PARAMETER ESTIMATION OF A ROTOR-BEARING SYSTEM USING GENETIC ALGORITHM AND SIMULATED ANNEALING

Castro, H. F., Pennacchi, P., and Cavalca, K. L.

Abstract: This work presents a technique that allows the calibration of mathematical models in rotating systems. The method is based on the application of a hybrid meta-heuristic search, which employs genetic algorithm and simulated annealing. The capabilities of the method are evaluated by means of experimental results obtained in the test-rig of Politecnico di Milano. The experimental setup consists of a rotor, supported by four elliptical journal bearings on a flexible foundation. This rotor is excited by unbalance forces. A finite element model, with Timoshenko beams, is used to model the system, considering the gyroscopic effect. A linear hydrodynamic force model is considered for the bearings. So, the estimated parameters of the system are the damping and stiffness coefficients of the bearings and the module and phase of the unbalance applied to the system. The objective function is based on the difference of experimental results and simulated results, considering the parameters to be fitted. This objective function has to be minimized in order to fit these parameters. Once the parameters (stiffness and damping coefficients of each bearing and unbalance module and phase) are estimated, it is possible to calibrate the mathematical model, and then to obtain reliable responses for the physical system studied. The method presented here is also suitable to the rotating machine design area as it presents a relatively simple methodology for the updating and validation of models of machines and structures.

Keywords: fault identification, model updating, genetic algorithm, simulated annealing, rotordynamics

INTRODUCTION

The rotordynamic analysis is becoming a previous phase of study to the design, due to the possibility of predicting problems during the operation of the system, as those caused by vibration amplitudes when a rotor, for example, is passing through a critical speed (Lalanne, 1990 and Vance, 1988).

Mathematical models were developed, in order to represent real machines with considerable confidence. So, several researches were aimed to determine best models to rotating machinery as turbogenerators and multi stage pumps, which are horizontal rotating machines of high load capacity. As the influence of supporting structure can have an important influence on rotating system behavior, it is necessary to consider the foundation in the model (Cavalc a et al., 2005, Pennacchi et al., 2006).

The experimental analysis is also a strong support in processes of predictive and preventive maintenance, because allows fault identification and the diagnosis of operational problems, before some failure of the system (Bachschmid et al., 2000 and 2002).

Bachschmid, Pennacchi and Vania (2002) show an identification of multiple faults in rotor systems. A model-based identification method for multiple faults is presented. In this work they model elements like rotor, bearings and foundation. The models of the faults are represented by harmonic components of equivalent force or moment systems.

This work makes use of linearized models for the hydrodynamic bearing analysis, by means of the evaluation of the hydrodynamic oil-film resultant forces. The supporting hydrodynamic forces model adopts the short bearing mathematical development (Childs, 1993). Using this approach, it is possible to obtain faster numerical solutions of the motion equations of the system. Experimental data can be utilized to update analytical model and estimate or improve unknown parameters.

A meta-heuristic method based in Genetic Algorithms (GA) and Simulated Annealing (SA) is proposed to fit bearings orbits by Castro et al. (2005). The present work intends to apply this meta-heuristic method on the adjustment of unbalance response, intending to search the linearized bearing coefficients or unbalance parameters and position.

MATHEMATICAL MODEL

Before introducing the rotor-bearing-foundation model, it is necessary to introduce the reference systems used in two-dimensional (2D) finite element (f.e.) model of the rotor and of the foundation, modeled by means of a modal representation.
Each node of the rotor model has four degrees of freedom (d.o.f.). If one considers the two subsequent nodes, the \( j \)th and the \( j+1 \)th, they define the element \( j \)th, see Fig. 1.

