

THE HEAT TRANSFER ANALYSIS OF THE TURBULENT FLOW OF VISCOELASTIC FLUIDS

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Abstract. A recently developed viscoelastic $k-\varepsilon$ turbulence model is used to analyze the reduction in heat transfer coefficient of viscoelastic fluid. The new dynamic turbulence model is able to link fluid rheology and turbulence and constitutes an alternative approach to the prediction of turbulent flow of drag reducing fluids. Here, the momentum equations are numerically solved for a fully developed pipe flow and the flow field solution is used to solve the thermal energy equation, in order to obtain the heat transfer coefficient along the developing boundary-layer. The numerical calculations of momentum compare favourably with experimental data, whereas the heat transfer simulations do show a significant heat transfer reduction in relation to that of Newtonian fluid, thus showing that the new formulation is able to predict both drag and heat transfer reductions in pipe flows.

Keywords. Turbulence, Viscoelastic, Heat Transfer, Fluid Flow

1. Introduction

Drag reduction is a fascinating subject that occurs in turbulent flow of some dilute polymer solutions and surfactant solutions, but is still poorly understood. One of the theories for explaining drag reduction in non-Newtonian fluids relates the suppression of turbulent fluctuations with some elastic properties (Lumley, 1977; Virk, 1975), especially the normal stresses.

More recently, experimental and DNS investigations have confirmed the role of fluid elasticity in drag reduction (Massah and Hanratty, 1997; Warholic et al, 1999), especially the effects of polymer extensibility and enhanced elongational viscosity (Sureshkumar et al, 1997; Dimitropoulos et al, 1998) but so far no single physical model has clearly emerged for explaining drag reduction. Still, as a possible cause for drag reduction an enhanced elongational viscosity has been quoted more often at least since the nineteen seventies (Lumley, 1977). Other tentative theories, such as stress or viscosity anisotropy, have been less successful and the corresponding results confusing. For instance, the numerical investigations of Den Toonder et al (1997) has predicted higher drag reductions for anisotropic purely viscous models than for anisotropic viscoelastic fluids, in contradiction with experimental findings.

In parallel with the reduction in turbulent momentum transport those fluids also exhibit a reduction in convective heat transfer. Although both phenomena (drag and heat transfer reduction) are important in many branches of chemical and mechanical engineering industries, the heat transfer has been far less investigated than the corresponding fluid dynamic problem (Matthys, 1996). This can be partially explained, considering that heat transfer experiments are in general more complex to perform accurately, but also because developments in predicting turbulent drag reduction with polymer solutions are required prior to attempting to predict heat transfer with the same fluids. The non-existence of a widely accepted turbulence model for drag reducing fluids has been the greatest obstacle to theoretical and numerical investigations on their heat transfer characteristics.

The major problem in the description of turbulent flow characteristics of viscoelastic fluids, is the correct inclusion of rheological parameters of the fluid in order to make it as general as possible. In the late nineteen seventies, some authors (Mizushima, 1977; Durst et al, 1977; Hassid and Poreh 1975, 1977, 1978) used experimental results of pipe drag reduction to adjust the constants, wall functions and damping functions in standard and low Reynolds number turbulence models and then were able to predict the characteristics of the same flows. However they could not successfully predict the behaviour of the same fluids in other flows or in pipe flows of different diameter and, as far as

we are aware, there was no further progress in deriving appropriate single-point closures. Since then most of the research in the field, based on DNS simulations of elastic fluids, has been directed at understanding the molecular configurations and corresponding fluid properties (for example, De Angelis et al, 2002).

Without knowing the correct physics behind drag reduction and the correct rheological constitutive equation for a given drag reducing fluid, which at present nobody really knows for sure, it will not be possible to arrive at the correct turbulence models. However, it is the duty of the engineer to improve the current situation since there is clearly the need to further advances in single-point closures for drag reducing fluids. These must include some, if not all, of the assumed relevant fluid rheological properties, and the best candidate is a strain-thickening elongational viscosity

Recently, Pinho (2002) and Cruz and Pinho (2002) proposed a new turbulence model for drag reducing fluids that was developed from a Generalised Newtonian Fluid, and is based on the classical, low Reynolds number $k - \epsilon$ model. The GNF constitutive model was modified to mimic some of the elastic fluid properties that are held responsible for drag reduction, namely enhanced elongational viscosity and a viscoelastic damping function was introduced. So far, the results have been quite successful and, consequently, in the present work this turbulence model is extended to deal with the heat transfer of a drag reducing fluid and its capabilities, in terms of heat transfer performance, are assessed. As part of the solution, the thermal energy equation for turbulent flow will be solved for a thermally developing pipe flow, but under the conditions of fully-developed hydrodynamic flow, ie, corresponding to high Prandtl number condition.

