Uncertainties in mechanical models of larynx and vocal tract for voice production

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Abstract: We have reviewed a number of studies of voiced sound articulatory synthesis, but all of them have been conducted using deterministic models. However, real voices are random processes. This paper is concerned with the discussion of uncertainties and random variables involved in the process of voice production, based on the maximum entropy principle, and using the mathematical-mechanical model proposed by Ishizaka and Flanagan. We discuss an implicit numeric method for synthesizing voice, which allows us to solve the associated inverse dynamics problem; that is, to identify the parameters of the model, given a specific voice signal. We can, then, analyze the sensitivity and random characteristics of the frequencies involved in the process, due to uncertainties in the parameters.

Keywords: Voice production, Uncertainties, Identification, Modeling, Signal Processing.

NOMENCLATURE

\( \alpha_{\text{reg}} \) = neutral glottal area, area, m²
\( a \) = section or glottal area, area, m²
\( \delta \) = coefficient in the Newmark method, dimensionless
\( A \) = random variable related to the area, area, m²
\( c \) = damping coefficient, damping, Ns/m
\( \tilde{c}_r \) = expression related to the damping, m3/N
\( \tilde{c}_r = \) Fourier coefficients
\( \tilde{c} \) = vector related to the Fourier coefficients
\( [C] \) = damping matrix, damping, Ns/m
\( d \) = stiffness, length, m
\( f_0 \) = fundamental frequency, frequency, Hz
\( f \) = force acting in the mass, force, N
\( h \) = transfer function of the vocal tract
\( k \) = linear stiffness, stiffness, N/m
\( k_c \) = linear stiffness, stiffness, N/m
\( [K] \) = stiffness matrix, stiffness, N/m
\( \ell \) = length of the tube or the vocal fold, length, m
\( \ell \) = expression related to the length, kg/m
\( L \) = random variable related to the length, length, m
\( m \) = mass of the vocal cord, mass, kg
\( m_{x} \) = mean value of \( X \)
\( [M] \) = mass matrix, mass, kg
\( N \) = number of points of the period
\( p_m \) = pressure acting in the mass, pressure, Pa
\( P_r \) = probability used for the confidence region, dimensionless
\( p \) = radiated pressure, pressure, N/m²
\( p_X \) = probability distribution of \( X \)
\( q \) = parameter to be identified, dimensionless
\( Q \) = random variable related to \( q \), dimensionless
\( q_v = \) quantile
\( r \) = acoustic resistance in the mouth, Pa/m³/s
\( \tilde{r} \) = radiation impedance, Pa/m³/s
\( s \) = nonlinear stiffness function, stiffness, N/m
\( t \) = damping function, damping, Ns/m³
\( T \) = period, time, s
\( \dot{u} \) = volume velocity in the frequency domain, density, m³/Hz
\( u \) = air volume velocity across the tubes, velocity, m³/s
\( \dot{u} \) = derivative of \( u \), m³/s²
\( v_c \) = sound velocity, velocity, m/s
\( x^+ \) = upper envelope related to confidence region
\( x^- \) = lower envelope related to confidence region
\( x \) = displacement of the mass, distance, m
\( w \) = vector containing \( x_1, x_2, u_0 \)
\( y \) = radius of the tubes, length, m
\( z_i \) = vector used in the Newmark method

Greek Symbols
\( \alpha \) = index used in the algorithm
\( \tilde{\alpha} \) = constant used in the Newmark method
\( \beta \) = index used in the algorithm
\( \Gamma \) = Gamma function
\( \delta \) = dispersion coefficient used in the probability distribution
\( \hat{\delta} \) = constant used in the Newmark method, dimensionless
\( \Delta t \) = sampling time, time, s
\( \eta_\text{pr} \) = nonlinear coefficient of the springs during collision, dimensionless
\( \eta_\text{pr} \) = nonlinear coefficient of the springs, dimensionless
\( \rho \) = air density, density, kg/m³
\( \mu \) = shear viscosity coefficient, viscosity, Pas
\( \mathcal{U} \) = function used in the coupling equation
\( \delta \) = coefficient used in the Newmark method
\( v \) = number of the Fourier coefficients
\( \sigma_X \) = standard deviation of \( X \)
\( \theta \) = realization
\( \xi \) = damping ratio, dimensionless