By defining the vector \( \mathbf{x}_{j}^{(j)} \) of generalized displacements of the \( j \)th node as:

\[
\mathbf{x}_{j}^{(j)} = [x_j, y_j, x_{j+1}, y_{j+1}]^T
\]  

(1)

the vector \( \mathbf{x}_r \) of generalized displacements of all nodes of the rotor is composed of all the ordered vectors \( \mathbf{x}_{j}^{(j)} \):

\[
\mathbf{x}_r = [\cdots x_j, y_j, x_{j+1}, y_{j+1}, \cdots]^T
\]  

(2)

![Figure 1. Reference system on a general rotor element \( j \).](image)

In similar manner, also the d.o.f.s of the foundation, horizontal and vertical displacements, which are relative to the number of bearings \( b \) of the rotor, can be ordered in a vector. If pedestals were used, it resulted:

\[
\mathbf{x}_f = [x'_f, y'_f]^T
\]  

(3)

However, the foundation can be modeled by a modal representation too. So, a vector \( \mathbf{\eta}_f \) is defined containing \( k_m \) ordered modes that are taken into account in the modal representation.

\[
\mathbf{\eta}_f = [\eta_{f1}^T \cdots \eta_{km}^T]^T
\]  

(4)

The foundation d.o.f.s can be obtained by means of the modal nodes by Eq. (5):

\[
\mathbf{x}_f = [\Phi] \cdot \mathbf{\eta}_f
\]  

(5)

where \([\Phi]\) is the eigenvector matrix.

The complete vector of the system d.o.f.s, considering the rotor and the modal foundation is given by Eq. (6).

\[
\mathbf{\xi} = \begin{bmatrix} \mathbf{x}_r \\ \mathbf{\eta}_f \end{bmatrix}
\]  

(6)

The rotor finite element model can be obtained by the application of Lagrange’s Equation and is represented by the mass matrix \([M]^{(r)}\), the damping matrix \([C]^{(r)}\) (which takes into account the gyroscopic effect) and the stiffness matrix \([K]^{(r)}\).

The rotor is supported by oil-film bearings that realize the coupling between the rotor train and the supporting structure. The oil-film bearing forces are modeled by means of linearized forces, which take into account damping and stiffness coefficients for each rotation speed. Therefore, the expression of the linearized forces of the oil-film of \( i \)th bearing on the journal located in the \( j \)th node, due to the rotor d.o.f displacements only, is:
\[
F^{(br)} = -\begin{bmatrix}
k^{(b)}_x(\Omega) & 0 & k^{(b)}_y(\Omega) & 0 & \dot{x}^{(r)}_i \\
0 & 0 & 0 & 0 & \dot{u}^{(r)}_j \\
k^{(b)}_x(\Omega) & 0 & k^{(b)}_y(\Omega) & 0 & \dot{y}^{(r)}_j \\
0 & 0 & 0 & 0 & \dot{v}^{(r)}_j
\end{bmatrix} = -\begin{bmatrix}
k^{(b)}_x(\Omega) \cdot \dot{x}^{(r)}_i \\
0 \cdot \dot{u}^{(r)}_j \\
k^{(b)}_x(\Omega) \cdot \dot{y}^{(r)}_j \\
0 \cdot \dot{v}^{(r)}_j
\end{bmatrix}
\]

while that of the forces on the supporting structure, due to the foundation d.o.f displacements only, is:

\[
F^{(br)} = -\begin{bmatrix}
k^{(f)}_x(\Omega) & k^{(f)}_y(\Omega) & 0 & 0 & \dot{x}^{(f)}_i \\
k^{(f)}_x(\Omega) & k^{(f)}_y(\Omega) & 0 & 0 & \dot{y}^{(f)}_j \\
k^{(f)}_x(\Omega) & k^{(f)}_y(\Omega) & 0 & 0 & \dot{y}^{(f)}_j \\
k^{(f)}_x(\Omega) & k^{(f)}_y(\Omega) & 0 & 0 & \dot{y}^{(f)}_j
\end{bmatrix} = -\begin{bmatrix}
k^{(f)}_x(\Omega) \cdot \dot{x}^{(f)}_i \\
k^{(f)}_x(\Omega) \cdot \dot{y}^{(f)}_j \\
k^{(f)}_x(\Omega) \cdot \dot{y}^{(f)}_j \\
k^{(f)}_x(\Omega) \cdot \dot{y}^{(f)}_j
\end{bmatrix}
\]

