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A NEW SONIC BOX METHOD FOR OSCILLATING WINGS WITH THICKNESS IN UNSTEADY TRANSONIC FLOW

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Abstract. *The problem of unsteady lifting surfaces with straight leading edges in sonic flow that was already studied in Cesar and Soviero (2001) is extended in this work to the case of finite thickness wings. The wings are supposed to be undergoing harmonic oscillation near the sonic flight speed in an inviscid, shock free flow. The thickness effect is accounted for in the analysis through the use of the local Mach number distribution over the wing surface at its mean position, as it was obtained in Alksene, and Spreiter (1961), by employing the local linearization concept and a suitable coordinate transformation. Sample results are shown for a rectangular wing, which are compared with that of analogous theory shown in other works, such as in Ruo et al. (1974). The generalized aerodynamic forces coefficients for a series of reduced frequencies and thickness as well as pressure distributions for finite wings of various aspect ratios are shown.*

Keywords*. Unsteady aerodynamics, Sonic flow, Panel method, Linearized theory, Finite thickness wing.*

1. Introduction

The adequate knowledge of the forces acting on three-dimensional wings in oscillatory motion is very important for the study of flutter and other aeroelastic responses of an aircraft, because the aeroelastic problem is frequently critical in the transonic range. A nonlinear partial differential equation with nonlinear boundary conditions govern the physical problem. The basic small perturbation equation governing the velocity potential for transonic flow over a thin wing is well known Landahl (1961). For low amplitude, high-frequency oscillation where the unsteady part is considered to be a small disturbance to the steady part, the steady state properties can be completely uncoupled from the unsteady equation, and the governing equation for unsteady transonic flow can be linearized.

In this work we develop an approximate method to account for the more important effects of finite wing thickness in order to predict transonic oscillatory aerodynamic parameters in a range of frequencies where linearization can be considered valid. The present study is limited to attached, shock-free flow and rectangular wings.

As told in Ruo et al. (1974), almost all unsteady transonic flow theoretical work lies within the framework of linearized theory where the thickness effect of the wing is neglected. An important consequence of these linearization is the suppression of deviations in local Mach number from freestream value. These deviations have appreciable effect on the propagation of pressure disturbances over the lifting surface, then significant improvement in the theory may be accomplished by “recoupling” the steady and unsteady flow parameters so that solutions may approximately consider variations in mean local Mach number caused by finite wing thickness. In the present work, this is achieved by considering all of the steady-flow parameters over the wing to be invariant within a small finite region. This latter assumption, equivalent to the concept of local linearization (Spreiter and Alksne, 1958; Spreiter and Alksne, 1964), permits the nonlinear differential equation for the potential to be reduced to a linear equation with variable coefficients containing the local Mach number. By means of an appropriate coordinate transformation, the equation becomes identical to the linearized transonic unsteady-flow equation with constant coefficients. Numerical results are then obtained through the new sonic-box method presented in (Soviero and Pinto, 2000; Vargas Cesar and Soviero, 2001).

The types of wings treated here are those having unswept trailing edges, without control surfaces. Because of the transformation method used, the mean steady flow everywhere over the wing must not be very different from that of the undisturbed stream.

Calculations using the present method were made to evaluate the thickness contribution to the unsteady aerodynamic forces by comparison with cases where thickness effect were not taken into consideration. The wing considered was a rectangular with bicircular-arc profile. Comparisons are made to the new sonic-box method, for cases without thickness effects, as well as to results from Ruo et al. (1974).

2. Problem Formulation

Consider a small thickness wing, immersed in an inviscid compressible fluid that translates with the undisturbed flow velocity U close to the speed a_∞ , the sound speed of the undisturbed flow, performing a small amplitude oscillation around its zero angle of attack position. The wing is assumed to be smooth and thin enough so that the small-perturbation velocity potential equation for transonic flow can be applied. The physical coordinates and a sample wing geometry are shown in Fig. (1).

