After buckling behavior of thin membranes and textiles based on tension field theory and experiments

Aura Conci
UFF - Federal Fluminense University, RJ – Brazil

Abstract

This paper presents a study on the use of the tension field theory for efficient representation of fabrics. A highly buckled membrane model provides the simplest approach for visualization of vertical fabrics. The differential equation of tension lines gives the outline of the wrinkles. An equation system is presented, which, when iteratively solved supplies the basic variables of realistic fabric displacements: the number and positions of wrinkles, and the wrinkle depth. The model can also represent important physical characteristics of textiles, as malleability, elasticity and weight. Experimental measures have been developed as a way to verify the adequacy of the equations and to subsidize empirical propositions for improvement of the model. How to use this approach for textile visualization is the objective of this work. Although the original theory is complex, the resulting algorithm is simpler and faster than many numerical treatments already used for prediction of the cloth aspect. The idea is to find a simple realistic model that turns possible the addition in the displacement field of other important necessities of the fashion industry like animations and textures.

Keywords: tension field, theory of membranes, wrinkle lines, fabric displacements, post-buckling behavior, deformation of textiles.

1 Introduction

Realistic visualization of fabric has been the subject of many works because it is essential in computer-aided apparel design and fashion industry [1]. The main difficulty in realistic simulation of cloth is that its surface tends to look artificial in usual representation [2]. For modeling the way in which a cloth hangs, folds and wrinkles must be represented, as well as cloth's physical properties such as rigidity, elasticity and yarn structure. These characteristics distinguish a fabric from other materials. Moreover, they are associated with the aesthetic aspect of the garments.

The task of realistic visualization of cloth is much more difficult than a representation of its surface texture and has been considered usually by complex aproaches [3]. Attempts to use finite element analysis and high-level mathematical techniques to model textiles demand vast computer power and yet still do not produce faithfully simulations of fabrics [4]. An experimental evaluation must be con-

Mechanics of Solids in Brazil 2007, Marcílio Alves & H.S. da Costa Mattos (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-30-7

sidered to maintain the aesthetic aspect. When fabrics are modeled as a physical object, even a single piece draping on an object (like table linings and curtains) may need a great amount of calculations [5]. The scale of the problem increases when fabric properties (elasticity, weight, etc.) must be taken into account and increase more when the problem moves from modeling only the appearance to experimental comparison of results. Computer model of fabrics has achieved a certain success (in computer graphics) but it has yet space to combine reality with mathematical models for an efficient graphical representation. This work analyzes the use of the tension field theory combined with experimental measurements for fabric representation. This combination makes possible to characterize textile details in macro scale without the huge amount of computation necessary to micro scale representation, as that of approaches based on fibers or threads.

2 Basic considerations

Observing the fabrics we see that the textile structure can be considered as a membrane. Fabric folds are produced by gravity when only part of the cloth is directly supported. The lines of strain presented on buckled membrane or highly thin plate are referred to as wrinkle lines [6], and they represent the folds and the crinkles in fabrics [7]. The relationship between the membrane surface and the load direction is very important in the mechanical behavior. There is no external loading in many applications of textiles and the weight acts everywhere in vertical direction (due to the Earth gravity). Fabrics undergo large displacements for small applied forces. Their displacement properties depend on the textile surface direction. We shall distinguish between two essential kinds of behavior on these structures:

(1) Normal load - a textile with loads applied in direction normal or perpendicular to its initial plane (typical examples of this first type of behavior are a carpet falling on the floor, a waving flag, or a cloth being lifted from a flat surface); and

(2) In plane load - a textile which carry loads in the same directions of its plane (typical examples of this second type of behavior are vertical suspended blankets, curtains or a dress on a mannequin and other fashion applications,).

The effect of gravity is generally the main load on a fabric. Therefore, in great number of representation, the first kind of behavior represents fabrics with initial planes in horizontal direction. In this case a satisfactory approximate theory of membrane bending by lateral loads can be used with great realism and small complexity. If the initial plane of the textile to be represented is mainly vertical, it corresponds to the second kind above (almost all the applications for fashion design), and the approach will have to account for the buckling of the membrane. Buckling behavior in all structures (bars, beams, membranes, plates or shells) is much more complex than bending [8]. This paper presents equations for simple and fast representation of the way in which a vertical cloth will fold (second kind of behavior) and compares the results obtained from these equations with real cloth.

