

## A DEADBEAT TYPE CONTROLLER FOR LOW FREQUENCY NOISE REDUCTION AND VIBRATION CONTROL

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**Abstract.** *Two main streams can be found in dealing with unwanted noise and vibration in the noise/vibration reduction area, they are known as "passive control" and "active control" techniques. At higher frequencies, passive noise control systems still are the natural choice. Passive control techniques include insulators, silencers, vibration damping treatments, absorptive devices, etc. Passive techniques work best at high frequencies and they play an important role in nearly any industrial, commercial or domestic machinery in today's increasingly noise-sensitive world. However, when used in low frequency applications, passive techniques usually include bulky and heavy equipment. This is because the size and mass of passive devices usually depend on the acoustic wavelength, making them thicker and more massive for lower frequencies. The control of low frequency noise and vibration has traditionally been difficult, expensive and in many cases not feasible through passive techniques. On the other hand, the lightweight and small size of active control techniques have become critically important, and in many cases, an irreplaceable benefit. Active noise control schemes are appropriate for low frequencies applications. In the low frequency range, they are more efficient and cost effective than passive schemes. As a consequence of the steadily improvement of digital computers speed performance in the last decades, several new digital control strategies have been proposed by the scientific community, among them, the deadbeat control strategy, initially proposed for regulation type problems. This paper presents a further application of the deadbeat control technique applied to the noise and vibration control problem. It is shown that for low frequency noise (or vibration) and with a fast enough sampling rate, the deadbeat control algorithm delivers a very fast and accurate controller response. The design procedure of the proposed deadbeat control algorithm is also presented and discussed. Finally, simulation results are included to verify the feasibility of the proposed technique and to assess the controller performance.*

**Keywords:** *Noise reduction; vibration control; deadbeat control.*

### 1. INTRODUCTION

The quality of life has become a main concern among the human population in modern societies. In face of the recently huge expansion of industrial areas, keeping healthy environments for working and living has become an important objective of development engineers. One invisible but not less important environment polluter is noise/vibration coming from the most spread kind of sources. Noise and vibration in human environments are responsible for physical and psychological illnesses among humans, bringing together substantial economical losses to the production lines. Two main streams can be found in dealing with unwanted noise and vibration in the noise/vibration reduction area, they are known as "active control" and "passive control" techniques. Active noise control schemes are appropriate for low frequencies applications. In the low frequency range, they are more efficient and cost effective than passive schemes. At higher frequencies, passive noise control systems still are the natural choice.

Passive control techniques include insulators, silencers, vibration damping treatments, absorptive devices, etc. Passive techniques work best at high frequencies and they play an important role in nearly any industrial, commercial or domestic machinery in today's increasingly noise-sensitive world. However, when used in low frequency applications, passive techniques usually include bulky and heavy equipment. This is because the size and mass of passive devices usually depend on the acoustic wavelength, making them thicker and more massive for lower frequencies. The control of low frequency noise and vibration has traditionally been difficult, expensive and in many cases not feasible through passive techniques because of the long wavelengths involved. On the other hand, the lightweight and small size of active control techniques have become critically important, and in many cases, an irreplaceable benefit.

Active noise/vibration control consists on noise/vibration field cancellation by electro-acoustical/mechanical means. In this case, the control system drives an actuator (speaker or vibration source) to produce a field that is a mirror image of the offending noise/vibration field (from the control view point that can be seen as some kind of pole-zero cancellation). The net result is the reduction of sound/vibration. The idea of using active noise control (ANC) as an alternative to passive control for low frequency was initially proposed Paul Lueg in 1933. The basic idea was to use a transducer to introduce a secondary disturbance into the system to cancel the existent noise. For several decades, the absence of proper technology and control acknowledge kept these ideas in stand by. Only in the early 80's, the active noise control became the focus of interest of the scientific community again. By that time the surging of small computer technology and other advances in control engineering made possible the implementation of the first active noise control

systems. During the last decade of the 20th century, new sensors and actuators technologies were developed and the decreasing cost of very powerful small computers, already existing, permitted an immense growth of the research work in this field. Recently, some interesting commercial applications have been successfully implemented.