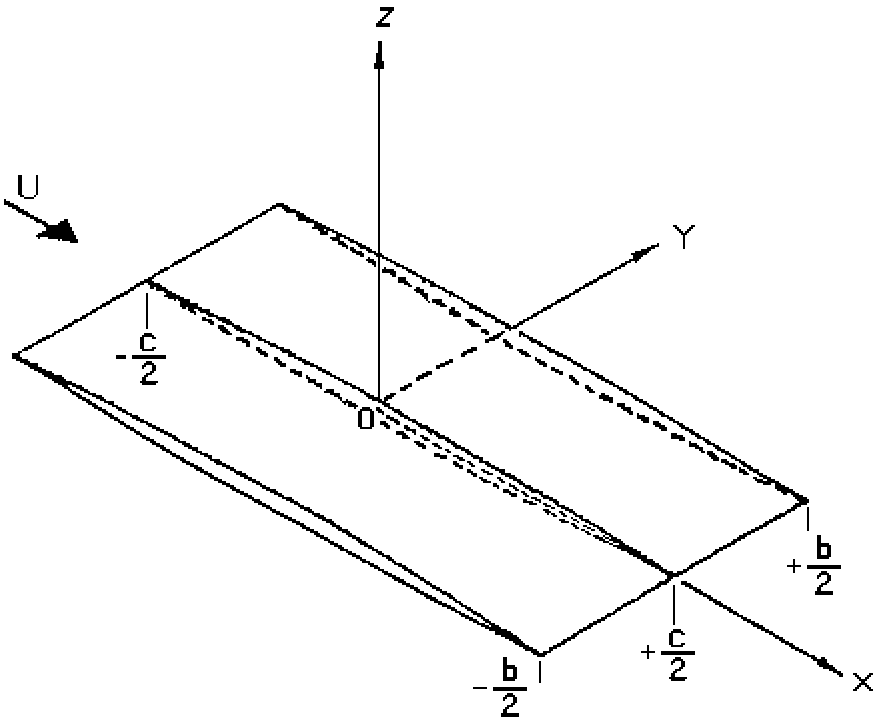


Figure 1. Coordinate system on the physical plane of the wing, *c* is the wing root chord and b is the wing span.

The chosen coordinate system has the *X*-axis in the direction of the flow. The undisturbed position of the wing is in the *XY* plane, with the *X*-axis formed by the straight line passing through the leading edge half span point and the trailing edge half span point, the origin is at the intersection point of the wing semi-span with the wing half chord.

The physical problem is governed by nonlinear partial differential equations and nonlinear boundary conditions.

The complex velocity perturbation potential Φ due to the harmonic small-amplitude motion of a thick wing is described by the nonlinear differencial equation given below:

$$\frac{\partial^2\Phi}{\partial X^2}+\frac{\partial^2\Phi}{\partial Y^2}+\frac{\partial^2\Phi}{\partial Z^2}-\frac{1}{a^2}\left[\frac{\partial^2\Phi}{\partial t^2}+\frac{\partial V^2}{\partial t}+\vec{V}\cdot\vec{\nabla}\left(\frac{V^2}{2}\right)\right]=0 \tag{1}$$

where a is the local speed of sound, V the local flow velocity lies in the positive X direction and Φ is made nondimensional relative to V and a reference length L, the root semichord of the wing; t is a non-dimensional time obtained by multiplying the physical time by U/L.

Equation (1) is linearized assuming that the local velocity vector differs only slightly in direction and magnitude from the free-stream velocity vector. This is the basic assumption of the small disturbance theory, that is equivalent to assume that the local Mach number M is close to the value of the free stream Mach number M_∞.

Assuming that the wing describes an harmonic motion then the small perturbation potential can be written as

$$\Phi(X,Y,Z,t)=\tilde{\Phi}(X,Y,Z)\exp[-i\omega t] \tag{2}$$

where ω is the angular frequency of the motion.

The complex velocity perturbation potential $\tilde{\Phi}$ due to the harmonic small-amplitude motion of a thick wing is described in the frequency domain by a linear differencial equation, similar to the Eq. (3), given by Landahl (1961) for a thin wing shown below

$$\frac{\partial^2\tilde{\Phi}}{\partial Y^2}+\frac{\partial^2\tilde{\Phi}}{\partial Z^2}-\frac{2Vi\omega}{a^2}\frac{\partial\tilde{\Phi}}{\partial X}+\frac{\omega^2}{a^2}\tilde{\Phi}=0 \qquad (3)$$

where $\tilde{\Phi}$ is made nondimensional relative to V and the reference length L.