3 Tension field theory

The tension field theory describes the highly wrinkled state of membranes subjected to displacements well in excess of that necessary to initiate buckling. The assumption of zero flexural membrane stiffness on post-buckling and wrinkles at right angles to the lines of tension characterizes this theory. Mansfield [6] considers an initially flat membrane of arbitrary shape and variable stiffness, under self-weight supported in a vertical plane. Slack in the membrane manifests itself in the formation of wrinkle lines whose determination is the prime objective. The tension field theory was conceived as a theory of wrinkling isotropic elastic membranes, but the basic assumptions do not depend on isotropy or elasticity [7]. The membrane may have anisotropy related to the weave and nonlinear elastic properties [9]. Pipkin's derivation [7] takes into account that there is resistance to changes in the angle between the weft and warp directions in fabrics. However the theory does not attempt to describe the field of displacements normal to the membrane surface. Rimrott and Ma [10] use this theory to prevent wrinkles in substratum of solar panels. They simulated, in laboratory, blankets with in-plane loading and their experiments showed that the number of wrinkles was a function of the critical stress that the material withstand. Theory and experiment are compared with good agreement [10]. However, like the other works using tension field the description of the 3D wrinkled displacement is not considered. This displacement is much important for realistic visualization of textiles and related to the out-of-plane displacements engendered by the buckling action of the compressive stresses [9]. Strictly speaking such relations are nonlinear and their exact analysis presents formidable difficulties [9]. Fortunately, textiles can easily be experimented in laboratory and their out-of-plane displacements can be measured to improve assumptions to derive the 3D equations [11].

3.1 The wrinkle model from tension field assumptions

A distinctive property of textile materials is their extremely low compressive stiffness. A general assumption is that a textile cannot support any compressive stress. For the case when the vertical membrane is suspended by fixed points, the wrinkle lines have to pass through these fixed points in order to satisfy equilibrium conditions [6], whose solution will depend on the boundary condition of each problem. From equilibrium conditions Mansfield [6] developed wrinkle lines equations for membranes (supported at points or edge-supported) with different boundary: cosine, parallelogram, triangular, pennant-shaped and wedge-shaped (see figure 1). Mansfield shows the wrinkle line equations occur in a repetitive sequence in case of strip supported at intervals (figure 2). The case of inclined supports was also considered and demonstrated that the pattern of wrinkle lines is obtained from the previous case by a simple shearing process (as the parallelogram on figure 1 that is derived from rectangular shape).

The simplest wrinkle line solution [6] is given by

$$y = a_o e^k \cos(\pi x/l) \tag{1}$$

where a_o represents the maximum height of the first wrinkle (amplitude of cosine curve), l is the distance between fixed points or supports (figure 3), and

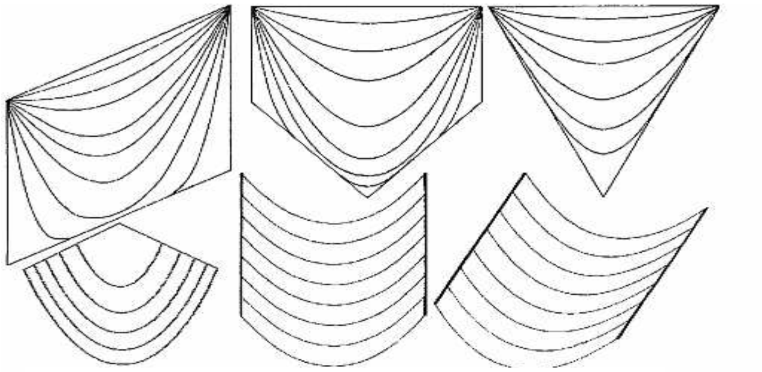

Figure 1: 2D wrinkle lines from tension field theory [6].

$$k = -(\pi^2 F)/(l^2 w) \tag{2}$$

where w is the textile weight per area (textile total weight, W, divided by the area of its surface) and F is the tension in the horizontal direction. This solution is exact for cosine-shaped textiles, that is when the lower boundary (first wrinkle line) is given by

$$y_o = a_o \cos(\pi x/l)$$

and the upper boundary (upper wrinkle line) is given by

$$y_n = a_n \cos(\pi x/l) \tag{3}$$

In the limiting case we have $a_n = 0$, which corresponds to a straight upper boundary. Note that F is zero on y_o, which agrees with equilibrium requirement of free stress lower boundary. When $a_n \to 0$ the horizontal component F grows to infinity.

