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THERMALLY DEVELOPING FORCED CONVECTION OF NON-NEWTONIAN FLUIDS INSIDE ELLIPTICAL DUCTS SUBJECTED TO BOUNDARY CONDITION OF SECOND KIND

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Abstract. Laminar forced convection inside tubes of various cross-section shapes is of interest in the design of low Reynolds number heat exchanger apparatus. Heat transfer to thermally developing, hydrodynamically developed forced convection inside tubes of simple geometries such as a circular tube, parallel plate or annular duct has been well studied in the literature and documented in various books, but for elliptical duct there are not much work done. The main assumptions used in this work are non-Newtonian fluid, laminar flow, constant physical properties, negligible axial heat diffusion (high Péclet number). Most of the previous research in elliptical ducts deal mainly with aspects of fully developed laminar flow forced convection such as velocity profile, maximum velocity, pressure drop and heat transfer quantities. In this work we examine heat transfer in hydrodynamically developed, thermally developing laminar forced convection flow of fluid inside elliptical tube, under a boundary condition of second kind. To solve the thermally developing problem we use the generalized integral transform technique (GITT) also known as Sturm-Liouville transform. Actually, such integral transform is a generalization of the finite Fourier transform where the sine and cosine functions are replaced by more general sets of orthogonal functions. The axes are algebraically transformed from the Cartesian coordinate system to the elliptical coordinate system in order to avoid the irregular shape of the elliptical duct wall. Then the GITT is applied to transform and solve the problem and to obtain the once unknown temperature field. Afterward it is possible to compute and present the quantities of practical interest, such as the bulk fluid temperature, the local Nusselt number and the average Nusselt number for various cross-section aspect ratios.

Keywords: *Non-Newtonian fluids, Forced convection, Integral Transform, Elliptical Tube*

1. Introduction

Problems involving non-Newtonian fluids constitute a field of interest in mechanical engineering of great economic importance. Researches of rheologic nature are necessary for the accurate knowledge of the deformation relations as well as for the understanding of the whole dynamics related to this class of fluids. Therefore, the development of techniques for the solution of problems in this area is relevant, especially those that permit to obtain parameters of interest in the design of thermohydraulic equipment. The contribution of the present work is related to the calculation of the heat transfer coefficients for the flow of one class of these fluids inside ducts of elliptical section.

The several methods and techniques of solution related to convection problems are mostly concerned with the flow of Newtonian fluids and, usually, are more accurate and more powerful when applied to the simplest problems where the results are already known. On the other hand, analytical solutions, and even solutions of hybrid nature (analytical-numerical), are more scarce for complex cases involving flow in ducts of irregular geometries submitted to variable boundary conditions or for situations where the fluid properties exhibit some dependence with temperature (Shah and London, 1978:1-4), etc. Solutions for problems that deal with the flow of non-Newtonian fluids are even scarcer.

It is worthy mentioning, however, that the Generalized Integral Transform Technique – GITT (Cotta, 1998) has been consistently developed for the solution of complex diffusive and diffusive-convective problems that can not be solved by the techniques of the Integral Transform or Separation of Variables. Among the problems solved successfully by means of the GITT are the flow in ducts of irregular shape (Aparecido et al., 1989), (Maia et al., 2000), (Maia et al., 2001), with time varying coefficients and boundary conditions (Cotta and Ozisik, 1986), (Cotta and Ozisik, 1987), problems involving space dependence for the heat transfer coefficients and boundary conditions (Vick and Wells, 1986) problems with thermally and hydrodynamically developing flow (Silva et al., 1992), diffusive problems with moving boundaries (Diniz et al., 1990), problems of non-Newtonian fluid flows (Santos et al., 1994), (Quaresma and Lima, 1998), (Macêdo and Quaresma,1998), etc.

Continuing the development of this line of research, it is presented here the calculation of heat transfer parameters for the flow of non Newtonian fluids, in problems of thermally developing flow, inside ducts of elliptical section, subjected to boundary condition of second kind. In the present work only fluids that follow the Power Law are considered because they constitute the great majority of the known non-Newtonian fluids. In particular, a convenient change of variables for the solution of this problem was used to determine the velocity profile inside the elliptical duct and also to transform the elliptical profile into a new geometry in order to simplify the application of the boundary conditions. Eventually, the generalized integral transform technique was applied to the energy equation corresponding to this problem, thus enabling to a obtain the temperature field in the flow and, consequently, the computation of the heat transfer parameter of interest.