Defining a new complex potential as

$$\phi=\tilde{\Phi}\exp[-(i\omega X)/(2aM)] \qquad (4)$$

Now, using the transformation

$$\mathrm{x}=X/\mathrm{L}\,,\quad \mathrm{y}=\mathrm{M\,Y/L}\,,\quad \mathrm{z}=\mathrm{M\,Z/L}\,, \qquad (5)$$

the governing differential equation of the problem, i.e., Eq. (3), can be rewritten as the classical diffusion equation Morse and Feshbach (1953), analogous to the subsonic (Soviero and Bortolus, 1992) to the supersonic (Soviero and Resende, 1997) and the sonic (Soviero and Pinto, 2000; Vargas Cesar and Soviero, 2001), formulations and reads:

$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-2ik\frac{\partial\phi}{\partial x}=0 \qquad (6)$$

where $k = \omega L/V$ is the reduced frequency and the mean steady local Mach number M is assumed to be known.

Equation (6) is a parabolic differencial equation and possesses source and doublet elementary solutions.

Our procedure here is similar to the one by Ruo et al. (1974), they made an adaptation of the linear theory to account for the effect of the wing thickness insofar, as he advises, it produces a nonuniform mean flow, including possibly a local supersonic region without a terminating shock. We assume, like they did, that the physical state is adequately described within a limited region by related linear equations in which all parameters involved have their local values taken as being invariant. This is the underlying assumption of the local linearization concept described in (Spreiter and Alksne, 1958; Rubbert and Landahl, 1963). This approach suggests that, in the case of unsteady flow, the calculations can be carried out with sufficient accuracy, using the linearized equations which contain the local values of the steady flow parameters. Landahl (1963) cites evidence for the validity of applying the concept of local linearization to the case of unsteady flow.

Ashley (1963) pointed out that a simple way of potentially improving the accuracy of unsteady flow calculations is to use the linearized velocity potential equation, but with the Mach number of the undisturbed flow replaced by the local Mach number which varies spatially due to thickness, mean angle of attack, and/or camberline shape. His work was extended by Sankaranarayanan and Vijayavittal (1970) to supersonic flow past delta wings where they found that the general effect of thickness is to reduce the flutter speed, and Kacprzynski (1968) who examined three dimensional effects in supersonic flow. These works are concerned with thickness effects in supersonic flow. Little information is available about the behavior of an oscillating finite wing in transonic flow.

What we are doing is to stretch our physical wing to work on a transformed plane with a transformed wing which is a thin wing whose local Mach number over its surface is known and has the same value of the correspondent local Mach number over the surface of the physical wing, both wings are in a free stream sonic flow.

To have the results for our real wing we need to return to the physical plane from the transformed plane, this is done applying the similarity law for unsteady transonic flow Eq. (7), Landahl (1961),

$$\phi(x,y,z;\sigma;k;M;\varepsilon)=M^{-1}\phi(x,y,z;M\sigma;k;M;\tilde{\varepsilon})\,, \qquad (7)$$

where σ is the semi-span to semi-chord ratio, ε is the thickness ratio defined by the thick to semi-chord ratio, $\tilde{\varepsilon}$ is the transformed wing thickness ratio, and $\tilde{\varepsilon}<<1$.

The linearized boundary condition on the wing surface is written in the transformed plane as

$$\omega(x,y)=\frac{\partial\phi}{\partial z}=\frac{\exp(ikx/2)}{M}\left[\frac{\partial h}{\partial x}+ikh\right] \qquad (8)$$

where $h(x,y)$ represents the wing surface nondimensional vertical displacement. The complex pressure coefficient is written as

$$C_p=-\frac{2}{UL}\exp\left(-\frac{ikx}{2}\right)\left[\frac{\partial\phi}{\partial x}+i\frac{k}{2}h\right] \qquad (9)$$

and pressure continuity is ensured if Eq. (9) is applied to both sides of the wake, that is,

$$\delta C_p = 0 = \frac{\partial \delta\phi}{\partial x} + i\frac{k}{2}\delta\phi \tag{10}$$

where $\delta\phi$ and δC_p are the complex velocity potential and pressure coefficient jump between the lower and upper surfaces of the wake, respectively. It is important to stress that the same condition applies whenever the trailing edges are subsonic.