This paper is organized as follows: In Section 2, the active noise control problem is commented. In Section 3, a brief review of deadbeat digital control is presented. In Section 4, the design procedure of deadbeat control applied to noise and vibration cancellation is considered. In Section 5, simulation results are included to illustrate the performance of the proposed control scheme. Finally, in Section 6, final comments and conclusion are presented.

## 2. THE ACTIVE NOISE CONTROL PROBLEM

A modern active noise control (ANC) system consists of one or more control sources used to introduce a secondary signal into the acoustic system. The introduced signal suppresses the unwanted noise originated from one or more primary noise sources. An algorithm implemented in a microprocessor-based controller usually generates the control signal. The control algorithm uses as inputs measurements of the incoming noise produced by the primary source.

A main issue to guarantee the good performance of the noise cancellation system is the suitable design of the acoustic part of the system. Having achieved a good acoustic/structural solution, a second step is the proper choice of the digital controller, that implements the control algorithm, and the proper specification and positioning of adequate actuators, sensors and transducers.

Active noise control techniques are the proper choice for noise fields that are spatially simple such as low-frequency noise waves traveling through a duct. Active noise controllers reach their best performances when the wavelength is longer than the dimensions of its environment, which characterizes a one-dimensional problem. A worst scenario case would be a complex geometry involving high frequency noise. Broadband noise, that is, noise containing a wide range of frequencies is more difficult to control than narrowband noise with a strong fundamental frequency.

Several active noise/vibration control applications can be found in industry: active control of vibration and noise in aircrafts such as helicopters, ships and submarines; active low frequency noise reduction in industrial environments such as vacuum pumps, forced air blowers, cooling towers and gas turbine exhaust; active vibration control of low frequency vibration in large space structures; active control of oscillations in tall buildings; active noise reduction in air conditioning systems and large electrical transformers; reduction of noise inside car cabins; dynamic suspension systems for vehicles; active headsets and earmuffs for aircraft pilots; etc; Hansen (1996).

“The appropriate control strategy is dependent upon the control objective, whether it be vibration control, radiated sound power control, sound transmission control or some other objective. For example, the physical control system for reducing aircraft interior noise is not the same as the physical control system for reducing noise transmission in an air handling duct or the system for vibration isolation of an electron microscope. Similarly, the electronic controller for an adaptive feedforward system is not the same as that electronic controller for a feedback system. However, the underlying principles of efficient design for each subsystem are the same,” Hansen (1996).

## 3. THE DEADBEAT CONTROL TECHNIQUE – A SHORT REVIEW

The flexibility, low cost, and increasing computational power and speed of new digital computers have allowed the development of new digital control algorithms not be restricted to discrete versions of analog designs. In particular, it is possible to formulate control laws that, under proper conditions, will produce the desired closed loop response. The main components of a standard feedback control system are: The plant that is the physical system to be controlled; typical examples in ANC are a headphone and the air inside it, or air traveling through an air-conditioning duct. The sensors that are devices, such as microphones and accelerometers, used to measure the disturbance. The actuators that are devices that physically perform the task of modifying the plant response; they usually are electromechanical devices such as speakers or vibration generators (shakers). And the control algorithm that defines the power to be delivered to the plant by the actuator; it can be implemented in digital computers, micro-controllers or any digital device able to run numerical algorithms.

The block diagram of a standard sampled data control system is shown in Fig. 1. Figure 2 shows a simplified block diagram of a feedback control loop.

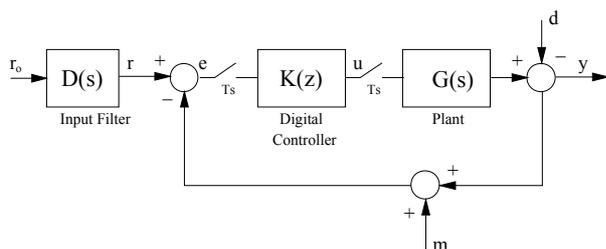


Figure 1. The digital control block diagram.