2. Analysis

For the formulation of the present problem hydrodynamically developed and thermally developing flow is assumed, with uniform temperature profile at the entry and in steady state. The viscous dissipation and the axial conduction are neglected and the fluid properties are assumed to be constant in the whole domain. Thus, the energy equation, according to the coordinate system presented in Fig. 1, is given by:

$$\rho c_p V(x,y)\frac{\partial T(x,y,z)}{\partial z} = k\left[\frac{\partial^2 T(x,y,z)}{\partial x^2} + \frac{\partial^2 T(x,y,z)}{\partial y^2}\right] \quad ; \qquad \{(x,y) \in \Omega \quad and \quad z > 0\} \tag{1}$$

where ρ is the fluid density, c_P is the specific heat, k is the thermal conductivity, $T(x,y,z)$ is the temperature field and $V(x,y)$ is the velocity profile for fully developed flows of fluids that follow the power law in ducts of elliptical section (Maia et al., 2002) given by

$$V(x,y) = \left(\frac{3n+1}{n+1}\right)\left(1-\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^{\frac{n+1}{2n}}\right)V_{av} \tag{2}$$

Here, n is the fluid behavior index and V_{av} is the flow average velocity.

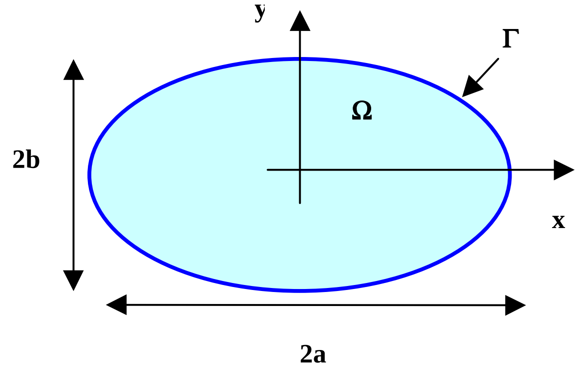

Figure 1. Geometry of the problem.

In the present work it is considered constant heat flux $\dot{q}''_o$ at the contour Γ

$$-k\frac{\partial T(x,y,z)}{\partial \eta} = \dot{q}''_o \ , \quad \{z>0, \ \eta \perp \Gamma\}. \tag{3}$$

The others boundary conditions of the problem can be written as:

$$\frac{\partial T(x,y,z)}{\partial x} = 0 \ , \ \{z>0, \quad x=0\}; \tag{4}$$

$$\frac{\partial T(x,y,z)}{\partial y} = 0 \;, \quad \{z>0, \quad y=0\}. \tag{5}$$

and the condition at the entrance is written as follows

$$T(x,y,z) = T_i \;, \qquad \{z=0,\ (x,y)\in \Omega\}. \tag{6}$$

2.1. Transformation of Coordinates

The temperature potential and other physical and geometrical parameters were written in dimensionless form as follows:

$$\theta(X,Y,Z) = \frac{T_o - T(x,y,z)}{\dot{q}''_o \; D_h / k} \;; \tag{7}$$

$$X = x/D_h \quad ; \quad Y = y/D_h \;; \quad Z = z/(D_h\, Pe) \;; \quad \eta^* = \eta \,/\, D_h \quad ; \tag{8}$$

$$\alpha = a/D_h \quad ; \quad \beta = b/D_h \;; \quad D_h = 4\,A_{sc}/P \;; \quad r = b/a \,; \tag{9}$$

$$A_{sc} = \pi\, a\, b \;; \quad P = 4a \int_0^{\pi/2} \sqrt{1-\kappa^2\, sen^2\theta}\; d\theta \;; \quad \kappa = \sqrt{a^2-b^2}\,/\,a \,; \tag{10}$$

$$U(X,Y) = \mathrm{V}(x,y)\,/\,\mathrm{V}_{av} \,. \tag{11}$$

Here, A_{sc} is the area of the elliptical section, P is the perimeter of the elliptical contour and r is the aspect ratio of the ellipsis. With these new variables the energy equation is rewritten as

$$U(X,Y)\,\frac{\partial \theta(X,Y,Z)}{\partial Z} = \frac{\partial^2\theta(X,Y,Z)}{\partial X^2} + \frac{\partial^2\theta(X,Y,Z)}{\partial Y^2}\,. \tag{12}$$

The dimensionless velocity profile adimensional defined by Eq. (11) can be written as

$$U(X,Y) = \frac{3n+1}{n+1}\left[1 - \left(\frac{X^2}{\alpha^2} + \frac{Y^2}{\beta^2}\right)^{\frac{n+1}{2n}}\right] \tag{13}$$

and the entry and boundary conditions

$$\theta(X,Y,Z) = 0 \quad , \ \{Z=0,\ (X,Y)\in\Omega\}; \tag{14}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial\eta^*} = 1 \,, \quad \{Z>0,\ \eta^* \perp \Gamma\}; \tag{15}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial X} = 0 \;, \ \{Z>0, \quad X=0\}; \tag{16}$$

$$\frac{\partial\theta(X,Y,Z)}{\partial Y} = 0 \;, \ \{Z>0, \quad Y=0\}. \tag{17}$$