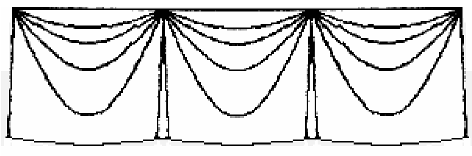

Figure 2: Repetitive folds in case of fabric strip supported at intervals.

Observing the behavior of actual suspended fabric it is possible to see that a finite number of wrinkles will be developed [11]. The amplitude of the wrinkles is strongly dependent on the horizontal motion of the supports but this is not the case for the number of wrinkles [12]. In experiments this number are also not dependent of warp directions. We have investigated carefully this problem because most of fabrics have anisotropy, but all results related with the wrinkles position and shape show not direction dependence [13]. A more important observation is that there is not randomness in the fold shapes [10–16]. In steady state, repetitions of experiments using the same initial conditions always agree with this observation [11–16].

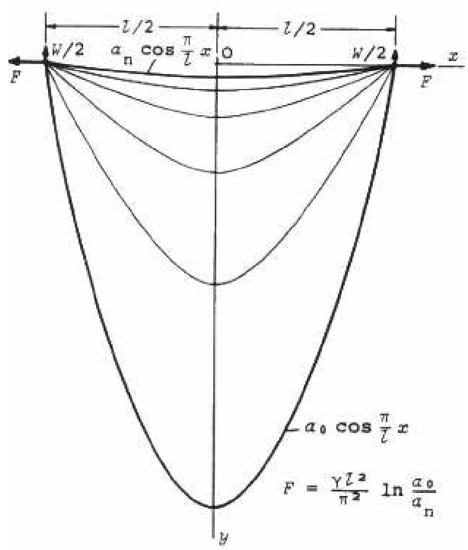

Figure 3: Cosine membrane and its wrinkle characteristics used in the equations.

4 Simulating 3D wrinkles

The tension field theory [9] considers only the 2D locus of the fabric folds (figure 1). Even if complicated types of crinkle are considered, as those treated by Pipkin [7] of two intersected families of fold lines or double folds, the wrinkle description is purely on plane. Of course the number of folds and their locus is the main aspect of the cloth representation. However, folds must be 3D in realistic textile visualization. Strictly speaking such problems are nonlinear and their exact analysis presents formidable difficulties [9]. Fortunately, textiles can easily be experimented in laboratory and their out-of-plane displacements can be studied to improve assumptions to derive the 3D equations [11]. To aid displacement descriptions, experiments have been organized with fabrics of different materials and several shapes [13].

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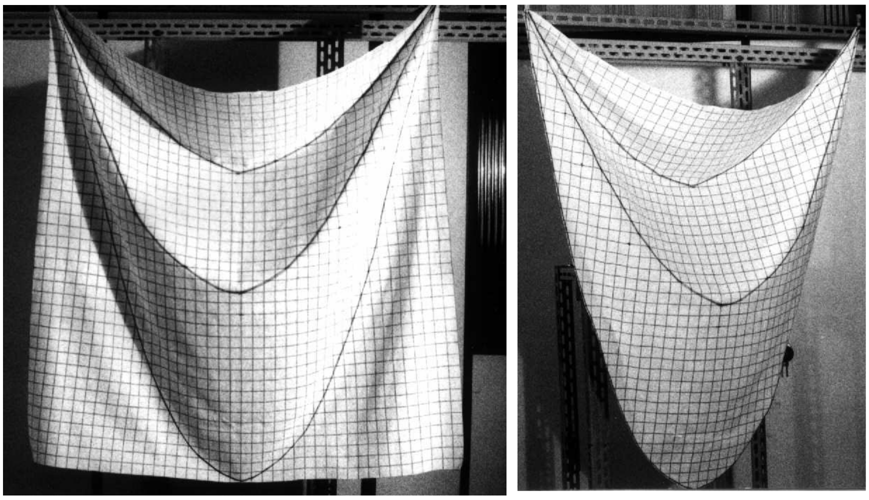

Figure 4: The same fold prevision from the proposed approaches are drawn on the real textiles. The match is inadequate only for the lowest wrinkle in non-cosine shapes. For these figures $h = L = 140$ cm, $t = 0.4$ mm. The material presents $E = 40$ MPa, $w = 17.7{\times}10^7$ N/mm^2. The calculated values are $n = 4$, Q=0.6 and $z_{max} = 7$ cm.