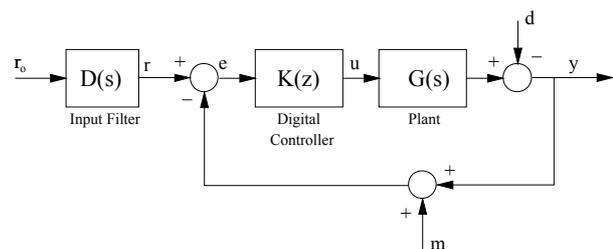


Figure 2. The simplified block diagram.

In this case  $d(t)$  is a disturbance,  $m(t)$  is a measurement noise,  $r(t)$  represents the desired output (set-point),  $e(t)$  is the error signal used by the control algorithm to compute  $u(t)$  that represents the power delivered to the plant.  $D(s)$  is the transfer function in Laplace domain of an input filter usually used to smooth sharp changes in the set-point,  $G(s)$  is the transfer function in Laplace domain that represents the plant and finally  $K(z)$  is the transfer function in the z-domain that represents the digital controller. In this case the closed loop transfer function,  $T(z)$ , from  $r$  to  $y$  is given by:

$$T(z) = \frac{Y(z)}{R(z)} = \frac{G(z)K(z)}{1 + G(z)K(z)} \quad (1)$$

Deadbeat control aims for the best possible response to a set point change. In control theory, the deadbeat control design problem consists of finding a control signal that applied to the plant input brings its output to zero in the smallest number of sampling intervals. It can be shown that for an  $n$ th-order linear time unvarying (LTI) system this number of steps is  $n$ , provide that the system is null controllable (it can be brought to the origin by some input). This can be achieved by finding a feedback controller,  $K(z)$ , such that all poles of the desired closed-loop transfer function,  $T_d(z)$ , are at the origin of the z-plane. The design is carry out using the synthesis equation defined by:

$$K(z) = \frac{1}{G(z)} \left[ \frac{T_d(z)}{1 - T_d(z)} \right] \quad ; \quad T_d(z) = z^{-P} \quad (2)$$

Notice that the desired discrete closed loop transfer function,  $T_d(z)$ , corresponds to a system with  $p$  poles at infinity in the Laplace domain. It is worth to remark some important facts on deadbeat control:

- The design technique uses the inverse of the plant model,  $G(z)$ . This is essentially how it enables the user to specify the desired closed loop response,  $T_d(z)$ . That is, it uses the inverse model to cancel out the undesired plant dynamics (replacing it with another dynamic characteristic) so that the desired closed loop response could be achieved. However, the use of an inverse model poses a problem when the process zeros lie outside the unit circle (a non-minimum-phase system). If  $G(z)$  has a zero outside the unit circle (non-minimum-phase), the  $K(z)$  will have a pole outside the unit circle and  $G(z)K(z)$  will have an unstable pole/zero cancellation (unwise design), in this case, the synthesis equation should not be used.
- The controller may not be causal if the desired closed loop response,  $T_d(z)$ , is chosen faster than the actual system can respond.
- Additionally, it can be shown that as the sampling rate increases, minimum-phase continuous systems could become non-minimum-phase ones in the z-domain. This usually happens when the continuous system has a pole-zero excess greater than 2. Thus, the sampling rate should be carefully chosen. In addition, selecting a faster sampling - without modifying the desired closed-loop response  $T_d(z)$  - causes the controller performance to be rough leading to poor inter-sampling behavior.
- The phenomenon known as ‘ringing’ can be caused by a controller negative real pole. The closer it is to the -1 point in the z-plane, the more severe the ringing will be. It is usually caused by some degree of model mismatch. In this case, the controller may produce an oscillating control signal with decreasing amplitude as time increases.