The orthogonal elliptical coordinate system (u,v) is utilized to transform the original domain, with elliptical contour in the plane (X,Y), in one domain with rectangular contour in the transformed plane (u,v)

$$X = \alpha^* cosh(u)\, cos(v) \quad ; \qquad Y = \beta^*\, sinh(u)\ sin(v) \quad ; \qquad Z = z \tag{18}$$

$$\alpha^* = \alpha / cosh(u_o) \quad ; \quad u_o = \text{arctanh}(\beta / \alpha) \quad , \quad \{ 0 < \beta < \alpha \} . \tag{19}$$

The metric coefficients h_u(u,v) and $h_v(u,v)$, and the Jacobian $J(u,v)$ of the transformation of the system of coordinates (X,Y) to the system (u,v) are given by

$$h_u(u,v) = h_v(u,v) = h(u,v) = \alpha^* \sqrt{sinh^2(u) + sin^2(v)} \quad \text{and} \tag{20}$$

$$J(u,v) = \frac{\partial (X,Y)}{\partial (u,v)} = \alpha^{*2} [sinh^2(u) + sin^2(v)] \quad . \tag{21}$$

With these new variables, the equation of the elliptical contour can be written as

$$\left[\frac{X}{\alpha^* \cosh(u_o)} \right]^2 + \left[\frac{Y}{\alpha^* sinh(u_o)} \right]^2 = 1 \cdot \tag{22}$$

The energy equation is rewritten as

$$H(u,v) \frac{\partial \theta(u,v,Z)}{\partial Z} = \frac{\partial^2 \theta(u,v,Z)}{\partial u^2} + \frac{\partial^2 \theta(u,v,Z)}{\partial v^2} , \tag{23}$$

with $H(u,v)$ given by

$$H(u,v) = J(u,v) U(u,v) \tag{24}$$

and $U(u,v)$, the velocity profile invariable (u,v),

$$U(u,v) = \frac{3n+1}{n+1} \left[1 - \left(\frac{\alpha^{*2}}{\alpha^2} \cosh^2(u) \cos^2(v) + \frac{\alpha^{*2}}{\beta^2} sinh^2(u) sin^2(v) \right)^{\frac{n+1}{2n}} \right] . \tag{25}$$

The entry and boundary conditions in the new coordinate system are

$$\theta(u,v,Z) = 0 \quad , \{ Z = 0, \ (u,v) \in \Omega \} ; \tag{26}$$

$$\frac{\partial \theta(u,v,Z)}{\partial u} = h(u,v) \quad , \{ Z > 0, \ u = u_o \} ; \tag{27}$$

$$\frac{\partial \theta(u,v,Z)}{\partial u} = 0 \quad , \{ Z > 0, \ u = 0 \} ; \tag{28}$$

$$\frac{\partial \theta(u,v,Z)}{\partial v} = 0 \quad , \{ Z > 0, \ v = 0, \pi/2 \} . \tag{29}$$

2.2. Homogenization of the Boundary Conditions

In order to apply the GITT it is convenient to carry out the homogenization of the boundary conditions. To accomplish this, consider the following change of variables

$$\theta(u,v,Z) = \theta^*(u,v,Z) + \frac{u^2}{2 u_o} h(u_o,v) . \tag{30}$$

In this new variable, the energy equation transforms into

$$H(u,v) \frac{\partial \theta^*(u,v,Z)}{\partial Z} = \frac{\partial^2 \theta^*(u,v,Z)}{\partial u^2} + \frac{\partial^2 \theta^*(u,v,Z)}{\partial v^2} + G(u,v) , \tag{31}$$

$$G(u,v) \; = \; \frac{h(u_o,v)}{u_o} + \frac{\alpha^* u^2}{2\,h(u_o,v)\,u_o}\{\cos(2v) - [\frac{\alpha^*}{2\,h(u_o,v)}\,sin^2(2v)\,]^2\;\}\cdot \tag{32}$$

The entry and boundary conditions are redefined as:

$$\theta^*(u,v,Z\,) = -\frac{u^2}{2\,u_o}h(u_o,v) \quad , \quad \{\,Z = 0,\;\;(u,v) \in \Omega\,\}\;; \tag{33}$$

$$\frac{\partial\theta^*\,(u,v,Z\,)}{\partial\,u} \;=\; 0 \qquad , \quad \{\,Z > 0\,, \quad u = u_o\,\}\;; \tag{34}$$

$$\frac{\partial\theta^*\,(u,v,Z\,)}{\partial u} \;=\; 0 \qquad , \;\{\,Z > 0\,, \quad u = 0\,\}\;; \tag{35}$$

$$\frac{\partial\theta^*\,(u,v,Z\,)}{\partial\,v} \;=\; 0 \qquad , \;\{\,Z > 0\,, \quad v = 0\,,\pi\,/\,2\;\}\;. \tag{36}$$