The treatment of the tridimensional thick wing problem is similar, though not the same, to the one done in Ruo et al (1974), where it was used the classical sonic box method reported in (Rodemich and Andrew, 1965; Olsen, 1966). Here we use the new sonic box method reported in (Soviero and Pinto, 2000; Vargas Cesar and Soviero, 2001).

The solution of problem just described is obtained from the integral equation that relates the potential jump across the lifting surface (and wake) to the downwash. For a planar configuration this integral is written as

$$\omega(x,y) = \frac{1}{4\pi}\iint \delta\phi \frac{ik}{(x-x_0)^2}\exp\left(-\frac{ik(y-y_0)^2}{2(x-x_0)}\right)dx_0 dy_0 \tag{11}$$

The integrand represents a doublet at (x_0, y_0) inducing a normal velocity in the wing plane (z=0) at the receiving point (x,y). The integral sign must be taken in its usual way along y_0 and in the sense of the finite part integration along x_0. The general formulation relative to integral Eq. (8) can be found in Morse and Feshbach (1953), and is a result of the application of Green's theorem to the diffusion equation (Eq. (6)). Its kernel function, the induced doublet velocity, comes from the unitary strength source velocity potential,

$$\phi(x,y,z) = \frac{1}{4\pi(x-x_0)}\exp\left(-ik\frac{(y-y_0)^2+(z-z_0)^2}{2(x-x_0)}\right) \tag{12}$$

for a point source placed at (x_0, y_0, z_0), with $x > x_0$, which must be differentiated twice along the z direction.

For $x \le x_0$, $\phi = 0$.

To have the correspondent values for the physical wing we apply Eq. (7) to the results obtained by Eq. (12).

3. Numerical Solution

The physical wing is approximated by a region composed of *nx* boxes along the root chord and *ny* boxes along the wing semi-span Fig. (2). The chords of these panels equals to *c/nx* and its spans are equal to *b/2ny*.

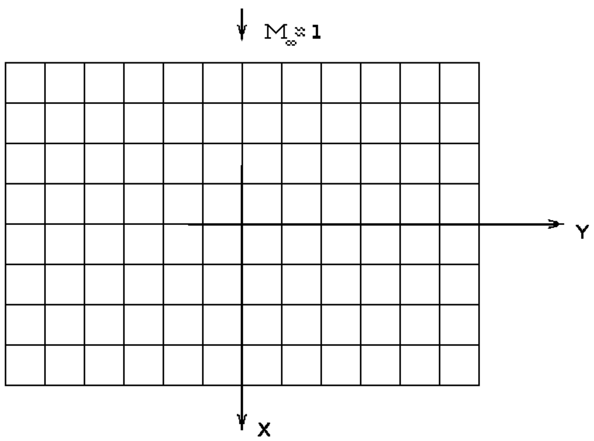


Figure 2. Mapping the wing by *nx* panels in the root chord direction and *ny* panels in the wing semi-span direction.

Using the method presented in Alksne and Spreiter (1960) to calculate the local Mach number, we apply a type of Prandtl-Glauert transformation (Eq. (5)) to obtain a transformed thin wing that has the similarity transonic law (Eq. (7)) relating its velocity potential in the transformed plane to that of the physical wing.