Subscripts
\( g \) = relative to glottal
\( n \) = relative to the number of the tubes
\( r \) = relative to the radiated sound
\( 1 \) = relative to the mass 1 in the model
\( 2 \) = relative to the mass 2 in the model

Superscripts
\( \exp \) = relative to the given signal
\( \text{mod} \) = relative to the model
\( \text{reg} \) = relative to the regenerated signal
INTRODUCTION

The voice production process has been studied by many researchers, and for many different reasons such as to obtain synthesis of voiced sounds (Ishizaka and Flanagan, 1972; Koizumi et al., 1987; Cataldo et al., 2006), to simulate pathological vocal-fold vibrations (Ishizaka and Isshiki, 1976; Zhang et al., 2005), to discuss nonlinearities related to the process (Steinecke and Herzel, 1995; Herzel et al., 1995; Lucero, 1999).

We are interested here in phonation, which is one of the larynx functions. The laryngeal complex is located between the pharynx and the trachea, and consists of a number of cartilages and muscles. In the larynx, we can find the vocal folds, small muscular cushions that adduct (come together) to close the laryngeal airway, or abduct (separate) to open this airway. The opening between the vocal folds is called the glottis, and the term glottal has come to be used as a general term for the laryngeal function. Sound results from the vibration of vocal folds, which alternately snap together and apart, colliding with one another in a basically (quasi-)periodic fashion.

The laryngeal function is highly similar within major groups of sounds produced. For example, vocal fold vibration differs little across vowels, which gain their distinctiveness by the shaping of the articulatory system above the larynx, i.e., the portion that goes from the glottis up to the mouth, called vocal tract. For this reason, the phonetic description of speech is based largely on supraglottal articulatory features.

The vocal folds, together with glottal airflow, constitute a highly nonlinear self-oscillating system. According to the accepted myoelastic theory of voice production proposed by Van den Berg (1968) and Titze (1980), the vocal folds are set into vibration by the combined effect of the subglottal pressure, the viscoelastic properties of the folds, and the Bernoulli effect. We cannot forget the coupling between the vocal folds dynamics and the vocal tract acoustics. The effective length, mass, and tension of the vocal folds are determined by muscle action; and in this way, the fundamental frequency (pitch) and the waveform of the glottal pulses can be controlled. The vocal tract acts as a filter which transforms the primary signals into meaningful voiced speech.

The two-mass model of the vocal folds (Ishizaka and Flanagan, 1972) has been widely used and the capability of this well-known model to reproduce the oscillation in detail has been successfully demonstrated. In addition to this model of the vocal folds, it is necessary to have a model for the vocal tract; and as usual, we use an acoustic tube for it. This complete model (vocal folds + vocal tract) will be subsequently referred as the IF72 model in this paper.

The system just discussed has been used for producing and studying voiced sounds in a deterministic way. However, the human voice production system is not deterministic. One of our purposes here is to take into account uncertainties, by using a probabilistic approach related to the parameters present in the IF72 model; or more generally, to the voice production process.

We divided our work, basically, into three parts:

(1) Modeling: we discuss an implicit numerical method to synthesize voiced sounds using the IF72 model.

(2) Inverse dynamics problem (deterministic): the fundamental frequency is controlled by the parameters of the vocal folds and the distinctiveness between the vowel sounds is related to the parameters of the vocal tract. Then, we discuss a way to solve the inverse problem; that is, given a recorded voice signal, how to identify the parameters of the IF72 model that best approach it.

(3) Randomization of the direct problem: the parameters of the vocal folds and the vocal tract are modeled as random variables by taking into account that the voice production is a stochastic process. An experimental database is constructed with experimental realizations of the stochastic process and, for each realization from that database, the random parameters of the model are identified by solving an optimization problem.

MODELING

Figure 1 shows a sketch of the IF72 model. Each vocal fold is represented by two (nonlinear) mass-damper-spring systems, coupled through a (linear) spring ($k_c$) and the vocal tract is represented by a standard two-tube configuration for vowel /a/ (Ishizaka and Flanagan, 1972; Titze, 1994).

The IF72 model assumes motion in the direction perpendicular to airflow only, which is in turn assumed to be quasi-steady and described by Bernoulli’s energy equation. Other assumptions may be found in the cited references.

