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# NATURAL CONVECTION HEAT TRANSFER IN AN ARRAY OF VERTICAL CHANNELS WITH TWO-DIMENSIONAL HEAT SOURCES: PLATES UNIFORM AND NON-UNIFORM HEATING

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***Abstract.*** *An experimental and numerical analysis was performed to investigate the conjugate conduction-convection heat transfer problem in an array of vertical, parallel plates, forming open channels with heated protruding elements attached to one of the walls. It was analyzed the situation of uniform and non-uniform heating of the plates. It was varied both the distance between plates and power dissipated per plate. In the situation of non-uniform heating, in each plate and for a specified total heat generation rate, one element was electrically supplied with a power level different from the others and the influence of the location of this element on the temperature distribution was investigated. The SIMPLEC algorithm, based on the finite volume method, was used for solving the pressure velocity coupling. Numerical and experimental temperature profiles were compared and good agreements was observed.*

***Keywords****. Natural Convection, Protruding Heat Sources, Non-Uniform Heating, Vertical Channels*

## 1. Introduction

In the last decades natural convection between vertical plates has been the focus of several studies due to its application in various thermal systems, such as in the cooling of electronic components, chemical processes and solar energy systems. The first work on natural convection in open vertical channels was the experimental study of Elenbaas (1942). For the case of channels with smooth plates, this heat transfer problem has been studied for several types of boundary conditions imposed at the channel walls, such as uniformly symmetric or asymmetric heated plates, one plate insulated and the other with a discrete heated section, and two discrete heated plates. Many of them were reviewed by Incropera (1988), and by Peterson and Ortega (1990), who carried out comprehensive reviews on thermal control of electronic equipment. The concept of partial heating, which is a particular case of non-uniform heating of the plates, has been explored in some studies. Wirtz and Haag (1985) obtained experimental results for symmetric isothermal heated plates with an attached unheated entry portion. Lee (1994) analyzed numerically channels formed by isothermal or isoflux plates with unheated extensions placed near the entrance or exit of the channel. The boundary layer approximation was used in the analysis. Campo et al (1999) reformulated the problem analyzed by Lee (1994) using an elliptic model for the conservation equations. There are also several works dealing with channels with multiple protruding heat sources, mainly in the last decade. Fujii et al (1996) analyzed numerical and experimentally the natural convection heat transfer to air from an array of vertical parallel plates with protruding discrete and densely distributed heat sources. Behind and Nakayama (1998) performed numerical simulations on natural convection considering the same geometry analyzed by Fujii el al (1996) for several values of substrate thermal conductivity and channel width. Bessaih and Kadja (2000) carried out numerical simulations on turbulent natural convection cooling of three identical heated ceramic components mounted on a vertical adiabatic channel wall. It was investigated the effect on the cooling of the spacing between components and of the removal of heat imposed in one of the components. The present work is aimed at analyzing the conjugated natural convection conduction heat transfer problem in an array of vertical channels with two-dimensional heat sources mounted in one of the plate surfaces. It was analyzed the situations of uniform and non-uniform heating of the plates. In the situation of non-uniform heating, in each plate and for a specified total heat generation rate, one element was provided with a power level different from the others, and the influence of the location of this element on the temperature distribution was investigated. Numerical solutions were obtained to the full elliptic two-dimensional steady-state Navier-Stokes equations using the SIMPLEC algorithm. It was varied both the distance between plates and total power dissipated per plate. The motivation for this work is that, in spite of the great number of studies on this subject, the problem of non-uniform heating in channels with protruding heat sources is not usually analyzed, although it is a common situation in electronic equipment.

## 2. Experimental Analysis

A schematic view of the experimental apparatus is showed in Fig. (1).