Experiences has been conducted comparing cosine with others types of boundaries and the results showed that the wrinkle pattern is visually identical until the bottom wrinkle [12]. Quadrilateral boundaries were the case with worse results. Figure 4 shows the drawings in real cloth of the wrinkled prevision. The real cloths have rectangular boundary and cosine shapes. Both drawings are obtained from considering the cosine expressions of the last section. The only position where the prevision and the real folds visually do not match (see figure 4) is for the rectangular shape on the wrinkle whose amplitude goes through the bottom boundary. But locus and shapes can be considered correctly described for textile computer graphics representation. These experiences indicated that the use of the 3D cosine boundary developed expressions is also adequate for other shapes [11]. The same happens with the assumption of the lateral displacement. The displacement in z direction (normal to the xy plane of figure 3 or the lateral displacement) varies linearly on any strip folded between a + or - direction, with maximum value for $x = 0$. The z component of the wrinkle line of order i can be represented

$$z_i = z_{max} \cos(\pi x)/l \tag{4}$$

Then, the fabric representation can be performed in three steps: (1) grid generation; (2) determination of the elements related to the folds; and (3) the grid displacement and its combination with other displacements (figure 5) and rendering techniques (but this is out of the scope of this work).

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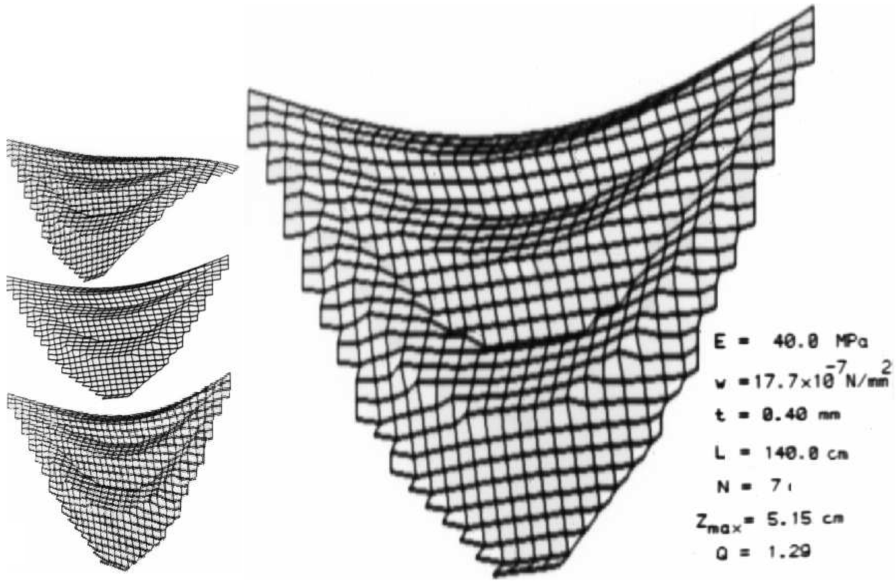

Figure 5: Results of the computer program implemented: The deformed grid for cosine membrane with the characteristics defined in the central line of Table 1, and examples of addition with other deformations and displacements.

4.1 First step: grid generation

This step represents the cloth modeled as a grid of three-dimensional coordinates. By increasing the density of the grid, greater resolution of the surface model may de obtained. A distinction must be made between free points and restricted points on the grid. The restricted points are those that fix a piece of fabric, their coordinates will not change. The remaining points are considered as free points, they will move with textile folds and their new position must be computed. The grid is treated as a coordinate system that describes the garment shape before the appearance of folds. Each grid point should be corresponded with a fabric point on x, y and z axes.

4.2 Second step: determination of fold positions

To determine the displacement of each point when the textile is suspended and the folds appear, the locus of the wrinkles must be computed. For this, some parameters must be calculated. They are the number of wrinkles, n, the amplitude of each wrinkle in vertical direction, a_i, and the maximum component of the wrinkle lines out of the initial plane, z_{max}. The placement of each point of the grid on the folded fabric, which will be determined in the next step, is related to these values.

The process of determining these parameters involves an iterative stage. It has been shown [10] for cosine boundaries (figure 3) that the wrinkle amplitudes grow in geometric progression. Hence, for each wrinkle i, the parameter a_i in equation (3) can be described as

$$a_0/a_1 = a_1/a_2 = \ldots = a_{n-1}/a_n = (a_0/a_n)^{1/n} = Q \tag{5}$$

So the vertical position of each fold is:

$$y_i = (a_o/Q^i)\cos(\pi x/l) \tag{6}$$

However, the value of the ratio Q, and the number of wrinkles n are not yet determined. Since the critical stress, σ_{cr}, cannot be exceeded, the number of wrinkles and their amplitudes will adjust themselves. This stress is limited by the buckling stress [8]. The ratio Q can be related to the maximum compressive stress in the cloth [11]:

$$wh(Q-1)^2/(tQ(Q+1)) \leq \sigma_{cr} = cEt^2/(l^2(1-\nu^2)) \tag{7}$$

where t is thickness of the cloth, h is its height or its initial maximum on the vertical dimension, c is a constant describing the load condition [8], E is the material Young's modulus and ν its Poisson's ratio.