The complete model (vocal folds and vocal tract) is called a source-filter model (Fant, 1960; Ishizaka and Flanagan, 1972) and it is a well known representation of speech production acoustics, layered on the theory of linear systems. A fundamental assumption is that the vocal tract can be decomposed into dynamic and mutually independent parts (in our case, two) that interact linearly.

We divide the source-filter system in two parts: the subsystem of the vocal folds (source) and the subsystem of the vocal tract (filter). They are coupled by the glottal volume velocity.
During phonation, the filter is excited by a sequence of airflow pulses with a frequency, say $f_0$, which we call the fundamental frequency of the voice signal. The full interaction between the glottal flow and vocal tract is included by solving the differential equations related to the airflow in the vocal tract, and the equations that specify the mechanical vibration mechanism of the vocal folds.

In summary, the output pressure (the voice produced) can be written, in the frequency domain, as

$$\hat{p}_r(\omega) = \hat{u}_g(\omega)\hat{h}(\omega)\hat{s}_r(\omega)$$

where $\hat{p}_r(\omega)$ refers to the radiated sound pressure, $\hat{u}_g(\omega)$ refers to the air volume velocity, $\hat{h}(\omega)$ represents the transfer function of the vocal tract and $\hat{s}_r(\omega)$ denotes radiation characteristic, in the mouth. Putting this equation into words, we could say that the radiated sound pressure waveform of speech is the product of the laryngeal spectrum, the vocal tract transfer function, and the radiation characteristic.

The subglottal pressure $p_s$ is the input of the subsystem source, which output is the air volume velocity $u_g$. Consequently, $u_g$ is the input of the subsystem filter, which output is the radiated pressure, denoted by $p_r$. If we consider the complete system, $p_s$ is the input and the output is the radiated pressure $p_r$.

The complete IF72 model, can be described, in a simplified form, by Eq. 2 and Eq. 3

$$\phi_1(w)|u_g|u_g + \phi_2(w)u_g + \phi_3(w)u_g + \frac{1}{\ell_1} \int_0^\ell (u_g(\tau) - w_3(\tau))d\tau - p_s = 0$$

$$[M]\ddot{w} + [C]\dot{w} + [K]w + h(w, u_g, u_r) = 0$$

where

$$w(t) = \begin{pmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w_4(t) \\ w_5(t) \end{pmatrix} = \begin{pmatrix} x_1(t) \\ x_2(t) \\ u_1(t) \\ u_2(t) \\ u_r(t) \end{pmatrix}$$

The functions $t \mapsto x_1(t)$ and $t \mapsto x_2(t)$ are the displacements of the masses, $t \mapsto u_1(t)$ and $t \mapsto u_2(t)$ describe the air volume velocity through the (two) tubes that model the vocal tract and $t \mapsto u_r(t)$ is the air volume velocity through the mouth. The function, that we call radiated pressure, $t \mapsto p_r(t)$, is defined by Eq. 5.

$$p_r(t) = u_r(t)r_r$$

where $r_r = \frac{128\rho v_c}{9\pi^2 y_2^2}$, $\rho$ is the air density, $v_c$ is the sound velocity and $y_2$ is the radius of the second tube.

And also,
 Uncertainties related to mechanical models for voice production described in the appendix.

\[
[M] = \begin{bmatrix}
m_1 & 0 & 0 & 0 & 0 \\
0 & m_2 & 0 & 0 & 0 \\
0 & \bar{\ell}_1 + \bar{\ell}_2 & 0 & 0 & 0 \\
0 & 0 & \bar{\ell}_2 + \bar{\ell}_r & -\bar{\ell}_r & 0 \\
0 & 0 & 0 & -\bar{\ell}_r & \bar{\ell}_r \\
\end{bmatrix}, \quad [C] = \begin{bmatrix}
r_1 & 0 & 0 & 0 & 0 \\
0 & r_2 & 0 & 0 & 0 \\
0 & 0 & r_1 + r_2 & 0 & 0 \\
0 & 0 & 0 & r_2 & 0 \\
0 & 0 & 0 & 0 & r_r \\
\end{bmatrix},
\]