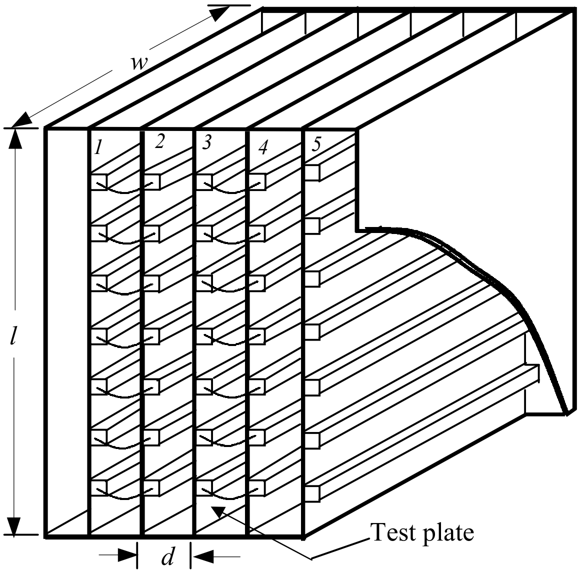


Figure 1 - Schematic view of experimental apparatus.

An array of five fiber glass plates was accommodated in a metallic structure that is used in telecommunications devices and that allows variation of the distance between plates. The plates are numbered from 1 to 5 for convenience. Each plate was 365mm heigth (*l)* and 340mm width (w*)*, with 1,5mm thickness and it had seven heat sources mounted on its surface. The protruding heat sources were constructed from two aluminum bars 12,25mm height, 340mm width and 6,125mm thickness, with one resistance wire between them. The elements resulted were screwed into the fiber glass plates and an equal spacing of 34,5mm was adopted. The protruding heat sources were connected in a way that any desired power level could be set to any given element, independently of the others. Power was supplied to the plates by regulated D.C. sources and both sides of the channels were closed to prevent lateral air flowing. In order to reduce the radiation heat transfer influence, the heat sources were polished with diamond paste. The structure was maintained about 1m from the ground and placed in a quiet room. Temperature measurements were obtained by using calibrated thermocouples 36 AWG type J, a switch and a digital thermometer. Special care was taken to embed the thermocouples in the aluminum and in the fiber glass surfaces. A very small hole was drilled in their surfaces, which was covered with a thin layer of thermal paste, and the thermocouples were fixed with epoxy adhesive. Experiments were performed varying the distance between plates and the total heat generation rate, $Q_T$, set the same for all plates during the tests. The distance between plates was ranged from 2 to 4cm, what corresponds to ratio (L = *l*/d) between 9 and 18, and the total heat generation per plate from 20 to 60W. The Rayleigh number was ranged from $1.x10^4$ to $8x10^5$.

Figure (2) represents a test plate. The symbols × and  indicate the points in the elements and plate surface where temperature was measured, respectively. The measurements were done at plate 3. Symbol **O** indicates air temperature measurement at the exit and at the entrance of the channel formed by plates 3 and 4. As indicated in Fig. (2), in each element, the temperature was measured in three points, and the average of these was taken as the temperature in the element. The temperature measurements were taken in steady-state condition, which was obtained after about 1h.

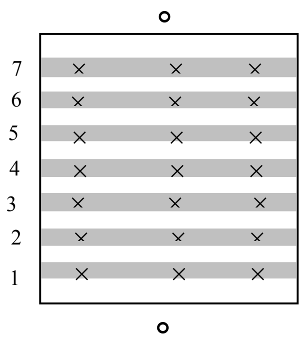


Figure 2 - Test plate.

In the situation of non-uniform heating of the plates, one protruding heat source was electrically heated with a power level twice the value supplied to the other protrusions. Experiments and numerical simulations were carried out setting the highest power to the protruding heat sources 1, 4 and 7, as indicated in Fig. (3).

In Fig. (3) it is schematically presented the cases of non-uniform heating of the plates.

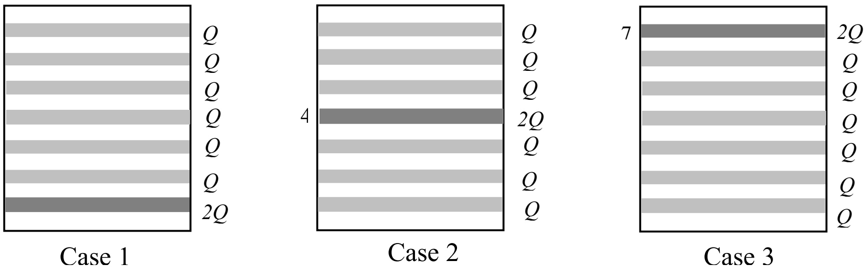


Figure 3 - Non-uniform heating cases.