#### 4. THE PROPOSED CONTROL SCHEME

Deadbeat control has been successfully applied to solve regulation type control problems. This paper analyses a extended application for the deadbeat technique, in this work, it is used to reduce unwanted output disturbance (notice that this is not the natural application for this type of controller). It is shown through simulation that if the sampling rate is properly chosen (fast enough), deadbeat control can be successfully applied to the active noise/vibration reduction/control problem. The conditions for good controller performance can be formulated as follows:

Given a sampling rate of  $T_s$ , conditions for good controller performance exist if the amplitude of the perturbed noise remains almost constant in a short time interval given by  $nT_s$ , where  $n$  is the order of the linear time unvarying system (plant). If such is the case, the noise control problem can be seen as a regulation problem, notice that this can be achieved using a very fast sampling rate.

This section presents a simple and straightforward control scheme for the unwanted noise/vibration reduction problem. The undesired noise/vibration is treated as a disturbance of a well-known stable system. Time delays, usually existing in this type of system, are naturally included in the formulation of the control law as well as the problems caused for non co-located sensors as actuators. Figure 3 shows the block diagram of the proposed control scheme. The proposed control scheme can be seen as a “smart” feedback actuator that delivers the proper signal that cancels the unwanted noise/vibration.

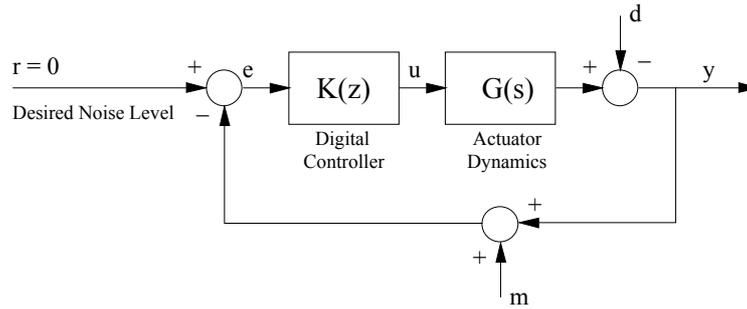


Figure 3. The proposed control technique.

**5. SIMULATION RESULTS**

In this work, a one-dimensional acoustic wave-guide 12th order model was used to assess the controller performance. For controller designing purposes the plant continuous model was chosen as a minimal phase linear model whose poles and zeroes are shown in Tab. 2. The lowest frequency of the plant is approximately 50 rd/s and the highest one is 1060 rd/s. So, the sampling rate was chosen as  $T_s = 1.5$  ms.

Table 2. Poles and Zeros of the Plant Model for Control Design Purposes.

| Poles  | Zeros  | Damping               | Frequency             |
|--|--|-----------------------|-----------------------|
| $-5.00 \times 10^{+0} \pm j 4.97 \times 10^{+1}$ | $-1.85 \times 10^{+1} \pm j 1.1034 \times 10^{+3}$ | $1.00 \times 10^{-1}$ | $5.00 \times 10^{+1}$ |
| $-1.78 \times 10^{+1} \pm j 2.11 \times 10^{+2}$ | $-2.22 \times 10^{+1} \pm j 6.482 \times 10^{+2}$  | $8.39 \times 10^{-2}$ | $2.12 \times 10^{+2}$ |
| $-1.21 \times 10^{+1} \pm j 6.17 \times 10^{+2}$ |  | $1.96 \times 10^{-2}$ | $6.17 \times 10^{+2}$ |
| $-5.71 \times 10^{+1} \pm j 6.66 \times 10^{+2}$ |  | $8.54 \times 10^{-2}$ | $6.69 \times 10^{+2}$ |
| $-7.59 \times 10^{+1} \pm j 8.69 \times 10^{+2}$ |  | $8.70 \times 10^{-2}$ | $8.72 \times 10^{+2}$ |
| $-2.72 \times 10^{+1} \pm j 1.05 \times 10^{+3}$ |  | $2.58 \times 10^{-2}$ | $1.06 \times 10^{+3}$ |

Figure 4 shows the plant impulse response and Fig. 5 presents the plant frequency response.