2.3. Application of the GITT

To solve the energy equation, Eq. (31), submitted to the conditions given by Eq. (33) to (36), the Generalized Integral Transformed Technique is applied. In order to accomplish this, consider the following auxiliary eigenvalue problem for the variable v (Aparecido, 1997)

$$\frac{d^2\psi(v)}{dv^2} + \mu^2\psi(v) = 0 \quad , \quad \{\,0 \le v \le \pi/2\,\} \quad , \tag{37}$$

$$\frac{d\psi(v)}{d\,v} \;=\; 0 \qquad , \quad \{\;v = 0\;\}\;\;; \tag{38}$$

$$\frac{d\,\psi(v)}{d\,v} \;=\; 0 \qquad , \quad \{\;v = \pi\;/\,2\;\}\,. \tag{39}$$

The eigenvalues and eigenfunctions associated to this problem are

$$\mu_i \;=\; 2\;(i-1) \quad , \quad i = 1,2,3...\,, \;\text{and} \tag{40}$$

$$\psi_i(v) \;=\; cos(\,\mu_i\,v\,) \tag{41}$$

The above eigenfunctions exhibit the property of orthogonality that permit the development of the following transform-inverse pair

$$\tilde{\theta}_i^*(u,Z\,) = \int_0^{\pi/2} K_i(v)\theta^*(u,v,Z\,)\,dv\,, \tag{42}$$

$$\theta^*(u,v,Z\,) \;=\; \sum_{i=1}^{\infty} K_i(v)\tilde{\theta}_i^*(u,Z\,), \tag{43}$$

where $K_i(v)$ are the normalized eigenfunctions given by

$$K_i(v) \;=\; \frac{\psi_i(v)}{N_i^{1/2}} \quad , \tag{44}$$

$$N_i = \int_0^{\pi/2} \psi_i^2(v)\,dv = \begin{cases} \pi/2 & , \quad i=1 \\ \pi/4 & , \quad i>1 \end{cases} . \tag{45}$$

Operating the internal product of $K_i(v)$ and of $\theta^*(u,v,Z)$ with Eq. (31) and (37), respectively, and by making use of the boundary conditions given by Eq. (36), (38) and (39) the following coupled system of partial differential equations of second order can be obtained:

$$\sum_{j=1}^{\infty} A_{ij}(u)\,\frac{\partial \tilde{\theta}_j^{\,*}(u,Z)}{\partial Z} + \mu_i^2 \tilde{\theta}_i^{*}(u,Z) = \frac{\partial^2 \tilde{\theta}_i^{*}(u,Z)}{\partial u^2} + C_i(u) , \tag{46}$$

$$A_{ij}(u) = \int_0^{\pi/2} K_i(v)\, K_j(v)\, H(u,v)\, dv , \tag{47}$$

$$C_i(u) = \int_0^{\pi/2} K_i(v)\, G(u,v)\, dv . \tag{48}$$

Let us now consider the following eigenvalue problem relative to variable u:

$$\frac{d^2\phi(u)}{du^2} + \lambda^2 \phi(u) = 0 \quad , \quad \{ 0 \le u \le u_o \} \tag{49}$$

$$\frac{d\phi(u)}{du} = 0 \qquad , \quad \{ u = 0 \} ; \tag{50}$$

$$\frac{d\phi(u)}{du} = 0 \qquad , \quad \{ u = u_o \} . \tag{51}$$

The eigenvalues and the eigenfunctions for this new problem are:

$$\lambda_m = (m-1)\,\pi / u_o , \tag{52}$$

$$\phi_m(u) = \cos(\lambda_m u) . \tag{53}$$

The above eigenfunctions are orthogonal and permit the development of the transform-inverse pair

$$\bar{\tilde{\theta}}_{im}^{*}(Z) = \int_0^{u_o}\int_0^{\pi/2} K_i(v)\, Z_m(u)\, \theta^*(u,v,Z)\, dv\, du \quad , \tag{54}$$

$$\theta^*(u,v,Z) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} K_i(u)\, Z_m(u)\, \bar{\tilde{\theta}}_{im}^{*}(Z) , \tag{55}$$

where $Z_m(u)$ are the normalized eigenfunctions

$$Z_m(u) = \frac{\phi_m(u)}{M_m^{1/2}} , \tag{56}$$

$$M_m = \int_0^{u_o} \phi_m^2(u)\, du = \begin{cases} u_o & , \quad m=1 \\ \dfrac{u_o}{2} & , \quad m>1 \end{cases} . \tag{57}$$