The transformed wing is a stretched wing mapped by *nx* rows, having *ny* columns in each row. Since we use the local Mach number to stretch each panel of a row and M differs a little from panel to panel, they will have different span in the transformed plane. To avoid having different span for each panel we map the transformed wing surface using *nx* panels along its root chord and *ñy* panels along its span. The transformed wing span is found considering it equal to the maximum value encountered comparing the sum of each transformed panel from the *nx* rows in which we have divided the transformed wing. The resulting transformed wing has a different number of panels in each row and could be treated like a delta wing with swept or unswept trailing edge, depending if the maximum value for the sum of the panels spans is encountered or not in the last row of the wing mapping. We then defined a new criteria to choose the number of columns we will have in the row that correspond to the maximum transformed wing span. We decide to divide the transformed wing semi-span by *dx* rounding the resulting to the nearer integer finding a value for the number of panels along the row of the transformed wing semi-span, *ñy*. Dividing the transformed wing semi-span by *ñy* we find ~~*dy*~~, which is the transformed wing panels span. Then using the criteria adopted by Rodemich and Andrew (1965), which states that the row of boxes is composed by only those boxes whose centroid lies on the projection of the wing surface at the xy plane. Recapitulating, we will have a transformed wing with panels having chords given by *c/nx* and spans given by ~~*dy*~~ = ~~*b*~~*/2ñy*, where ~~*b*~~ is the transformed wing span.

Since the number of panels per row is different in the physical and in the transformed wing we adopt the criteria described as follows. Each time the centroid of the transformed wing correspond to a point inside a certain panel of the physical wing the parameters of that transformed panel will be related to the parameters of this physical panel. The panels formed by these approximation of the transformed wing are rectangular and have a constant distribution of normal doublet over its surface. These distribution is given by the value of the normal doublet superficial density.

The tangential condition of the flow is satisfied in the control points of each panel, which coincide with the geometric center of the panels. Only panels upstream induce velocity on the panels downstream. Once the influence region of a point in the transonic flow is a cone with ninety degrees between the generatrix and its axis, the panels induce velocity to the others that are along the span. The resulting change in potential over the control points of each panel is given by the sum of the contributions of all the panels upstream, all the panels along its row, the panel itself and the undisturbed flow upstream.

Solutions of the problem are obtained by solving integral Eq. (11) for $\delta\phi$ by using the boundary conditions Eqs. (8) and (10) for the wing and wake surfaces, respectively. Both surfaces are mapped through the use of small rectangular elements of unknown constant density doublets, as for the case of the sonic Mach box formulation, Rodemich and Andrew (1965). The boundary conditions are enforced at control points located at each panel geometrical center. One can identify four kinds of integration domain, Fig. (3), in order to obtain the influence coefficients at P. Region I is completely upstream of the limiting Mach lines drawn from P, region II is only partially upstream of the limiting Mach lines, and region III is completely downstream of the limiting Mach lines, corresponding to zero influence. Region IV corresponds to the self-induced influence coefficient. In region I numerical integration is straightforward because the integrand is never singular.

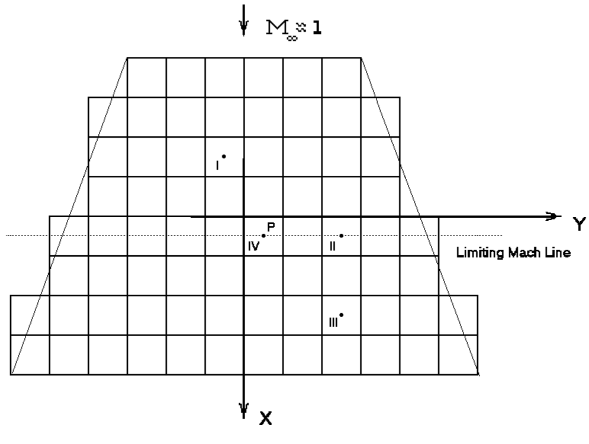


Figure 3. Transformed wing with aspect ratio Λ = 2.0 and thickness ratio σ = 0 at M_∞ = 1.0, mapped with *nx* panel along root chord and *ñy* panels along its semi-span.

The influence coefficient for regions I and IV is, Soviero and Pinto (2000),

$$F(x,y) = -\frac{1}{2\pi}\left[\frac{\exp(\lambda_4)}{(s-y)} + \frac{\exp(\lambda_3)}{(s+y)}\right] + \frac{1}{2}\sqrt{\frac{ik}{2\pi x_p}}\left[erf(\lambda_2) - erf(\lambda_1)\right], \tag{13}$$

where

$$s = b/4n_y \qquad \lambda_{1,2} = (y \pm s)\sqrt{\frac{ik}{2x_p}}, \quad \lambda_{3,4} = -\frac{ik(s \pm y)}{2x_p}, \tag{14}$$

Denoting W_i as the induced velocity over the panel i, F_{ij} as the influence coefficient of panel j over panel i and $\delta\phi_j$ the superficial doublet density of panel j, follows the system that represents the problem

$$\lfloor F_{ij} \rfloor \cdot \{\delta\phi_j\} = \{W_i\}. \tag{15}$$

Solving the system we find the values of $\delta\phi_j$ which belong to the transformed wing to have the correspondent values for the physical wing we apply Eq. (7).