2. THE CONSTITUTIVE EQUATION

As a first step, it is necessary to consider a constitutive equation for the viscosity. An algebraic form for the viscosity function can be a Bird-Carreau type of equation containing a shear-rate dependent term, that is multiplied by a strain-rate dependent term. While the former gives the appropriate variation of the shear-viscosity, the latter mimics some of the strain-thickening effects that are held responsible for drag reduction.

$$\mu = \mu_0 \left[1 + (\lambda_s \dot{\gamma})^2 \right]^{\frac{n-1}{2}} \left[1 + (\lambda_e \dot{\epsilon})^2 \right]^{\frac{p-1}{2}} \quad (1)$$

However, for simplicity in the derivation of the turbulence model, a power law based equation was preferred (Pinho, 2002) and, consequently, the same form (Eq. 2) is adopted here.

$$\mu = \mu_v(\dot{\gamma}) \mu_e(\dot{\epsilon}) = K_v \left[(\dot{\gamma})^2 \right]^{\frac{n-1}{2}} K_e \left[(\dot{\epsilon})^2 \right]^{\frac{p-1}{2}} \quad (2)$$

In equation (2) the viscometric function $\mu_v(\dot{\gamma})$ is obtained by fitting the viscometric viscosity η_v with a power law whereas the non-dimensional elastic function $\mu_e = 1/3 \eta_e(\dot{\epsilon})/\eta_v(\dot{\gamma})$ at $\dot{\gamma} = \sqrt{3} \dot{\epsilon}$ in order to respect the limiting physical behavior.

For this Generalized Newtonian fluid it is now necessary to derive the corresponding conservation equations, bearing in mind that there are turbulent fluctuations in the viscosity, because of its dependence on the flow kinematics.

3. THE TURBULENCE MODEL

The details of the derivation of the transport equations are presented in Pinho (2002) and here only the main features of the model are presented. The mean flow conservation equation for the GNF fluid, with the two-dimensional boundary layer simplifications, is :

$$\frac{\partial \rho U^2}{\partial x} + \frac{1}{r} \frac{\partial \rho r UV}{\partial r} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \left(\bar{\mu} \frac{\partial U}{\partial r} - \rho \overline{uv} \right) \right) \quad (3)$$

It is important to note here that a mean value of the molecular viscosity must be used, which is defined in Eq. (4a), since the viscosity functions no longer linear. This average molecular viscosity reduces to a constant value of $\bar{\mu} = K_v K_e$ when the shear and strain rate dependencies are reduced ($n = 1, p = 1$).

$$\bar{\mu}_h = (C_{\mu\rho})^{Z_1} 2^{Z_2} k^{Z_3} \epsilon^{Z_4} B^{Z_5} \quad (4a)$$

where:

$$m = \frac{n + p - 2}{n + p}$$

$$B = \left[\frac{K_v K_e}{A_e^{p-1}} \right]^{1-m} 2^{\frac{(n-1)-m(n+1)}{2}} \rho^m$$

$$Z_1 = \frac{3m(m-1)A_2}{8 + 3m(m-1)A_2}$$

$$Z_2 = \frac{4m(m-1)A_2}{8 + 3m(m-1)A_2}$$

$$Z_3 = \frac{6m(m-1)A_2}{8 + 3m(m-1)A_2}$$

$$Z_4 = \frac{[8 - 3m(m-1)A_2]m}{8 + 3m(m-1)A_2}$$

$$Z_5 = \frac{8}{8 + 3m(m-1)A_2}$$

and

$$A_e = 10$$

$$A_2 = 0,45$$

Since this average molecular viscosity was derived with high Reynolds number arguments, it must be corrected in the vicinity of a wall to where f_v is a molecular viscosity damping function. In this model, this damping function is made equal to f_μ .

$$\bar{\mu} = f_v \bar{\mu}_h + (1 - f_v) K_v [(\dot{\gamma})^2]^{\frac{n-1}{2}} \quad (4b)$$

To determine the shear stress the Boussinesq approximation is invoked by which

$$-\overline{\rho uv} = \rho \nu_t \frac{\partial U}{\partial x} \quad (5)$$

The eddy diffusivity ν_t is given by the Prandtl - Kolmogorov equation, which is modified for low Reynolds number effects with the damping function f_μ :

$$\nu_t = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}} \quad (6)$$

In Eq. (6) k stands for the turbulence kinetic energy and $\tilde{\varepsilon}$ is the modified rate of dissipation of turbulent kinetic energy which is used here as in most near wall, low Reynolds number k - ε models (Patel et al, 1985). It is related to the true rate of dissipation of turbulent kinetic energy ε by:

$$\varepsilon = \tilde{\varepsilon} + D \quad (7)$$

where D takes a specific form for each turbulence model.