For the computation of the transformed potential $\bar{\tilde{\theta}}_{im}^{*}(Z)$, the procedure is similar to that used to transform the energy equation relatively to the axis v. By making the internal product of $Z_n(u)$ and of $\tilde{\theta}_i^{*}(u,Z)$ with Eq. (46) and

(49), respectively, and using the boundary conditions, Eq. (34), (35), (50) and (51), the following coupled system of ordinary differential equations of first order is obtained

$$\sum_{n=1}^{\infty} \sum_{j=1}^{\infty} B_{ijnm} \frac{d\tilde{\bar{\theta}}^{*}_{jn}(Z)}{dZ} + (\mu_i^2 + \lambda_m^2)\, \tilde{\bar{\theta}}^{*}_{im}(Z) + D_{im} = 0 , \tag{58}$$

$$B_{ijnm} = \int_0^{u_o} Z_m(u)\, Z_n(u)\, A(u)\, du = \int_0^{u_o} \int_0^{\pi/2} K_i(v)\, K_j(v)\, Z_m(u)\, Z_n(u)\, H(u,v)\, dv\, du , \tag{59}$$

$$D_{im} = -\int_0^{u_o} Z_m(u)\, C_i(u)\, du = -\int_0^{u_o} \int_0^{\pi/2} K_i(v)\, Z_m(u)\, G(u,v)\, dv\, du . \tag{60}$$

Parameters B_{ijmn} and D_{im} can be integrated and, therefore, known. The solution of this system of equations, when submitted to the transformed entry condition

$$\tilde{\bar{\theta}}^{*}_{im}(0) = \int_0^{u_o} \int_0^{\pi/2} K_i(v) Z_m(u) \theta^{*}(u,v,0)\, dv\, du = -\int_0^{u_o} \int_0^{\pi/2} K_i(v) Z_m(u) \frac{u^2}{2u_o} h(u_o,v)\, dv\, du , \tag{61}$$

allows to obtain the transformed potential $\tilde{\bar{\theta}}^{*}_{im}(Z)$. Therefore, the temperature potential $\theta(u,v,Z)$ can be calculated numerically by Eq. (58), truncating the expansion for a given order $i = N$ e $m = M$

$$\theta(u,v,Z) = \sum_{i=1}^{N} \sum_{m=1}^{M} K_i(v)\, Z_m(u)\, \tilde{\bar{\theta}}^{*}_{im}(Z) + \frac{u^2}{2\,u_o} h(u_o,v) \tag{62}$$

Obviously, the higher N and M the greater the accuracy of the results.

2.4. Calculation of the Bulk Temperature and of the Nusselt Number

The bulk temperature of the fluid at a given duct section can be determined by means of an energy balance

$$\dot{q}''_o\, P\, z = \rho\, \bar{u}\, A\, c_p\, (T_{av} - T_e) . \tag{63}$$

Using the dimensionless variables defined by Eq. (7) to (11), the bulk temperature in its dimensionless form is determined

$$\theta_{av}(Z) = 4\, Z . \tag{64}$$

However, the dimensionless bulk temperature can also be determined as

$$\theta_{av}(Z) = \frac{D_h^2}{A_{sc}} \int_{\Omega} \theta(X,Y,Z)\, U(X,Y)\, d\Omega . \tag{65}$$

Thus, in the plane (u,v), $\theta_{av}(Z)$ is given by

$$\theta_{av}(Z) = \frac{D_h^2}{A_{sc}} \int_0^{\pi/2} \int_0^{u_o} \{ [\sum_{i=1}^{N} \sum_{m=1}^{M} K_i(v)\, Z_m(u)\, \tilde{\bar{\theta}}^{*}_{im}(Z)] + \frac{u^2}{2\,u_o} h(u_o,v)\, H(u,v) \}\, du\, dv . \tag{66}$$

Eq. (64) and (66) are appropriate to verify the accuracy of the numerical results when the expansion is truncated in orders $i = N$ and $m = M$, because they should provide the same results when utilizing an infinite number of terms in the series expansion.

The average wall temperature is obtained by integration of the temperature profile at the wall

$$\theta_{w,av}(Z) = \frac{4\, D_h}{P} \int_0^{\pi/2} \sum_{i=1}^{N} \sum_{m=1}^{M} K_i(v)\, Z_m(u_o)\, \tilde{\bar{\theta}}^{*}_{im}(Z)\, h(u_o,v)\, dv . \tag{67}$$

The Nusselt number is defined by $Nu(z) = \dfrac{h(z)\,D_h}{k}$, with h(z) written in the form

$$h(z) = \frac{q_o''(z)}{\overline{T}_w(z) - T_{av}(z)} . \tag{68}$$

Using the dimensionless variables

$$Nu(Z) = \frac{1}{\overline{\theta}_w - \theta_{av}} \tag{69}$$

The average Nusselt number is obtained integrating numerically Eq. (69)

$$\overline{Nu}(Z) = \frac{1}{Z}\int_0^Z Nu(Z')\,dZ' . \tag{70}$$

The thermal entry length, L_{th}, (Shah & London, 1978 : 50), is defined as the position where the local Nusselt number is 5% higher than the Nusselt number in the region where the flow is fully developed. Thus,

$$L_{th} \equiv \text{ positive root of } \{1.05\ Nu(\infty) - Nu(Z) = 0\} . \tag{71}$$

