 2001. In Portugese.

[2] Silva, C., *Lifetime extension of elasto-plastic pipelines using polymer composite sleeve reinforcement systems*. Master's thesis, UFF, 2002. In Portuguese.

[3] Maddox, S.J., Assessing the significance of flows in welds subjects to fatigue, 1974. Wel. 401-s.

[4] Paris, P. & Sih, G., Fracture toughness testing and its application. *ASTM STP*, **381**, p. 30, 1965.

[5] Appendix c – evaluation of flow in austenitic piping and appendix h – evaluation of flows in ferritic piping. ASME XI. Section I.

[6] Stephens, D., & Francini, R., (eds.), *A review and evaluation of remaining strenght criteria for corrosion defects in transmission pipelines, ETCE2000/OGPT-10255*, ETCE/OMAE2000 Joint Conference - Energy for the New Millenium: New Orleans -USA, 2000.

[7] Cosham, A. & Kirkwood, M., (eds.), *Best practice in pipeline defect assessment*, IPC00-205 Internationa Pipeline Conference: Calgary - Canada, 2000.