The turbulence kinetic energy equation is not deeply affected by the new definition of the viscosity and it can be written as follows, with the boundary layer simplifications:

$$\frac{\partial U k}{\partial x} + \frac{1}{r} \frac{\partial r V k}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \left(\frac{\bar{\mu}}{\rho} + \frac{v_t}{\sigma_k} \right) \frac{\partial k}{\partial r} \right) + P - \varepsilon \quad (8)$$

For the modified rate of dissipation of turbulence kinetic energy the transport equation is:

$$\frac{\partial U \tilde{\varepsilon}}{\partial x} + \frac{1}{r} \frac{\partial r V \tilde{\varepsilon}}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \left(\bar{v} + \frac{v_t}{\sigma_\varepsilon} \right) \frac{\partial \tilde{\varepsilon}}{\partial r} \right) + f_1 C_{\varepsilon 1} \frac{\tilde{\varepsilon}}{k} P - f_2 C_{\varepsilon 2} \frac{\tilde{\varepsilon}^2}{k} + E + \frac{v_t}{\sigma_\varepsilon \bar{v}} \frac{\partial \tilde{\varepsilon}}{\partial r} \frac{\partial \bar{v}}{\partial r} \quad (9)$$

In Eqs. (8) and (9) we adopted a modified version of the model proposed by Nagano and Hishida (1987), with the kinematic viscosity ν substituted by the average kinematic viscosity $\bar{\nu}$ and with a different damping function f_μ . The various viscous extra terms and damping functions take the following form:

$$D = 2\bar{\nu} \left(\frac{\partial \sqrt{k}}{\partial y} \right)^2 \quad (10)$$

$$E = \bar{\nu} v_t \left(1 - f_\mu \right) \left(\frac{\partial^2 U}{\partial y^2} \right)^2 \quad (11)$$

$$f_1 = 1 \quad (12)$$

$$f_2 = 1 - 0,3e^{-R_T^2} \quad (13)$$

$$R_T = \frac{k^2}{\bar{\nu} \tilde{\varepsilon}} \quad (14)$$

The coefficients were taken from Nagano and Hishida's model which are basically those of the standard model and are listed in Tab. (1).

Table 1- values of the parameters assigned to Nagano and Hishida's low Reynolds k - ε model.

C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$
0,09	1,0	1,3	1,45	1,9

The damping function f_μ has to be modified in order to consider the non-Newtonian behaviour of the fluids. The complete deduction of the damping function f_μ used here can be found in Cruz and Pinho (2002) and is given by:

$$f_v = f_\mu = \left(1 - \left(1 + \left| \frac{1-n}{1+n} \right| y^+ \right)^{-\left| \frac{1+n}{1-n} \right| \frac{1}{26}} \right) \left(1 - \left(1 + \left| \frac{p-1}{3-p} \right| y^+ C^{\frac{1-p}{2-p}} \right)^{-\left| \frac{3-p}{p-1} \right| \frac{1}{26}} \right) \quad (15)$$

where

$$y^+ = y \sqrt{\frac{\tau_w}{\rho}} \quad (16)$$

Constant C is equal to 9 and was obtained in Cruz and Pinho (2002) by comparing predictions with experimental data of Escudier et al (1999) for an aqueous solution of 0,125% PAA.

The thermal energy equation adopted here for the turbulent flow simulations is the same used by Durst and Rastogi (1977), which is written as:

$$\frac{\partial UT}{\partial x} + \frac{1}{r} \frac{\partial rVT}{\partial r} = \frac{\partial}{\partial r} \left(\frac{v_t}{P_r} \frac{\partial T}{\partial r} \right) + \alpha \left(\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \quad (17)$$

According to the literature (Loulou et al 1992 and Shin, 1996) the variation of thermal conductivity with shear rate is weak and so, here the thermal properties of the fluid are assumed constant.

4. THE NUMERICAL PROCEDURE AND RESULTS

The set of equations (3) to (15) was solved numerically using a finite volume formulation to obtain the fully-developed pipe flow solution. Since the flow problem is fully decoupled from the thermal problem, the momentum solution was then used as input to solve the thermal energy balance for the development of the thermal boundary layer, a situation that is physically consistent with a high Prandtl number flow. Here, however, the emphasis is not on investigating the characteristics of a high Prandtl number flow, but on assessing the capabilities of the current turbulence model to predict heat transfer reduction.