4. Results

Numerical calculations for a rectangular thick wing of aspect ratio 2.0 and thickness ratio ε = 0.0521 are described and the results obtained are compared with those reported in the work of Ruo et al. (1974), also results for a thin rectangular wing of aspect ratio 2.0 is present to be compared with the calculation method employed by Ruo et al. and to give an idea of the thickness effect on the aerodynamic forces. In their work Ruo et al. (1974) uses the method of Rodemich and Andrew (1965), they present analytical formulas for the square box, but nevertheless, some approximations are invoked. The work of Landahl (1961), which is also used here by comparison, uses series expansion for a small value of a Hankel function and the result is dependent on aspect ratio and reduced frequency.

Lift coefficient amplitude and phase angle compare fairly well to the ones given in former works (Landahl, 1961; Ruo et al., 1974) as can be seen in Figs. (4a), (4b), (5a), (5b), and (6).

The aerodynamic coefficients showed in Figs. (4a), (4b), (5a) and (5b) are defined as follows:

L_{11} is the lifting coefficient due to plunge or translation.

L_{22} is the pitching moment coefficient due to pitch.

The pitching axis passes through the most forward point of the wing being perpendicular to the x axis.

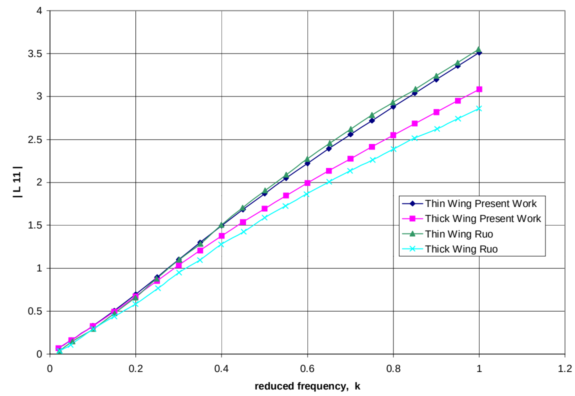


Figure (4.a) amplitude coefficient for an aspect ratio 2.0 rectangular wing in lift due to translation at M_∞ = 1.0.

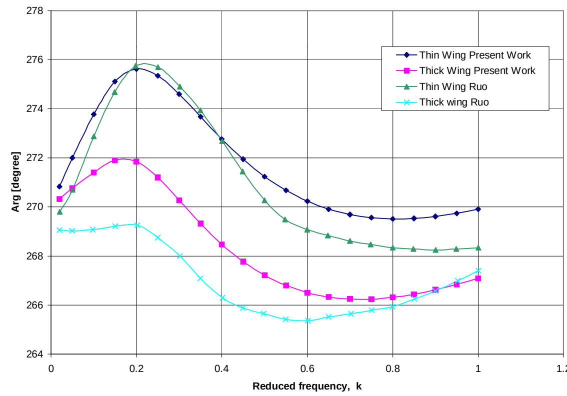


Figure (4.b) argument for an aspect ratio 2.0 rectangular wing in lift due to translation at M_∞ = 1.0.

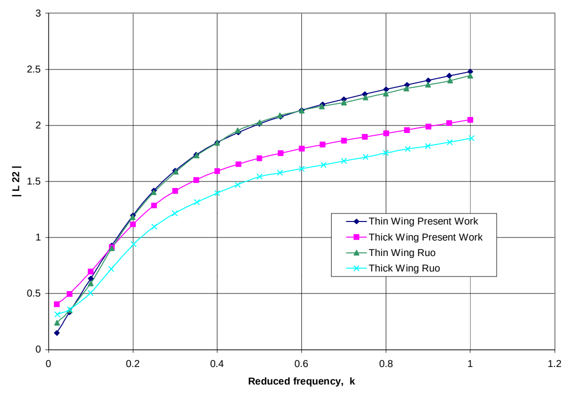


Figure (5.a) amplitude coefficient for the moment due to pitch for an aspect ratio 2.0 rectangular wing at M_∞ = 1.0.