In order to consider the coupling effect of the oil-film forces between the rotor and the foundation, it is necessary to define the coupling matrices \(K^{(fr)}\), \(K^{(rf)}\), \(K^{(ff)}\) and \(K^{(rr)}\), and the corresponding damping matrices \(C^{(fr)}\), \(C^{(rf)}\), \(C^{(ff)}\) and \(C^{(rr)}\).

The modal representation of the foundation is given by the modal matrices \(\tilde{M}^{(f)}\), \(\tilde{C}^{(f)}\) and \(\tilde{K}^{(f)}\), which are diagonal matrices.

The system equation of motion, considering the rotor, bearings and foundation is given by (Cavalca et al., 1991 and 1992):

\[
[\tilde{M}] \ddot{\xi} + [\tilde{C}] \dot{\xi} + [\tilde{K}] \xi = F_e
\]

where:

\[
[\tilde{M}] = \begin{bmatrix} M^{(r)} & 0 \\ 0 & M^{(f)} \end{bmatrix}
\]

\[
[\tilde{C}] = \begin{bmatrix} C^{(r)} + [C^{(r)}] \times \Phi \\ -[\Phi]^T [C^{(r)}] + [\Phi]^T [C^{(r)}] \times \Phi \end{bmatrix}
\]

\[
[\tilde{K}] = \begin{bmatrix} k^{(r)}(\omega) & k^{(r)}(\omega) \\ k^{(r)}(\omega) & k^{(r)}(\omega) \end{bmatrix}
\]

The modal representation of the foundation is given by the modal matrices \(\tilde{M}^{(f)}\), \(\tilde{C}^{(f)}\) and \(\tilde{K}^{(f)}\), which are diagonal matrices.
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\[
\begin{bmatrix}
\vec{F}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
K^{(c)} - K^{(r)} & \times \Phi
\end{bmatrix} + \\ -[\Phi]^T \times \begin{bmatrix}
K^{(e)}
\end{bmatrix}
\begin{bmatrix}
K^{(f)} - K^{(e)} \times \Phi
\end{bmatrix}
\end{bmatrix}
\tag{16}
\]

Moreover, the unbalance force vector \( \vec{F}_u \) of amplitude \((m r)^{(k)}\) and phase \(\phi^{(k)}\) at the rotating speed \(\Omega\) acting on the \(j\)th node has the following representation:

\[
\vec{F}_u = [0 : 1 \ 0 \ i \ 0 : 0]^T \times (m r)^{(k)} \times \Omega \times e^{i \phi^{(k)}}
\tag{17}
\]

Finally, the system response in the frequency domain can be obtained as:

\[
\bar{X} = (\Omega^2 [M] + i \Omega [C] + [K])^{-1} \vec{F}_u
\tag{18}
\]

METAHERISTIC METHODS

Simulated Annealing (SA)

This algorithm explores the analogy between the gradual cooling of a metal into a minimum energy crystalline structure and the search for a minimum in an optimization process, based on Metropolis algorithm (Metropolis, 1953).

The search for a minimum requires the definition of boundary constraints of the problem. It also requires a cost evaluation method of a particular solution, which can be used in all optimization problems. The algorithm carries out an iterative search for a better solution in the neighborhood of the current solution. A new solution can become the starting point for a successive step trying to find better solutions, or even to avoid local minimum. This procedure must be carried on until reaches the stop criteria.

When there are no local solutions that improve the quality of the solution, the algorithm stops at the local solution. The local optimum trap makes the local search a heuristic restriction for many combinatory optimization problems. This is because it strongly depends on the initial point. A desirable property of any algorithm is its ability to obtain the global optimum independently of the starting point.