In the case of non-uniform heating, the total heat generation rate in the plate, $Q_T$, plate is given by

$$Q_T = Q(6+2) = 8Q \tag{1}$$

**3. Numerical Analysis**

Figure (4) shows the physical model and coordinate system. An infinite number of plates is placed in a vertical parallel arrangement with equal spacing, *d*. Each plate has the same height, *l,* and thickness, *b*. On one surface of the plates there are seven two-dimensional protruding heat sources mounted, separated by the distance $s_p$. All heat sources have equal dimensions. The total heat generation in each plate is set to be the same. The solution domain is chosen to be the region bounded by the broken line in Fig. (4).

The flow is assumed to be at steady state, laminar and two-dimensional. The air thermo-physical properties are assumed to remain constant, except for the density in the buoyancy term of the momentum equation, which is assumed to follow the Boussinesq approximation. The heat conduction in the plates and in the heat sources is taken into account. It is admitted uniform heat generation within the heat sources. The radiative heat transfer among the plates and the ambient is not accounted. The harmonic mean formulation suggested by Patankar (1980) was used to handle abrupt variations in thermophysical properties, such as the thermal conductivity across the interface of two different media.

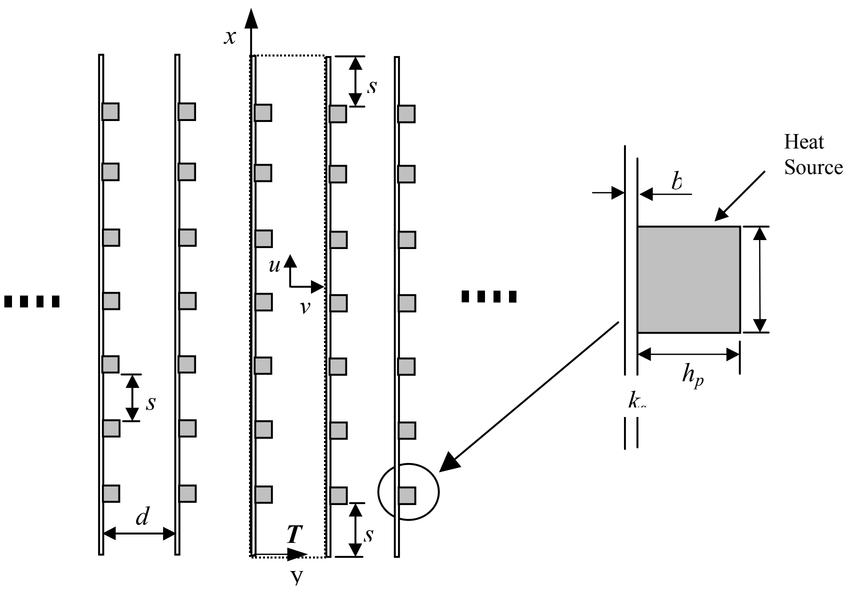


Figure 4 - Physical model and coordinate system.

The governing equations are expressed in dimensionless form as follow:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

Momentum equation in *X* direction

$$U\frac{\partial U}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{Pr}{Ra}\right)^{1/2}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + \theta \tag{3}$$

Momentum equation in *Y* direction

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{Pr}{Ra}\right)^{1/2}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) \tag{4}$$

Energy equation

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{Pr}{Ra}\right)^{1/2}\frac{k_i}{k_{air}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+f\ x\ S^* \tag{5}$$

where:

- f=1 for the protruding heat sources and f=0 for the rest of the domain;
- $k_i$ is thermal conductivity of the correspondent region. i=1,2 and 3, for air, plate and heating sources, respectively;
- The source term S*, depends on the configuration being given by,

$$S^*=\frac{2L}{npt\times XPT\times YPT\times\sqrt{Pr\,Ra}} \tag{6}$$

for the case of uniform heating of the plates, and in the case of non-uniform heating of the plates by,

$$S^*=\frac{4L}{(npt+1)\times XPT\times YPT\times\sqrt{Pr\,Ra}} \tag{7}$$

for the protruding heat sources with the highest power dissipation level, and by,

$$S^*=\frac{2L}{(npt+1)\times XPT\times YPT\times\sqrt{Pr\,Ra}} \tag{8}$$

for the other elements. Where *npt* is the number of protruding heat sources, *XPT* and *YPT* are the dimensionless element dimensions.