3. Results and Discussion

For the calculation of the transformed potential $\tilde{\overline{\theta}}_{im}(Z)$, the expansion, given by Eq. (62), was truncated for several orders *M* and *N*. It was observed that the convergence is relatively more slow when the aspect ratio $b/a \to 1$ and when the behavior index $n \to 0$. For truncations after orders of M = 30 and N = 30 it was verified that the values of the Nusselt numbers calculated for $Z > 0.0001$ converged around 4 digits, or more, for all cases analyzed. Thus, the results in this work were obtained with the truncation of the expansion in M=30 and N=30.

The integration involved in the calculations of the perimeter, parameters B_{ijmn} and D_{in}, the bulk temperature and other parameters, were performed numerically by the Gauss Method (36 points). The system of differential equations for the transformed potential, Eq. (58), was solved using the routine DIVPAG of the Library IMSL, (Visual Numerics, 1994). The heat transfer parameters were obtained for several ellipsis eccentricities (b/a = 1/8, 1/4, 1/2, 8/10, 9/10 and 99/100) and for several behavior indexes (n = 0.1, 0.15, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0, 0.3 and 5.0).

The difference between the bulk temperature obtained by Eq. (64) and the bulk temperature calculated by Eq.(66) is less than 10^{-5} for $Z > 0.0001$ for all the cases analyzed. Tables 1 and 2 present results obtained for the average wall temperature and for the Nusselt number as a function of the behavior index n of the fluid for flow in ducts of elliptical section with aspect ratio b/a = 0.5. The behavior of the local Nusselt number in the flow is shown in Fig. 2 as a function of the behavior index n, for aspect ratio b/a = 0.5, and in Fig. 3 as a function of the aspect ratio b/a, for behavior index n = 0.5.

Results obtained for the Nusselt number in the region where the flow is thermally fully developed can be seen in Table 4. It may be observed in Fig. 4 that the limit Nusselt number is more sensitive to the variation of the aspect ratio, when b/a < 0.5, and to the variation of the behavior index n for the pseudo-plastic fluids ($n < 1$). It can also be observed that the Nusselt number is smaller for dilatant fluids ($n > 1$) when compared to pseudo-plastic fluids due to the fact that the apparent viscosity increases with n for the fluids that obey the Power Law. Results were found in the literature only for the particular case of flow of Newtonian fluids, $n = 1$, (Iqbal et al., 1972). As it can be verified, on Table 3, there is an excellent agreement between the results for this case.

Table 1. Average wall temperature for aspect ratio *b/a* = 0.5

Z	*n* = 0.1	*n* = 0.15	*n* = 0.2	*n* = 0.5	*n* = 1.0	*n* = 1.5	*n* = 2.0	*n* = 5.0
0.00010	0.02550	0.02790	0.02958	0.03430	0.03683	0.03788	0.03845	0.03958
0.00020	0.03275	0.03573	0.03783	0.04374	0.04693	0.04825	0.04897	0.05040
0.00050	0.04591	0.04987	0.05267	0.06063	0.06496	0.06676	0.06774	0.06969
0.00100	0.05973	0.06461	0.06809	0.07805	0.08351	0.08578	0.08703	0.08949
0.00200	0.07844	0.08441	0.08870	0.10116	0.10805	0.11092	0.11250	0.11563
0.00500	0.11471	0.12237	0.12799	0.14465	0.15404	0.15798	0.16016	0.16448
0.01000	0.15612	0.16524	0.17206	0.19273	0.20461	0.20964	0.21242	0.21798
0.02000	0.21801	0.22872	0.23689	0.26235	0.27736	0.28377	0.28733	0.29447
0.05000	0.36141	0.37419	0.38417	0.41648	0.43624	0.44483	0.44964	0.45933
0.10000	0.57223	0.58607	0.59698	0.63290	0.65535	0.66521	0.67075	0.68199
0.20000	0.97675	0.99088	1.00205	1.03904	1.06233	1.07260	1.07840	1.09017
0.50000	2.17733	2.19147	2.20265	2.23968	2.26301	2.27331	2.27912	2.29094
1.00000	4.17733	4.19147	4.20265	4.23968	4.26301	4.27331	4.27912	4.29094