Another way to avoid the local optimum trap is to start a search from other initial solutions, and then use the best solution as the solution to the algorithm. It is noted that the repetitive local searches converge asymptotically to the optimum solution using all solutions as starting points. However, this is neither feasible nor desirable when dealing with huge problems because they require considerable computational time.

The paradigm of the Simulated Annealing offers us an escape from the local optimum by analyzing the boundary of the current solution, and by accepting solutions that worsen the current solution with a certain probability. This is aimed at finding a better way to obtain the global optimum. (See Fig. 2).

![Figure 2 - Evolution of Objective Function using Simulated Annealing.](image-url)

Annealing is a thermal process of melting a solid by heating and gradually cooling it. In the liquid phase, the molecules are scattered randomly, allowing them to reach the lowest possible level of energy – the stable state. The physical process of annealing can be modeled successfully using simulation methods of condensed matter physics. The temperature (a control parameter) is slowly cooled after a number of searches in the neighborhood of the current state. For this reason, some analogies are drawn between a particle physics system and a combinatory optimization problem. The solutions in an optimization problem are equivalent to states in a physical system. The cost of a solution is equivalent to the energy of a state. Choosing a solution in the neighborhood of an optimization problem is equivalent to the perturbation of a physical state. The global optimum of a combinatory problem is equivalent to the fundamental state of a system of particles. A local optimum of a combinatory problem is equivalent to a meta-stable structure in a
system of particles. With an iterative implementation, it is possible to obtain an algorithm for combinatorial optimization problems.

**Genetic Algorithm (GA)**

The genetic algorithm is a search strategy that employs random choice to guide a highly exploitative search, striking a balance between exploration of the feasible domain and exploitation of “good” solutions. This strategy is analogous to biological evolution. From a biological perspective, it is conjectured that an organism structure and its ability to survive in its environment (“fitness”), are determined by its DNA. An offspring, which is a combination of both parents DNA, inherits traits from both parents and other traits that the parents may not have, due to recombination. These traits may increase offspring fitness, yielding a higher probability of surviving more frequently and passing the traits on to the next generation. Over time, the average fitness of the population improves.

In GA terms, the DNA of a member of a population is represented as a string where each position in the string may take on a finite set of values. Normally, this “DNA” is represented by a binary string. It makes possible to work with integer and real numbers together in the same optimization process. Therefore, a decoding transforms this variable in binary numbers. However, it is possible to use different kind of codes, such as genes, that are represented by integer and real numbers.

Members of a population are subjected to operators in order to create offspring. Commonly used operators include selection, reproduction, crossover, and mutation. The selection operator compares the individuals of the population. The individuals that are closest to the optimum point have a major probability to produce a new offspring by reproduction, crossover and mutation.

More details of Genetic algorithms can be found in the work of Holland, 1975 and Goldberg, 1989.

**Hybrid Algorithm**

Currently, the Simulated Annealing and the Genetic Algorithm are two stochastic methods largely used in different optimization problems. The hybrid method has generated promising results in many applications, mainly in highly complex problems (Castro et. al., 2005 and Mori et. al., 2004).

While the SA uses the local movement to generate a new solution only by modifying the old one, the GA generates solutions by combining two different solutions. However, this does not necessarily make the algorithm better or worse than the others.

It is important to note that the GA and the SA are also bounded by the assumption that good solutions are probably found “near” the best know solutions, rather than chosen from a whole set of solutions. Otherwise they would not carry out a better search than that the random search.

The hybrid method was developed by arranging the algorithms in series. First the Genetic Algorithm was used, and then the Simulated Annealing. However, the algorithms parameters influences changes from each one separately, because in hybrid algorithm the genetic algorithm generates the first input to the simulated annealing. In this case, it is not necessary to reach the optimal GA solution. A sensitivity analysis of GA/SA parameters is carried out simultaneously, indicating the ideal parameters of both algorithms, defining the stop criteria of both algorithms.