A unique form is used to express the energy equation in the fluid and the solid regions. It was possible by using a very high value for the dynamic viscosity in the solid regions in order to make the velocities in these regions equal to zero.

The dimensionless variables in the above equations are defined by

$$X=\frac{x}{d},\quad Y=\frac{y}{d},\quad L=\frac{l}{d},\quad B=\frac{b}{d},\quad H_p=\frac{h_p}{d},\quad L_p=\frac{l_p}{d},$$
$$\theta=\frac{T-T_i}{q''d/k_{air}},\quad U=\frac{u}{u_o},\quad V=\frac{v}{u_o},\quad P=\frac{p-p_h}{\rho u_o^2} \tag{9}$$
$$Ra=\frac{q''qd^4\beta}{k_f\nu_f\alpha_f},\quad Pr=\frac{\nu}{\alpha}$$

where *q''* is defined based on the total surface area of the plate as

$$q''=\frac{Q}{2A}=\frac{Q}{2Lw} \tag{10}$$

and the reference velocity, $u_o$ is defined by

$$u_o=\left(d^2 g\beta\frac{q''}{k_{air}}\right)^{1/2} \tag{11}$$

The boundary conditions are:

Channel entrance (X = 0): $\theta = V = 0$; $P = -0.5U_m^2$

Channel Walls (Y = 0 and Y = B + 1): U = V = 0

A periodic boundary condition is imposed with respect to the temperature at the plate surface, i.e: $\theta(X,0) = \theta(X,B+1)$

Channel exit (Y = l/d): $\frac{\partial U}{\partial X}=\frac{\partial V}{\partial X}=\frac{\partial \theta}{\partial X}=P=0$

The pressure values at the channel entrance and exit were obtained from potential flow theory.

The governing equations were discretized using the control volume formulation described by Patankar (1980), where velocity control volume are staggered with respect to the pressure and temperature control volumes. Coupling of the pressure and velocity fields was treated using the SIMPLEC algorithm (Van Doormal and Rathby, 1984), with the power-law scheme. The conjugate problem of conduction and convection was dealt by using the harmonic averaging thermal conductivity at the interfaces solid-fluid, Patankar (1980). The periodic boundary condition imposed with respect to the temperature at the plate surface was handled by using the CTDMA algorithm (Cyclic TriDiagonal Matrix Algorithm), from Patankar et al (1977), to solve the discretized energy equation. No specification of the wall temperature is required in this formulation. This hypothesis was considered to be valid because the measured longitudinal temperature (*x* direction) in the inner three plates are almost the same. Negligible contact resistance between heat source and plate was assumed. This hypothesis was considered because experimental measurements showed that the temperature difference between the protrusions top surface and base was very small. The equations were solved in a non-uniform grid crowded near the solid walls. The number of nodes was varied from 622~56 to 622~64, depending on the distance between plates. The convergence of the iterative procedure was tested by the following criterion

$$\frac{\left|\Gamma_{i,j}^{n}-\Gamma_{i,j}^{n-1}\right|_{max}}{\left|\Gamma_{i,j}^{n}\right|_{max}}\leq 5\times 10^{-6} \qquad (12)$$

where $\Gamma$ stands for $U$, $V$, $\theta$ and the maximum residual in the continuity equation.

## 4. Results and discussions

Figures (5) and (6) show temperature excess profiles, for the situation of uniform heating of the plates, for several values of power, and distance between plates equal to 2cm and 3.5cm. The temperature excess, $\Delta T$, is defined as the difference between the heat source temperature, $T_p$,and the air inlet temperature, $T_o$.

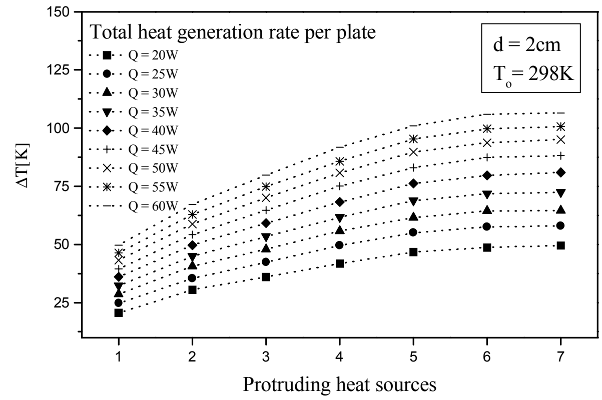


Figure 5 - Temperature excess profiles - Uniform heating - d = 2cm.