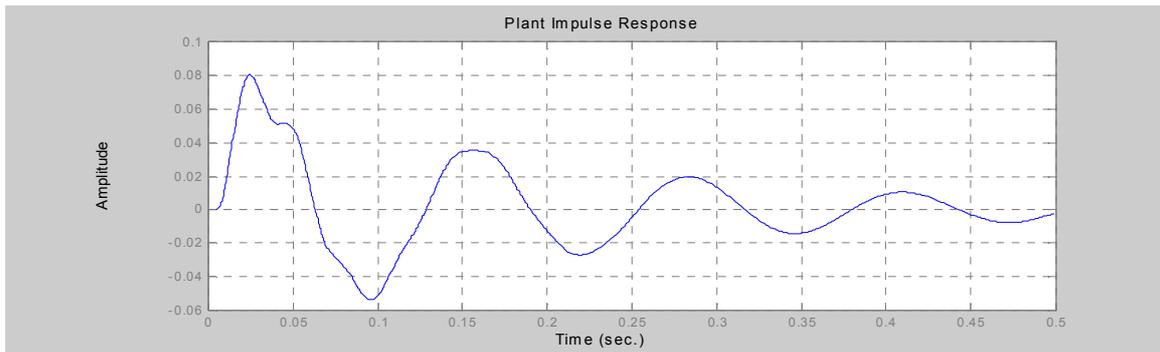


Figure 4. Plant Impulse Response.

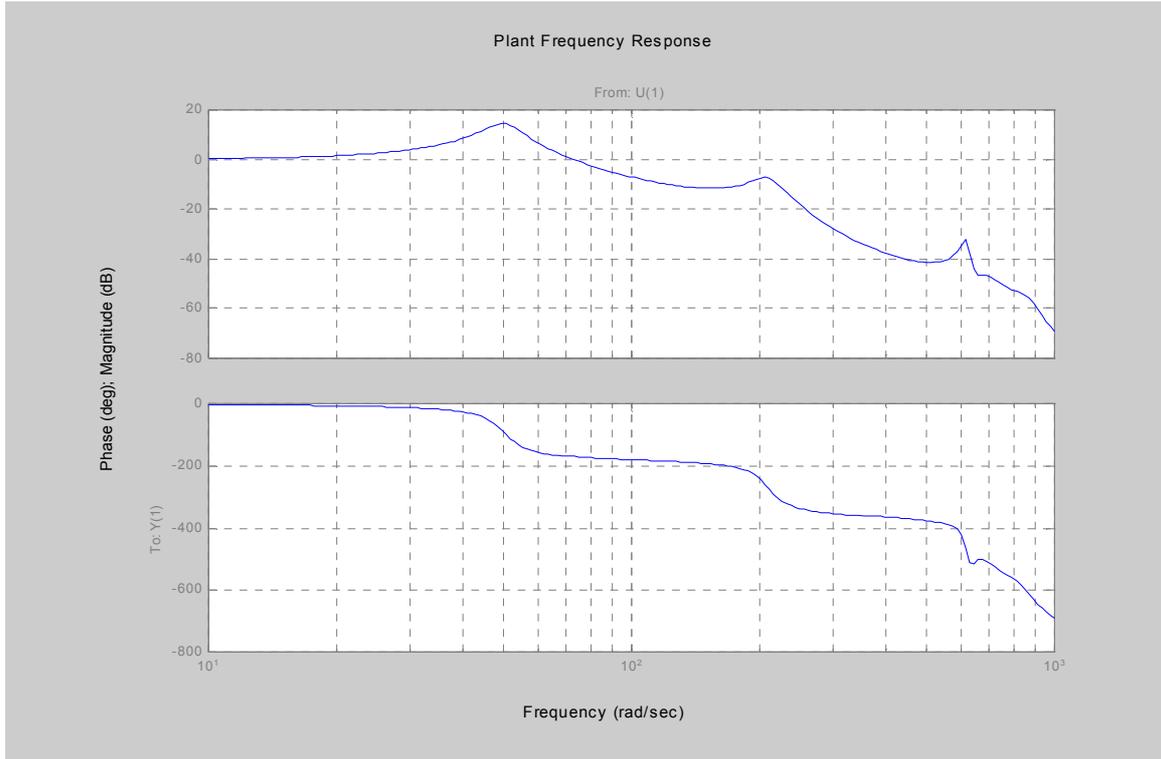


Figure 5. Plant Frequency Response.