Under these conditions the second term on the left-hand-side of Eq. (17) vanishes, and a finite difference formulation was used to compose the following ordinary differential equation:

$$-UT_I = -UT_{I+1} + \Delta x \left(\frac{d}{dr} \left(\frac{v_t}{P_r} \frac{dT_{I+1}}{dr} \right) + \alpha \left(\frac{1}{r} \frac{dT_{I+1}}{dr} + \frac{d^2 T_{I+1}}{dr^2} \right) \right) \quad (18)$$

which was numerically solved for each step *x* using the MATLAB framework.

Figure (1) compares predictions of the Fanning friction factor with the corresponding experimental measurements of Escudier et al (1999) for an aqueous solution of 0,125% polyacrylamide (PAA). As mentioned above, parameter *C* was made equal to 9 based on these comparisons, but the value worked equally well when predicting the behaviour of other aqueous polymer solutions based on xanthan gum and carboxymethylcellulose sodium salt. The values of *n*, *p*, *K_e* and *K_v*, listed in Table (2), were obtained from least-square fitting to the shear and elongational viscosity data of the same fluid, presented in the paper of Escudier et al (1999).

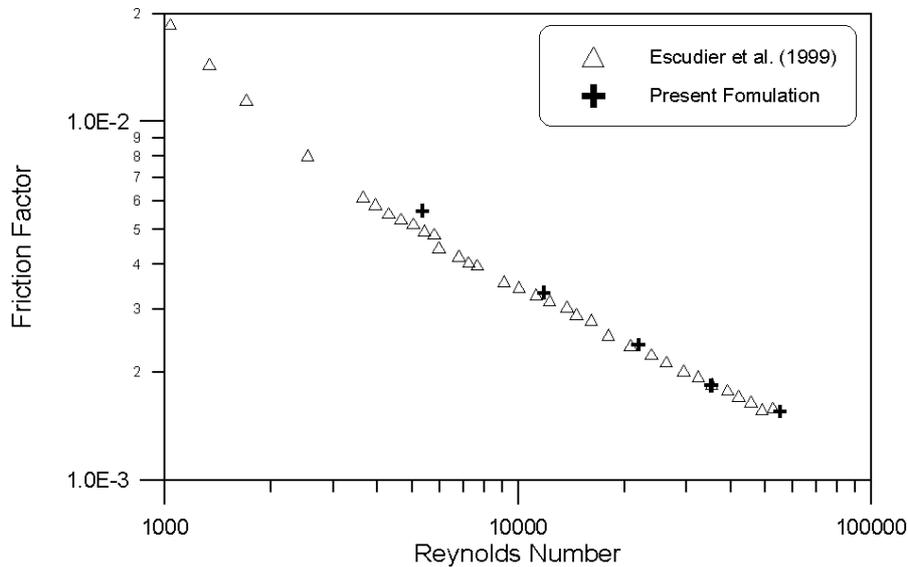


Figure 1. Comparison of the predictions of the Fanning friction factor with results for 0,125% PAA obtained by Escudier et al (1999) in a pipe flow.

Table 2 – Parameters of the viscosity model (Eq. 2) for an aqueous solution of 0.125% PAA

<i>n</i>	<i>p</i>	<i>K_e</i>	<i>K_v</i>
0,425	1,479	1,9393	0,2491

Results of the solution of the thermal energy equation are presented in Fig. (2) as the axial variation of the Nusselt number for flows at a wall Reynolds number of 37000 (the Reynolds number is based on the wall viscosity). The thermal diffusivity, inlet temperature and wall temperature used are listed in Table (3). First, the thermal behavior of a Newtonian fluid flowing at the same Reynolds number was calculated and is used here as a reference for the comparisons, since there are no experimental data for the heat transfer characteristics of this fluid.

It is clear from Fig. (2) that the present formulation of the turbulence model is able to predict some heat transfer reduction, and yet no modification to the standard closure of the Reynolds flux was introduced. The heat transfer reduction follows the reduction in drag since, in the present formulation, the thermal diffusion coefficient is obtained through the turbulent Prandtl number concept, here assumed to take on a constant value of 0,9. Note, however, that in quantitative terms both reductions are quite different. Whereas the reduction in heat transfer is of approximately 6%, that in drag was equal to 68%.