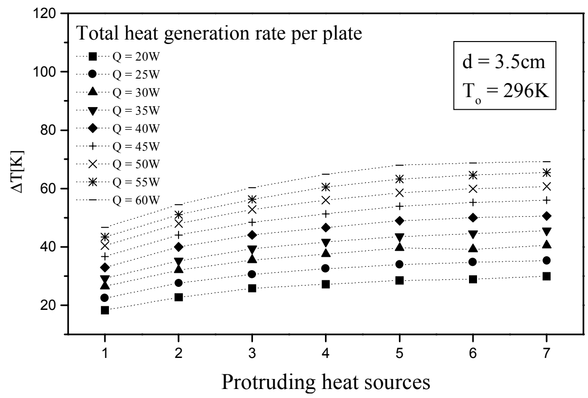


Figure 6 - Temperature excess profiles - Uniform heating - d = 3.5cm.

Figures (7) and (8) show temperature excess profiles, for the case 1 of non-uniform heating of the plates, for several values of power, and distance between plates equal to 2 cm and 3.5cm.

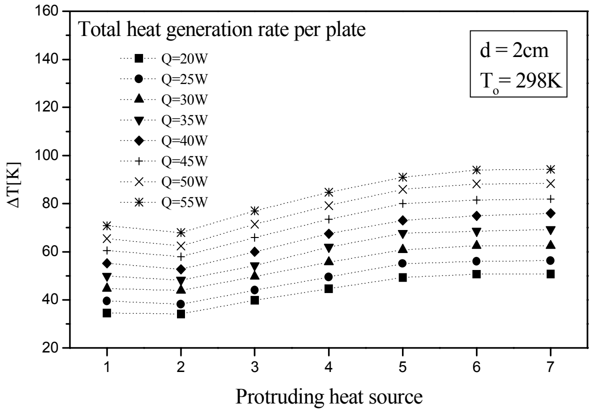


Figure 7 - Temperature excess profiles - Non-uniform heating - case 1- d = 2cm.

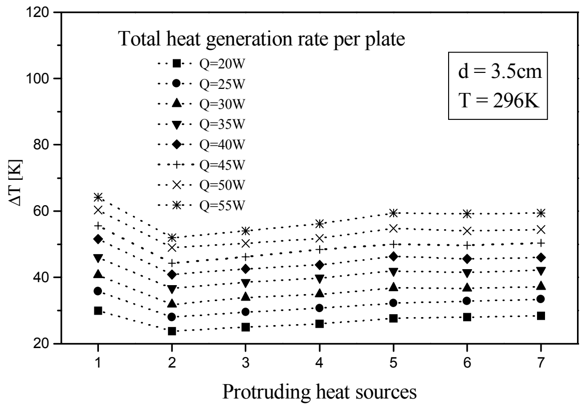


Figure 8 - Temperature excess profiles - Non-uniform heating - case 1 -d = 3.5cm.

Observing Figs. (5) to (8), it can be noticed that temperature profiles along the plate present slight variations with the increase in the heat rate dissipated per plate. However, the temperature gradient along the plate becomes less accentuated with the increase in the distance between plates. This behavior was observed for all heating conditions analyzed.

Figures (9) to (12) present isotherms for the distance between plates equal to 2cm and total heat generation rate of 25W. In Fig. (9) it is shown isotherms for the situation of uniform heating and in Figs. (10) to (12) for the non-uniform heating conditions of the plates. The channel is displayed in the horizontal direction for convenience.

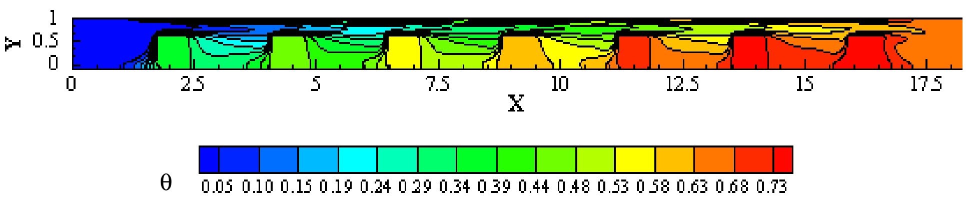


Figure 9 - Isotherms - $d$ = 2cm - $Q_T$ = 25W – uniform heating - $u_o$ = 0.11m/s.