For a sampling rate of 1.5 ms, the plant discrete model (in the z-domain),  $G(z)$ , was determined including a zero-order hold in the direct path of the loop. For these choices,  $G(z)$  has a pole-zero excess of seven. Thus, the controller was designed using Equation (2) with  $T_d(z) = z^{-7}$  (to reach a causal control algorithm as commented in Section 3). Figure 6 shows the plant and controller poles and zeros in the “z” domain.

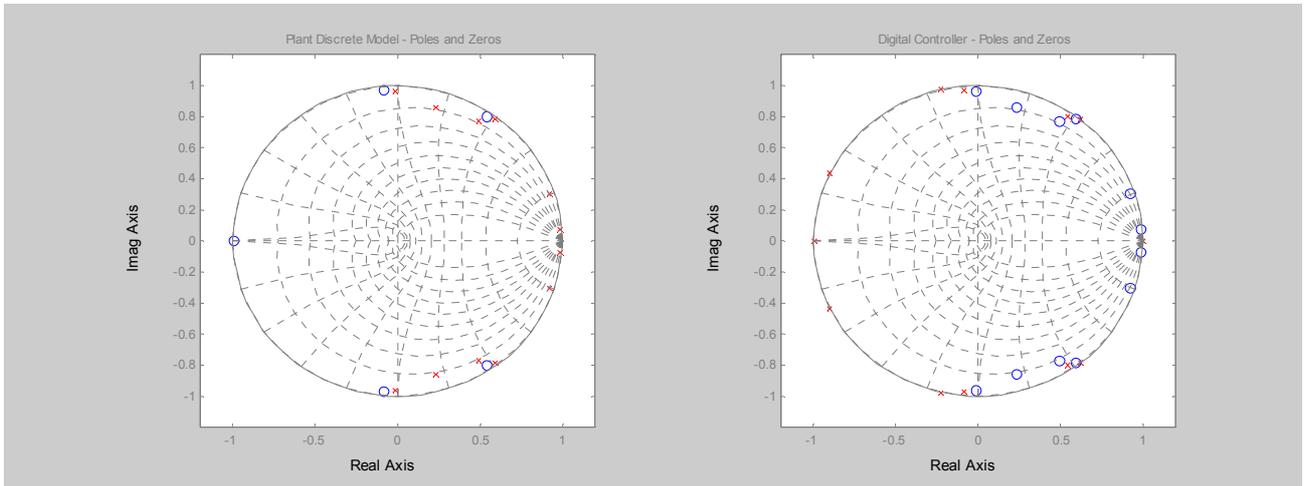


Figure 6. Plant and Controller Poles and Zeros.

The controller performance was tested for disturbance frequencies in the range of 1 to 50 rd/s. The results are presented in Figs. 7 to 10. It can be observed that the controller performance improves as the disturbance frequency decreases. Notice, however, that even for a disturbance frequency of 50 rd/s (also a resonant frequency of the plant) the noise attenuation is greater than 70% (Fig. 7) showing an excellent controller performance.