Table 2. Local Nusselt number for aspect ratio *b/a* = 0.5

Z	*n* = 0.1	*n* = 0.15	*n* = 0.2	*n* = 0.5	*n* = 1.0	*n* = 1.5	*n* = 2.0	*n* = 5.0
0.00010	39.833	36.357	34.265	29.500	27.447	26.681	26.279	25.521
0.00020	31.299	28.625	27.005	23.288	21.677	21.074	20.758	20.161
0.00050	22.776	20.891	19.736	17.056	15.882	15.442	15.211	14.774
0.00100	17.943	16.499	15.604	13.504	12,577	12.228	12.045	11.697
0.00200	14.196	13.087	12.391	10.735	9.9953	9.7163	9.5694	9.2909
0.00500	10.559	9.7687	9.2603	8.0226	7.4606	7.2474	7.1349	6.9212
0.01000	8.6119	7.9846	7.5725	6.5475	6.0749	5.8948	5.7996	5.6187
0.02000	7.2456	6.7238	6.3739	5.4840	5.0670	4.9075	4.8232	4.6627
0.05000	6.1955	5.7408	5.4296	4.6194	4.2329	4.0844	4.0058	3.8561
0.10000	5.8060	5.3742	5.0766	4.2936	3.9163	3.7707	3.6934	3.5462
0.20000	5.6578	5.2388	4.9491	4.1833	3.8120	3.6683	3.5920	3.4462
0.50000	5.6391	5.2227	4.9347	4.1723	3.8022	3.6588	3.5827	3.4371
1.00000	5.6390	5.2227	4.9347	4.1723	3.8022	3.6588	3.5827	3.4371

Table 3. Limit Nusselt number as a function of the aspect ratio and of the behavior index.

Z	*n* = 0.1	*n* = 0.15	*n* = 0.2	*n* = 0.5	*n* = 1.0	*n* =1.5	*n* = 2.0	*n* = 5.0	Ref. (Iqbal)
0.125	1.8188	1.5894	1.4416	1.0921	0.9432	0.8891	0.8611	0.8091	0.9433
0.250	3.9066	3.5263	3.2709	2.6274	2.3331	2.2225	2.1645	2.0553	2.333
0.500	5.6390	5.2227	4.9347	4.1723	3.8022	3.6588	3.5827	3.4371	3.802
0.800	6.1570	5.7451	5.4578	4.6865	4.3052	4.1562	4.0766	3.9239	-
0.900	6.2026	5.7913	5.5042	4.7326	4.3507	4.2012	4.1214	3.9682	-
0.990	6.2173	5.8056	5.5183	4.7462	4.3639	4.2143	4.1344	3.9810	4.364

Table 4. Thermal entry length as a function of the aspect ratio and of the behavior index.

Z	*n* = 0.1	*n* = 0.15	*n* = 0.2	*n* = 0.5	*n* = 1.0	*n* =1.5	*n* = 2.0	*n* = 5.0
0.125	1.35	1.32	1.30	1.24	1.21	1.20	1.19	1.18
0.250	0.31	0.30	0.29	0.29	0.29	0.29	0.29	0.29
0.500	0.079	0.079	0.080	0.081	0.081	0.084	0.084	0.085
0.800	0.042	0.042	0.043	0.044	0.047	0.047	0.048	0.049
0.900	0.040	0.040	0.040	0.042	0.044	0.045	0.046	0.047
0.990	0.039	0.039	0.039	0.041	0.043	0.044	0.045	0.046

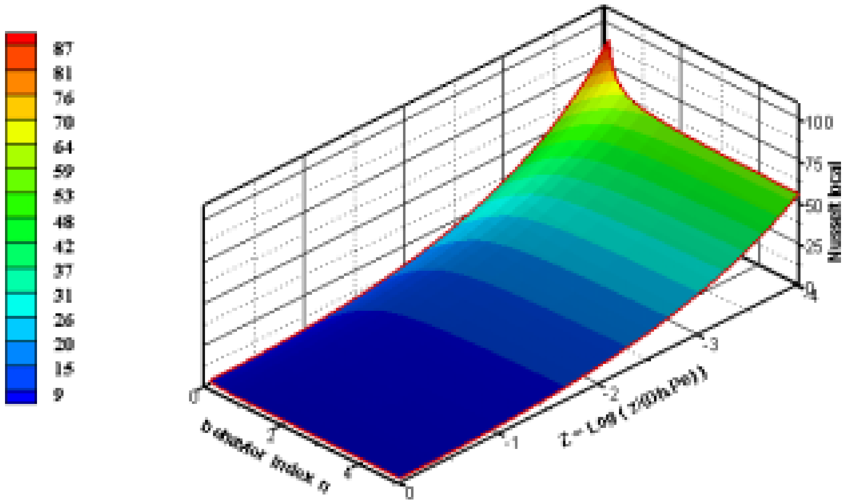

Figure 2. Local Nusselt number as a function of the behavior index for aspect ratio *b/a* = 0.5.