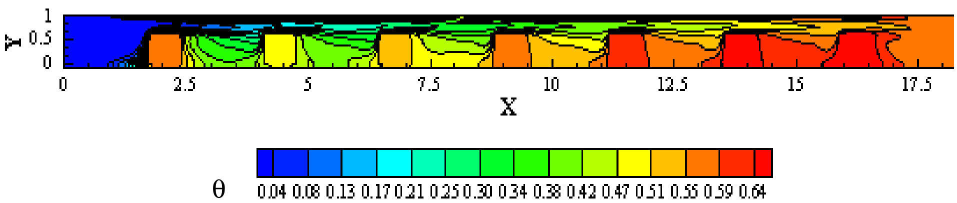


Figure 10 - Isotherms - $d$ = 2cm - $Q_T$ = 25W – non-uniform heating - case 1 - $u_o$ = 0.116m/s.

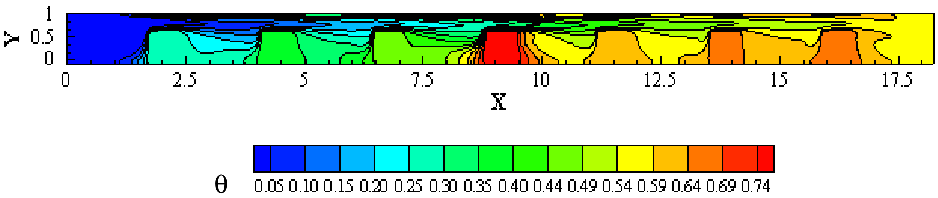


Figure 11 - Isotherms - $Q_T = 25$W - $d = 2$cm - non-uniform heating - case 2 - $u_o = 0.10$ m/s.

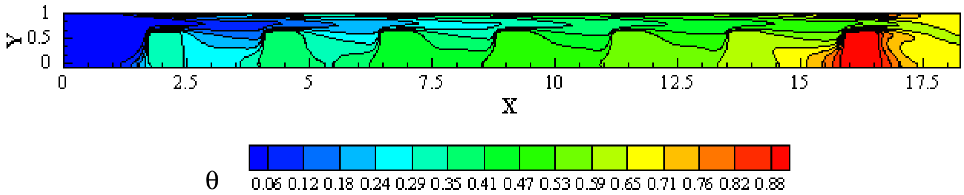


Figure 12 - Isotherms - $Q_T = 25$W - $d = 2.5$cm - non-uniform heating - case 3 - $u_o = 0.096$m/s.

From Figs. (9) to (12) it can be noticed that the isothermal lines are crowded on the surfaces that are parallel to the main flow direction and near the protrusions bottom corner. This means that the heat transfer rates are higher in these regions. In Figs. (10) to (12) it can be observed also that the isothermal lines are even more crowded around the protrusion with the highest dissipation level, which means a intensification of the heat transfer rate in these elements. This intensification can be confirmed observing the numerical Nusselt number profile, based in the distance between plates and the air inlet temperature, presented in Fig. (13).

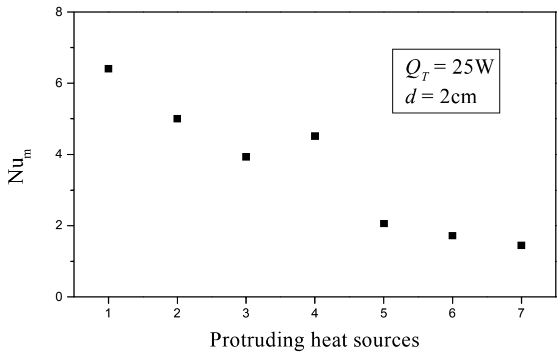


Figure 13 - Averaged Nusselt number in the protruding heat sources - non-uniform heating - case 2 - $Q_T = 25$W.