An important issue concerning heat transfer of viscoelastic fluids is the true value of the turbulent Prandtl number. It is known (Matthys, 1996) that the amounts of maximum drag and heat transfer reductions are not identical as they should if the Reynolds analogy held. By comparing some predictions with experimental data, Matthys (1996) has suggested that the turbulent Prandtl number for viscoelastic fluids is much higher than for Newtonian fluids. It is important to note, however, that such high values of the turbulent Prandtl number were obtained from modified Newtonian eddy viscosity models, which may not be entirely applicable to viscoelastic turbulent flows, as pointed out by Matthys (1996). Nevertheless, such findings are in agreement with experimental results (Kostic, 1994) that show the heat transfer reduction to be higher than the drag reduction, at least in the asymptotic condition of maximum reductions. Therefore, for turbulent flow of viscoelastic fluids the Reynolds analogy does not work.

Table 3. Thermodynamic and thermal properties of the fluid and numerical parameter

α	Δx	Radius	T_{wall}	T_{inlet}
$6 \times 10^{-5} \text{ m}^2/\text{s}$	0,01 m	0,025 m	350 K	300 K

To assess the effect of the turbulent Prandtl number (Pr_t) in the current turbulence model, two extra simulations were carried out for values of Pr_t of 3 and 15 and the corresponding results, plotted in Fig. (2), show the reduction in Nu as it should. The Nusselt number reduction in relation to the newtonian case is equal to 8,5% and 9,55% for Pr_t of 3 and 15, respectively, which means that the sensitivity of Nu decreases significantly with Prandtl number.

However, it is clear that the amount of heat transfer reduction is significantly less than the amount of drag reduction even at high turbulent Prandtl numbers. This suggests that other modifications to the thermal energy equation and turbulence model are required to account for viscoelastic effects in heat transfer, but this assertion also needs to be validated by experiments for the same fluids. In fact, the preliminary investigations conducted by Cruz and Pinho (2002) have shown that for this turbulence model the drag reduction is basically due to the molecular viscosity and the effect of f_v and is little affected by the turbulent viscosity. Since the energy equation is not affected by the molecular viscosity, but only by the turbulent viscosity the reduction in heat transfer became less than the reduction in drag.

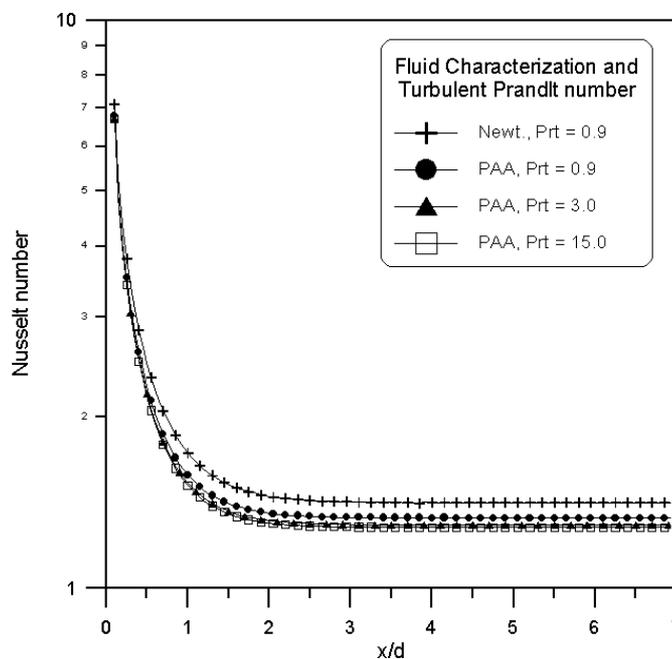


Figure 2. The influence of the turbulent Prandtl number variation on the Nusselt number for viscoelastic and newtonian fluids

Regarding the turbulent Prandtl number, its determination has been rather difficult due to the lack of a consistent theory for predicting the characteristics of drag reducing fluids. The main difficulty so far has been that the available models required, as input, prior knowledge of some intrinsic flow properties, such as the friction velocity. In contrast,

the present formulation needs no such previous flow information, and attempts to describe the flow field with no information other than the rheological parameters of the fluid.

5. Conclusion

In the present work a recently proposed turbulent-viscoelastic fluid formulation was used to analyze the heat transfer reduction in viscoelastic pipe flow. The results have shown a reduction in the heat transfer coefficient in parallel to a reduction in drag. It was also shown that an increase in the turbulent Prandtl number could raise the heat transfer reduction, as suggested by experimental data, but further work is necessary to improve the Reynolds flux closure model and to quantify the true value of the turbulent Prandtl number in order to improve the prediction of intense heat transfer reductions.

In any case, these results indicate that the present formulation has the potential to be used on the solution of engineering problems because it only requires information on the fluid rheology, thus overcoming the difficulties of previous formulations which needed some intrinsic flow properties as input.

6. References

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