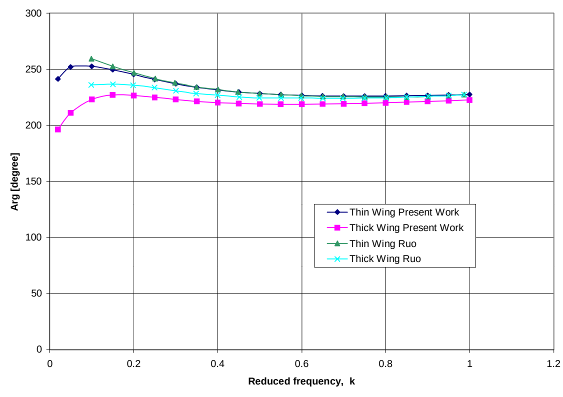


Figure (5.b) argument for an aspect ratio 2.0 rectangular wing in moment due to pitch at M_∞ = 1.0.

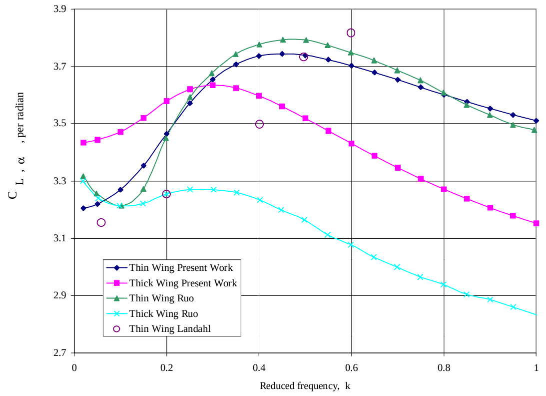
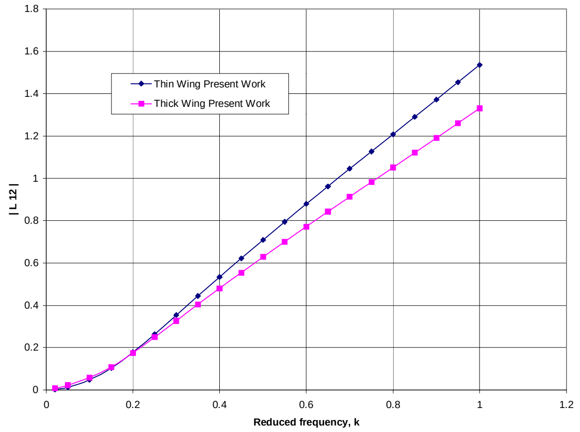


Figure 6. Lift-slope curve variation with reduced frequency and thickness for Λ= 2.0 rectangular wing at M_∞ = 1.0.

The stability derivative presented in Fig. (6) is obtained by using the following equation:

$$C_{L,\alpha} = -\frac{1}{k}\,\mathrm{Im}\left[L_{11}\right] \tag{16}$$



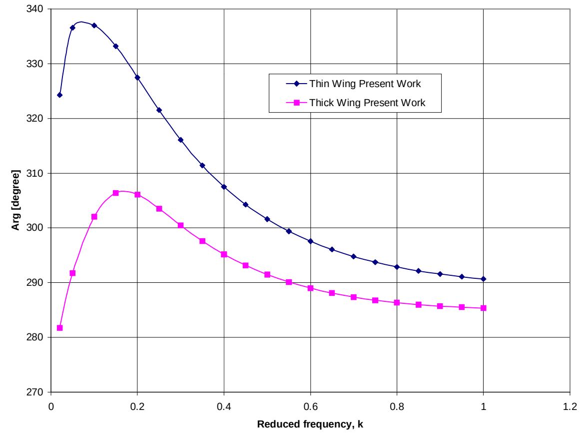
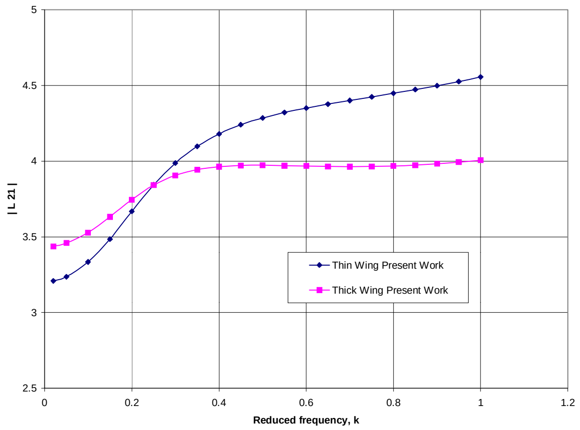