A more elevated Nusselt number value was observed in all the three cases of non-uniform heating and for all values of distance and power simulated. This occurs because the increment in the dissipated power level is not followed by a proportional temperature increment.

Comparing the air velocity values in the channel in the three cases of non-uniform heating, it can be noticed a decrease in the air flow rate as the most heated protuberance moves towards the channel exit.

Figures (14) and (15) show streamlines for the same values of total heat generation rate and distance between plates.

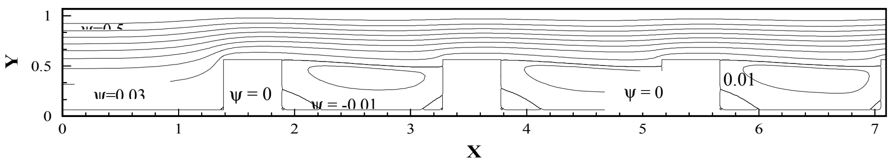


Figure 14 - Streamlines between the channel entrance and the forth protrusions - $Q_T = 25$W - $d = 2.5$cm.

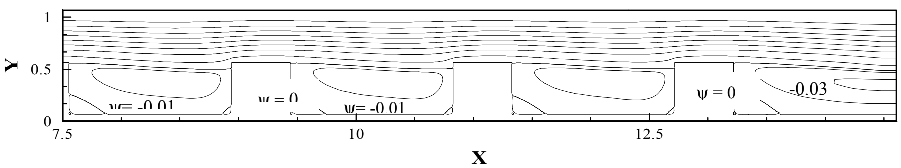

Figure 15 - Streamlines between the forth and the last protrusions - $Q_T$ = 25W - $d$ = 2.5cm.

It is observed that recirculation regions are formed between the protrusions, and that the flow pattern around each protrusion is almost the same in the region between the second and the fifth protrusions. It was not observed relevant variation in the flow pattern with heating conditions of the plates.

In Fig. (16) and (17) it is compared numerical and experimental values of dimensionless temperature excess for the value of total heat generation rate per plate equal to 25W and plate spacing equal to 2cm.

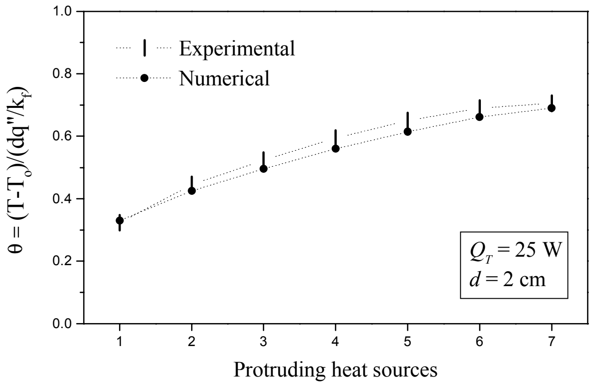


Figure 16 - Numerical and experimental values of temperature excess - uniform heating of the plates.

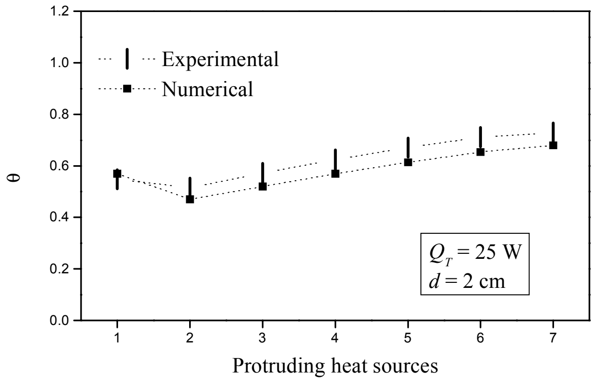


Figure 17 - Numerical and experimental values of temperature excess - non-uniform heating of the plates - case 1.

As can be noted from Fig. (16) and Fig. (17), good agreement was observed between numerical and experimental results. For the distances 2cm and 2.5cm, differences of about 8% were observed, while for the greatest distances, 3cm and 3.5cm differences were around 15%.

In Fig. (18) and Fig. (19) it is compared experimental temperature excess profiles for the tree cases of non-uniform heating of the plates. In Fig. (16) it is shown temperature excess curves for the distance 2cm and power 25W, and in Fig. (17) for the distance 3cm and power 40W.