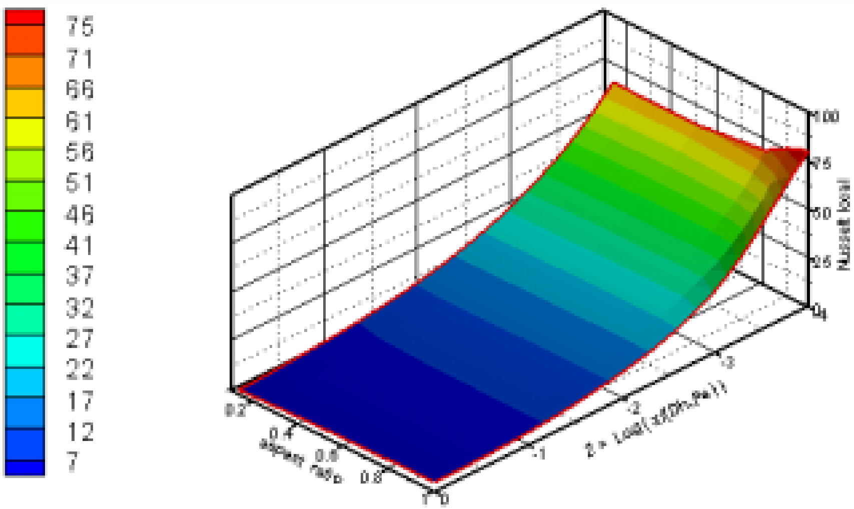

Figure 3. Local Nusselt number as a function of the behavior index *n* = 0.5.

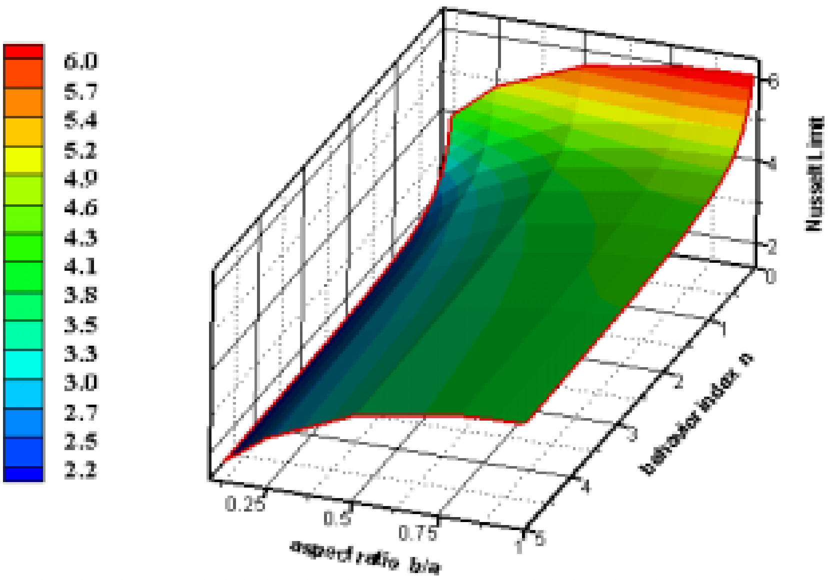

Figure 4. Limit Nusselt number as a function of the aspect ratio and of the behavior index.

Finally, Table 4 exhibits the results obtained for the thermal entry length L_{th}. It may be observed that the thermal development is less sensitive to the variation of the flow behavior index and strongly dependent of the aspect ratio *b/a*.

The thermal development diminishes with the aspect ratio, and the thermal entry length increases substantially when $b/a \to 0$.

4. Conclusion

In the present work, the heat transfer parameters for laminar flow of non-Newtonian fluids inside ducts of section elliptical section, subjected to boundary condition of second kind were determined. The problem considered here was the thermally developing flow of fluids that follow the Power Law. In order to enable the application of the boundary conditions, an adequate change of variables was performed by transforming the real elliptical contour of the duct section into a rectangular contour in the new coordinate system. As the energy conservation equation in this new system has non-separable variables, the GITT was applied to obtain the temperature distribution in the flow. The expansion in eigenfunctions for the temperature potential resulted in a relatively slow convergence, being necessary to truncate the series with at least 30 terms for variables u and v, to obtain a satisfactory convergence. Parameters of interest such as the average wall temperature, local, average and limit values of the Nusselt number and thermal entry length were obtained for several aspect ratios b/a of the elliptical section and for several indexes of behavior n of the flow. The results presented here exhibit a strong dependence of the heat transfer parameters with the aspect ratio. The index of behavior of the flow n has a markedly influence in pseudo-plastic fluids in the region where $n \to 0$. The results obtained showed an excellent agreement with those found in the literature, for Newtonian fluid only (n = 1).

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