Since the Genetic Algorithm has many solutions (population), it would be a good idea to use the method as a starting point for using the Simulated Annealing. Thus, the SA will have a pre-checked starting point in a universe of solutions.

**EXPERIMENTAL SETUP**

For the identification of unbalance parameters or bearing coefficients, a finite element model of the test-rig built at Politecnico di Milano during the EU funded research program MODIAROT (Brite Contract BRPR-CT95-0022 Model based DIAgnostics in ROTating machines) was considered. This test-rig can be used to study and analyze different malfunctions affecting the dynamic behavior of rotors. The test-rig, shown in Fig. 3 and Fig. 4, is composed of two rigidly coupled rotors driven by a variable speed electric motor and supported on four elliptical-shaped oil film bearings. Both rotors are made of steel and the rotor train is long about 2 m and has a mass of about 90kg. The rotor has three critical speeds within the operating speed range 0-6000 rpm. The model of the rotor has been tuned and the stiffness and damping coefficients of the bearings are determined with great accuracy as described in Bachschmid et al., (2000).

The rotor system is mounted on a flexible steel foundation, with several natural frequencies in the operating speed range. In this case, the foundation has been modeled by means of a modal representation and further details are reported in Provasi, Zanetta and Vania, (2000). Two proximity probes in each bearing measure the relative shaft displacements, or the journal orbits; two accelerometers on each bearing housing measure its vibrations, and two force sensors on each bearing housing measure the forces which are transmitted to the foundation. The absolute vibration of the shaft is calculated by adding the relative displacement measured by the proximitors to the absolute bearing housing displacement, which is obtained integrating twice the acceleration measured by the accelerometers.
The finite element model used for parameter estimation using the metaheuristic method has 47 nodes (Bachschmid, Pennacchi and Vania, 2002), each node of the model has four degrees of freedom. The bearings are in nodes 4, 17, 25 and 44.

Using a software written for this purpose, the rotor system response to the application of an unbalance (as described in a previous section) has been simulated for different rotating speeds. In particular, the effect of a unitary unbalance, with 8 angular equally spaced phases, has been simulated for all the rotor nodes and the responses where stored for the horizontal and vertical displacements in the bearings.

![Figure 3 - MODIAROT test-rig of Politecnico di Milano.](image)

![Figure 4 - Sketch of MODIAROT test-rig shaft. Dimensions are in millimeters.](image)

![Figure 5 - Finite Element Model.](image)

**PARAMETER ESTIMATION**

Two parameter estimation processes were made using the proposed methodology. In the first estimation, the bearing coefficients were searched for some rotation speed. In the second process, the unbalance magnitude and phase are determined. In that case, the parameters are estimated for all the speed rotation at which the vibration measurements have been measured on the test rig.

The search method proposed works with an objective function, containing all the control variables. That function correlates the experimental response with the simulated response, considering the fitting variable values, which determines an error function.

**Bearing coefficients estimation**

In order to determine the proximity between the experimental response and simulated response with adjusted parameters, an objective function that takes into account the difference between each response has to be considered for a determined rotation speed.

\[
f(\Omega) = \sum_{b=1}^{4} \left[ \left( W_x \frac{X_{\exp}^b(\Omega) - X_{\adj}^b(\Omega)}{X_{\exp}^b(\Omega)} \right) + W_\varphi \frac{\varphi_{\exp}^b(\Omega) - \varphi_{\adj}^b(\Omega)}{\varphi_{\exp}^b(\Omega)} \right]_{\text{vertical}} + \sum_{b=1}^{4} \left[ \left( W_x \frac{X_{\exp}^b(\Omega) - X_{\adj}^b(\Omega)}{X_{\exp}^b(\Omega)} \right) + W_\varphi \frac{\varphi_{\exp}^b(\Omega) - \varphi_{\adj}^b(\Omega)}{\varphi_{\exp}^b(\Omega)} \right]_{\text{horizontal}}
\]