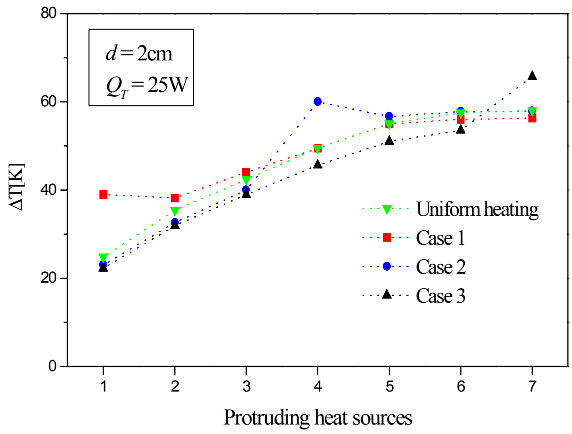


Figure 18 - Temperature excess profiles - $d$ = 2cm - $Q_T$ = 25W.

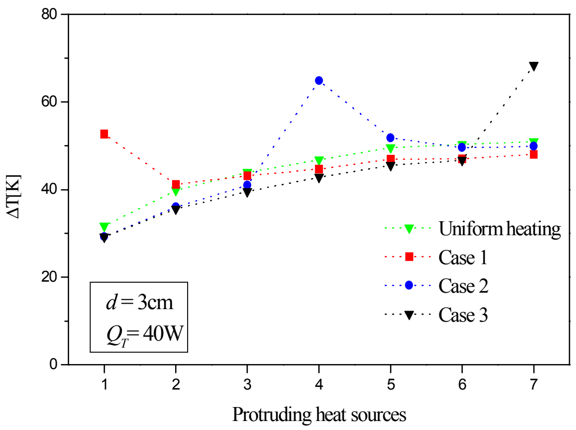


Figure 19 - Temperature excess profiles - d = 3cm - $Q_T$ = 40W.

Comparing the temperature profiles for cases 2 and 3, it can be noted that, for the first three heated sources, the curves are almost coincident. However, comparing the curves for case 1 the case 2, it can be noticed that the temperatures of the second and third elements are more elevated in case 1 than in case 2, and comparing the curves for cases 1 and 3, it can be observed that the temperature of the first six elements is also more elevated in case 1. This behavior was observed for all values of distance and power analyzed. Based on those results, it can be concluded that the position of the most heated protuberance only slightly affects the temperature of the upstream elements. On the other side, the thermal wake from the upstream elements cause a temperature increase on the downstream protuberances. Comparing maximum temperature values on the plate in non-uniform heating cases 1, 2 and 3 and uniform heating, it is observed a higher value in case 3, followed by case 2 with the lowest temperature excess value corresponding to the uniform heating situation. This behavior was noticed numerically and experimentally in all distances except for the 2cm plate spacing situation, where the uniform heating maximum temperature excess value was higher than in case 1. Therefore, when there are temperature limitations, positioning a higher dissipating element in the channel entrance region can be an effective way to achieve a higher power dissipation per plate than a uniform heating situation.

## 5. Conclusions

Natural convection heat transfer from an array of vertical, parallel plates, forming open channels containing heated protruding elements attached to one of the walls, was analyzed both numerically and experimentally. The obtained results suggest the following conclusions:

The experimental results show that the temperature excess profiles present slight variation with the increase in the heat rate dissipated per plate. However, the temperature gradient along the plate becomes less accentuated with the increase in the distance .

Good agreement between numerical and experimental results was verified. The differences between numerical and experimental results ranged from 8% to 15%. The highest difference was verified for the highest values of distances between plates and power dissipated per plate.

In the non-uniform heating situation it was observed that the position of the most heated protuberance only slightly affects the temperature of the upstream elements. On the other side, the thermal wake from the upstream elements cause a temperature increase on the downstream protuberances. Comparing maximum temperature values on the plate in cases 1, 2,3 and uniform heating, the lowest value was observed in the uniform heating situation, except for the distance of 2cm, where the uniform heating maximum temperature excess value was higher than in case 1. Therefore, when there are temperature limitations, positioning a higher dissipating element in the channel entrance region can be an effective way to achieve a higher power dissipation per plate than a uniform heating situation.

## 6. Acknowlewdgents

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