Figure (7.a) amplitude coefficient for the moment due to plunge for an aspect ratio 2.0 rectangular wing at M_∞ = 1.0.

Figure (7.b) argument for the moment due to plunge for an aspect ratio 2.0 rectangular wing at M_∞ = 1.0.



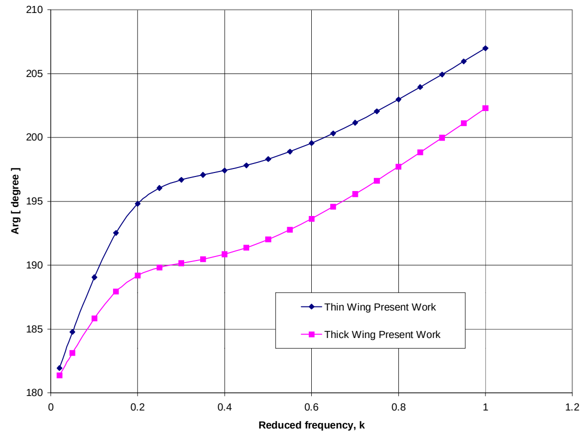


Figure (8.a) amplitude coefficient for an aspect ratio 2.0 rectangular wing in lift due to pitch at M_∞ = 1.0.

Figure (8.b) argument for an aspect ratio 2.0 rectangular wing in lift due to pitch at M_∞ = 1.0.

Figures (7a) and (7b) show the moment due to plunge coefficient (L_{12}) and its argument; Figs. (8a) and (8b) show the lift due to pitch coefficient (L_{21}) and its argument.

Forty boxes along the root chord and nineteen boxes along the transformed wing semi-span are used, which leads to about six hundred boxes on the half-wing.

Figures (4.a), (4.b), (5.a), (5.b) and (6) show unsteady results for a rectangular thick wing oscillating in plunge. Lift coefficient amplitude and phase angle compare fairly well with the ones given in Ruo et al. (1974).

The results shown in Figs. (7.a), (7.b), (8.a) and (8.b) are coherent with what was expected for a wing having a very small thickness ratio. We didn't find other works results to compare with the ones shown in Figs. (7.a), (7.b), (8.a) and (8.b).

For these calculations a grid of 40 x 19 square panels on the half transformed wing was used and computation time was less than one minute on a Pentium 233 MHz.

5. Conclusion

Preliminary results have been present in order to compare numerical solutions already studied by Ruo et al. (1974).

The new way to calculate influence coefficients, $\{F_{ij}\}$, which are critical in any panel method scheme, with apposite complex error functions, is as efficient as the former method of Rodemich and Andrew (1965) used by Ruo et al. (1974), with the additional advantage of being exact and applicable for rectangular panels of any aspect ratio.

The steady Mach number distribution used in this calculation uses the same method employed in Ruo's work and as it happens to their calculations we see the local Mach number rapid variation near the leading edge of the wing.

In spite the wing coordinates plan form is severely distorted, the box size used has not the same problem pointed by Ruo in his work, to be too large to define variations adequately, once we re-dimension the transformed wing panel adequate then to the transformed wing geometry.

Further configurations as physical wings with swept leading edge, a delta wing, a wing with swept trailing edge and a high aspect ratio wing typical of a transonic aircraft are being studied.

5. Acknowledgement

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