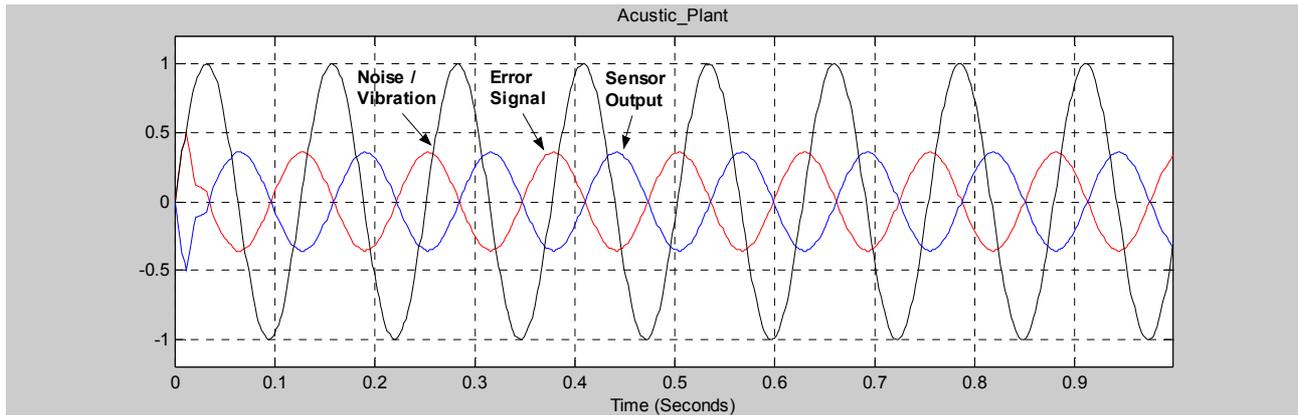


Figure 7. Close Loop System Response to a Sinusoidal Disturbance with  $\omega_d = 50$  rd/s (the worst case).

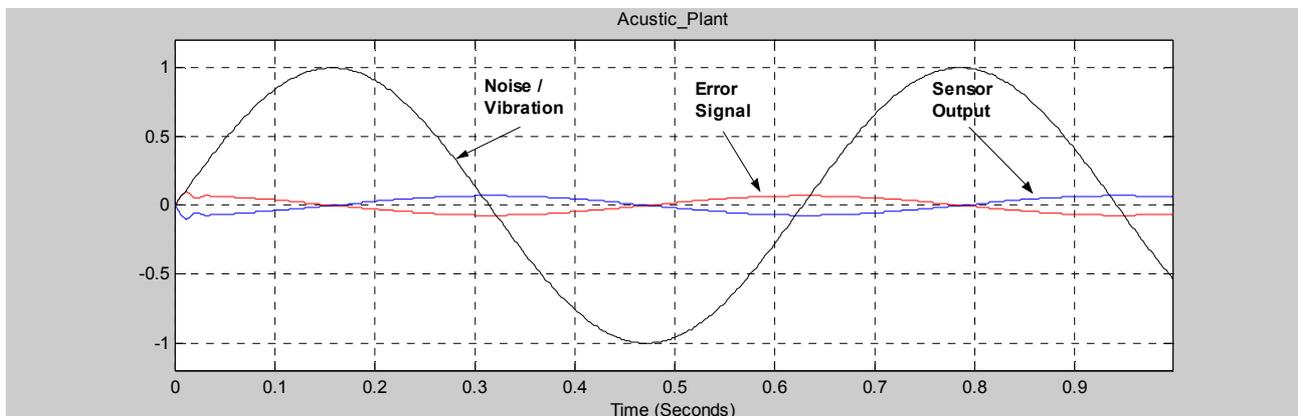


Figure 8. Close Loop System Response to a Sinusoidal Disturbance with  $\omega_d = 10$  rd/s.