(19)

Where \( w_X \) and \( w_\varphi \) are weight for magnitude and phase respectively, which were considered equal to 1, \( w_b \) is a weight for each bearing that was assumed the same for all bearing with a value of 0.25, \( X_{\exp}^b(\Omega) \) and \( X_{\adj}^b(\Omega) \) the magnitude of the experimental and adjusted response and \( \varphi_{\exp}^b(\Omega) \) and \( \varphi_{\adj}^b(\Omega) \) the phase of experimental and adjusted response.

Figures 6 and 7 show the response in bearing 3 and 4 in vertical and horizontal directions.
Figure 6 – System response in bearing 3 for bearing coefficient estimation

Figure 7 – System response in bearing 4 for bearing coefficient estimation
As can be observed in Fig. 6 and 7, the proposed algorithm can reach acceptable results, because the adjusted response is close to the experimental response.

The estimate of the stiffness coefficient curve $k_{xx}$ in bearing 1 is shown in Fig. 8. All other coefficients of each bearing were estimated in similar way.

In future works, some special attention should be given to the determination of the estimated bearing coefficient curve, in order to improve the fitting process.

\[ f = \sum_{k=1}^{4} \left( w_x \left( \sum_{\Omega} \left( X_{x_{k_{\Omega}}}^{\exp} - X_{x_{k_{\Omega}}}^{\adj} \right)^2 \right)^{\frac{1}{2}} + w_y \left( \sum_{\Omega} \left( \phi_{y_{k_{\Omega}}}^{\exp} - \phi_{y_{k_{\Omega}}}^{\adj} \right)^2 \right)^{\frac{1}{2}} \right) \]

In that case, the influence of the magnitude and phase in objective function is different. It means that proportional magnitudes of $w_x$ and $w_y$ were assumed, in order to guarantee the same weight in the objective function. So, it was considered $w_x$ equal to 1 and $w_y$ equal to $8.75 \times 10^{-7}$.

Table 1 shows the expected values of unbalance parameter and reached values. The system responses, comparing the experimental and adjusted results, in bearing 1 and 2 are shown in Fig. 9 and 10.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Expected value</th>
<th>Reached values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position (node)</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Magnitude [kg⋅m]</td>
<td>0.00036</td>
<td>0.0003687</td>
</tr>
<tr>
<td>Phase [rad]</td>
<td>-1.5708</td>
<td>-1.5717</td>
</tr>
</tbody>
</table>
As shown in Tab. 1 and Fig. 9 and 10, the metaheuristic search method can be considered as a good tool to identify unbalance position and determine its magnitude and phase.
CONCLUSION

A metaheuristic search method is proposed to be applied in model updating and fault identification. This method is based on Genetic Algorithm, which is used to make a first estimative of the searching parameters and Simulated Annealing, which is used to improve the parameters determined by the Genetic Algorithm.

Experimental unbalance response was used to calibrate the mathematical model of the rotor. In a first calibration, the bearing coefficients were determined for some speed rotation and, after that, a polynomial was fitted to this point. In order to determine these coefficients, an objective function that takes into account the difference between the adjusted and experimental response for a determined rotation speed. The search process was applied for some specific rotation speed. It could be observed that bearings coefficients have significant influence in the frequency response fitting.

The unbalance position and parameters were also determined by the proposed method. For this purpose, another objective function considers the difference between the experimental and adjusted response for all measured speed rotation, because the estimated parameters have to be constant for any speed rotation. In that case, the adjusted parameters values were very close to the expected values, showing that the method is efficient for unbalance identification.

In future works, the estimation of bearing coefficients can be improved. A first proposal can be searching the coefficients of a polynomial curve, instead of search the bearing coefficients, and then fitting a polynomial to this process. The unbalance identification presented satisfactory results and the search method should be applied in other kind of fault identification.

REFERENCES


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