As a first approach Q can be obtained from (7) by enforcing equality and using the maximum compression stress of thin plates [6]. The other two unknowns n and z_{max} are obtained by fabric inextensibility assumptions [11]. The initial length L and the wrinkled upper boundary length can be related by

$$L = \int_{-l/2}^{l/2} \sqrt{1 + \left(\frac{dy_n}{dx}\right)^2 + \left(\frac{dz_n}{dx}\right)^2}\, dx$$

or from (4) to (6)

$$L = \int_{-l/2}^{l/2} \sqrt{1 + \left(\frac{\pi}{l}\right)^2 \left(\frac{a_0^2}{Q^{2n}} + z_{\max}^2\right) \sin^2 \frac{\pi x}{l}}\, dx \tag{8}$$

The inextensibility of the vertical fibers leads, for $x = 0$, to

$$h = \sum_{i=1}^{n} \sqrt{(a_{i-1} - a_i)^2 + (2z_{\max})^2}$$

or from (5) and (6)

$$h = \sum_{i=1}^{n} \sqrt{\frac{a_0^2}{Q^{2(i-1)}} \left(1 - \frac{1}{Q}\right)^2 + (2z_{\max})^2} \tag{9}$$

An approximation of n can be determined by means of the equation (5):

$$n = Int\left[\log\left(a_0/a_n\right)/\log Q\right] \tag{10}$$

in which *Int [.]* is the integer approximation *[.]* , because the number of wrinkles must be integer. However, a_0 and a_n are both unknown. They are the amplitudes of the lower and upper boundaries of the folded fabric (figure 3). As a first approximation $a_0 = 1.01h$ and $a_n = 0.01h$ can be used (for practical purposes) beginning the iterative approach. The obvious $a_n = 0$ is out of the question in equation (10) (indetermination). These values are used as an initial approximation. As soon as n and Q become know, the amplitude a_i of each wrinkle line can be obtained from (5). A tentative value must be used for z_{max} (as $0.01h$) in the equation (8) and (9).

In this work a numerical integration (Simpson) is used to solve equation (8). Note that its result is a first approximation of horizontal textile length, L', where the real value L is known. The obtained value of L' is tested against the real value and new values of z_{max}, and n are obtained using above equations, until the convergence of L' to L. The tolerance value of this convergence is more or less arbitrary, depending on the desired accuracy. In every case on the next section 5% is used as tolerance value.

4.3 Third step: wrinkle line positions and grid displacement

The third step performs a 3D displacement on all points on the surface. After determining of the basic parameters Q, z_{max} and n, the wrinkles lines and all locations of fold points can be obtained by using equations (3), (4) and (5). Knowing the position of the folds, the 3D visualization of the deformed grid is straightforward from interpolation between consecutive folds and the surface is ready for projections, shadings, shadows and textures details. Figure 5 shows on the main illustration the deformed grid and in the small illustrations the results of the deformed grid added to other displacements or deformations (as in animation sequences).

5 Results and conclusions

Only the elements related to the wrinkle locus are here presented. First, numerical results consider the length influence on wrinkles. To study this the same textile with increasing length, h, in one direction is used in simulations. The other characteristics were kept and they are $t = 0.25$ mm, $E =$ 165 MPa, $w = 15{\times}10^{-7}$ N/mm^2 and $L = 140$ cm. The support displacement is 10 cm, so l= 120 cm (figure 3). Table 1 shows the parameters obtained by the approach for each h. The number of wrinkles increases when the length increases. For rectangular shapes, these results can easily be compared with laboratory experiments. Figure 5 shows the wrinkle line positions and grid displacement obtained for this fabric with cosine shape and $h = 140$ cm.