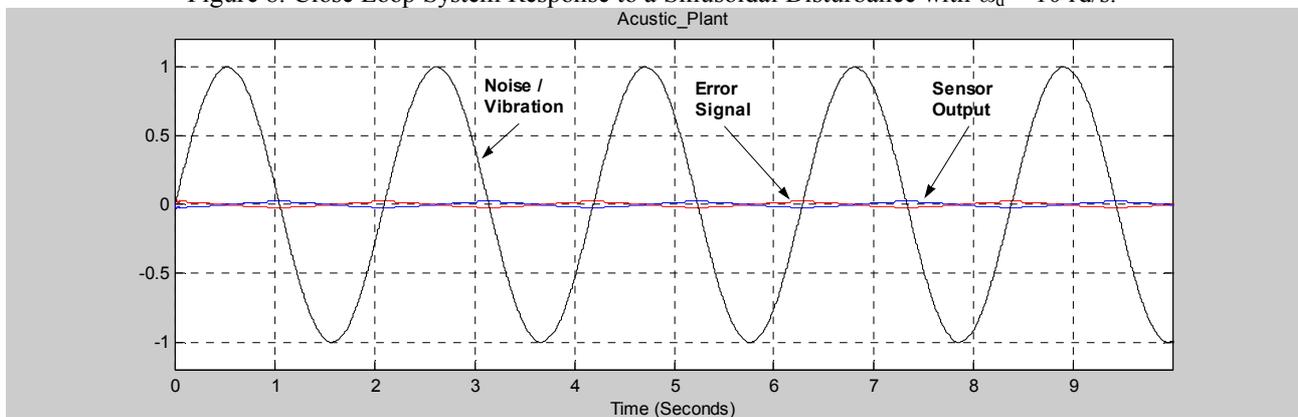


Figure 9. Close Loop System Response to a Sinusoidal Disturbance with  $\omega_d = 3$  rd/s.

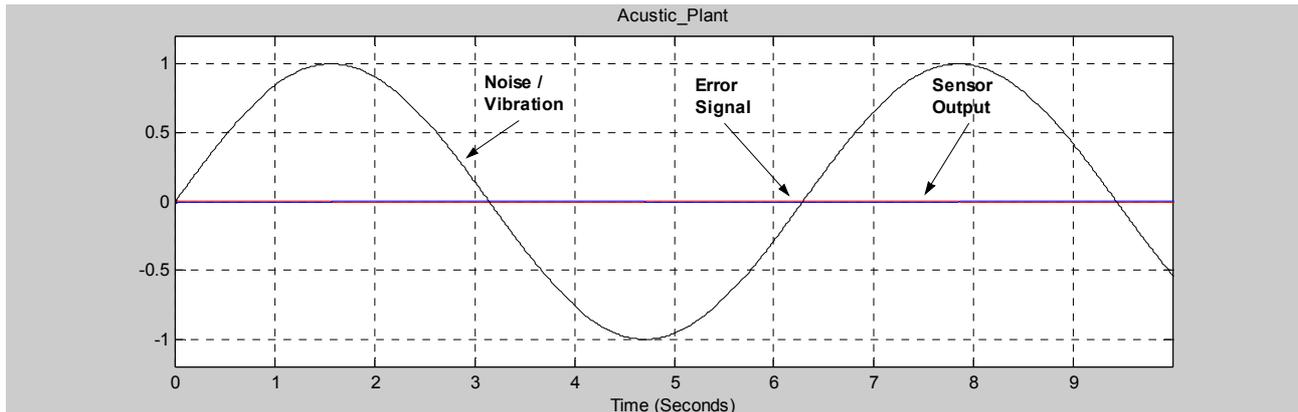


Figure 10. Close Loop System Response to a Sinusoidal Disturbance with  $\omega_d = 1$  rd/s (the best case).

## 6. FINAL COMMENTS AND CONCLUSIONS

A new digital control procedure has been introduced as an alternative for the control of new noise/vibration active control. The proposed technique is based on the deadbeat control algorithm. The algorithm was applied to a one-dimensional acoustic wave-guide 12<sup>th</sup> order model with resonant frequencies at 50, 212, 617, 669, 872 and 1060 rd/s, the sampling rate was chosen as 1.5 ms. The performance of the proposed algorithm showed to be highly dependent on the proper choice of the sampling rate. In the studied case, the algorithm performed well for disturbance frequencies up to 50 rd/s producing a noise reduction of about 70% at that frequency. The results have shown a promising outstanding performance of the proposed algorithm for the reduction and control of low frequency noise and vibration.

Finally, it is worth to mention that the lower the noise/vibration frequency the worse the passive techniques results would be and the better the proposed technique performance is. This fact shows a promising feature of the proposed technique in the hybrid controller applications area.

## 7. ACKNOWLEDGMENTS

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