Obtained fold characteristics for textiles with 145×90 cm and increasing weight are shown in Table 2. The other characteristics are $t = 0.35$ mm, $E = 150$ MPa and support displacements of 8.5 cm. These results show that the number of wrinkles increases with the weight. A series of simulations have been done for textiles with 90×75 cm and increasing thickness, t, as shown in Table 3. The other characteristics for the results (of Table 3) are $w = 12.8 \times 10^{-7}$ N/mm^2, $E = 185$ MPa, and support displacement of 7 cm. The effect of increasing stiffness is clearly seen. Some results obtained for textiles with length of 125×85 cm and increasing Young's modulus, E, are shown in Table 4. The other characteristics are $t = 0.23$ mm, $w = 17 \times 10^{-7}$ N/mm^2 and support displacement of 9 cm. It shows that, when the elasticity increases, the ratio Q increases, but the number of wrinkle decreases.

Table 1: Results for a increasing length.

h (cm)	n	z_{max}(cm)	Q
90	5	5.4	1.31
140	7	4.8	1.29
235	8	6.0	1.34

Table 2: Fabric with increasing weight.

w (10^{-7} N/mm^2)	n	z_{max}(cm)	Q
10	4	5.60	1.78
33	6	4.60	1.46
54	7	4.40	1.39

Table 3: Fabric with increasing thickness.

t (mm)	n	z_{max}(cm)	Q
0.09	7	3.45	1.33
0.26	4	4.15	1.65
0.40	2	1.10	2.60

Table 4: Fabric with increasing elasticity.

E (MPa)	n	z_{max}(cm)	Q
55	8	3.70	1.34
110	7	4.25	1.30
185	6	4.55	1.47

Beside numbers, side by side comparison of real cloth and simulation of the same cloth is a better argument for the visual similarity obtained with the approach. Figures 6 and 7 show the locus of the folds drawn over real cloths. The wrinkle lines behavior is considered in cosine formulation for both cases (according to the experimental investigations).

Results of this approach show that it is possible to represent the fold peculiarities related with textile geometry and physical characteristics. The introduced out-of-plane analysis of the displacements allows to represent the post-wrinkling state. The derived equations provide the basic variables for realistic appearance of clothes: number of wrinkles and their position. The resulting deformed description can be used then with a variety of rendering for realistic textile representation [1, 2].

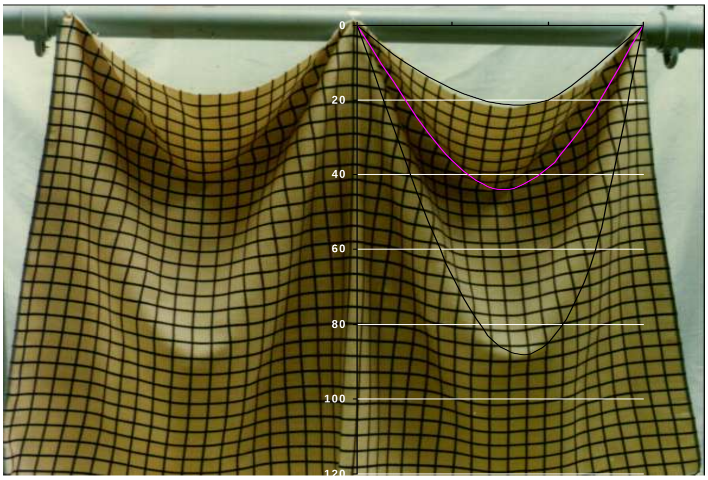

Figure 6: Folds obtained by the approach drawn over a real cloth supported at intervals (darkest lines represent negative folds).

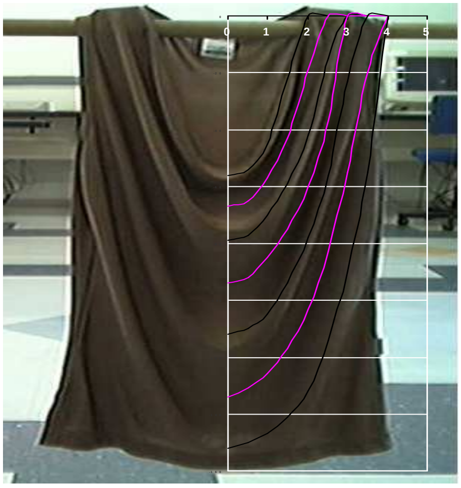

Figure 7: Folds obtained by the approach drawn over a real shirt (darkest lines represent negative folds).

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Mechanics of Solids in Brazil 2007, Marcílio Alves & H.S. da Costa Mattos (Editors)
Brazilian Society of Mechanical Sciences and Engineering, ISBN 978-85-85769-30-7