\[
[K] = \begin{bmatrix}
k_1 + k_c & -k_c & 0 & 0 & 0 \\
-k_c & k_2 + k_c & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\tau_1} + \frac{1}{\tau_2} - \frac{1}{\tau_1} & 0 & 0 \\
0 & 0 & -\frac{1}{\tau_1} & c_1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad h(w, u_g, u_{\bar{g}}) = \begin{bmatrix}
s_1(w_1) + t_1(w_1) - f_1(w_1, u_g, u_{\bar{g}}) \\
s_2(w_2) + t_2(w_2) - f_2(w_1, w_2, u_g, u_{\bar{g}}) \\
0 \\
0 \\
0 \\
\end{bmatrix},
\]

where

\[
\bar{\ell}_n = \frac{n}{\tau_{\bar{\ell}_n}}, \quad \bar{\ell}_r = \frac{\bar{\ell}_n}{\rho \bar{\ell}_r}, \quad r_n = \frac{\bar{\ell}_n}{\tau_{\bar{\ell}_n}}, \quad \omega = \sqrt{\frac{m}{m_1}}, \quad u_n = \pi y_n, \quad c_n = \frac{\bar{\ell}_n}{\rho \bar{\ell}_r}, \quad \bar{\ell}_n \text{ is the length of the } n \text{ th tube, } y_n \text{ is the radius of the } n \text{ th tube, and } \mu \text{ is the shear viscosity coefficient.}
\]

The functions \( w \mapsto \phi_1(w) \), \( w \mapsto \phi_2(w) \), \( w \mapsto \phi_3(w) \) and \( (w, u_g, u_{\bar{g}}) \mapsto h(w, u_g, u_{\bar{g}}) \); and also the functions \( s_1(w_1), s_2(w_2), t_1 = s_1(t_1), t_2 = s_2(t_2), (w_1, u_g, u_{\bar{g}}) \mapsto f_1(w_1, u_g, u_{\bar{g}}) \) and \( (w_1, w_2, u_g, u_{\bar{g}}) \mapsto f_2(w_1, w_2, u_g, u_{\bar{g}}) \) are described in the appendix.

In order to solve the system (Eq. 2 and Eq. 3); that is, find \( u_g \) and \( w \), given \( p_i \), a centered finite difference scheme is used for Eq. 2 and an unconditionally stable Newmark scheme is used for Eq. 3. All of the values used here are the same used by Ishizaka and Flanagan (1972).

Let \( \Delta t \) be the sampling time and \( w_i = w(i\Delta t) \), \( w_i = \bar{w}(i\Delta t) \), \( u_g_i = u_g(i\Delta t) \) and \( u_{\bar{g}}_i = u_{\bar{g}}(i\Delta t) \).

Then, for all \( i \geq 1 \), we can write

\[
\phi_1(w_i)u_g_i + \phi_2(w_i)u_g_i + \phi_3(w_i)\frac{1}{\Delta t}(u_g_i - u_{g,i-1}) + \frac{1}{\bar{\ell}_i} \Delta t \sum_{k=0}^{i-1} (u_{g,i} - w_{1,k}) - p_3 = 0
\]

and

\[
[A]w_i + h \left( w_i, \frac{w_i - w_{i-1}}{\Delta t}, \frac{u_g_i - u_{g,i-1}}{\Delta t} \right) = z_i
\]

where

\[
[A] = [K] + \bar{a}_0[M] + \bar{a}_1[C]
\]

\[
z_i = [M](\bar{a}_0w_{i-1} + \bar{a}_2\bar{w}_{i-1} + \bar{a}_3\bar{w}_{i-1}) + [C](\bar{a}_1w_{i-1} + \bar{a}_4\bar{w}_{i-1} + \bar{a}_5\bar{w}_{i-1})
\]

and

\[
\begin{align*}
\bar{w}_i &= \bar{a}_0(w_i - w_{i-1}) - \bar{a}_2\bar{w}_{i-1} - \bar{a}_3\bar{w}_{i-1} \\
\bar{w}_i &= \bar{w}_{i-1} + \bar{a}_0\bar{w}_{i-1} + \bar{a}_2\bar{w}_{i-1} \\
\bar{a}_0 &= \frac{1}{\Delta t}, \quad \bar{a}_1 = \frac{\delta}{\Delta t}, \quad \bar{a}_2 = \frac{1}{\Delta t} \\
\bar{a}_4 &= \frac{\delta}{\Delta t} - 1, \quad \bar{a}_5 = \frac{\delta}{\Delta t} - 2, \quad \bar{a}_6 = \Delta t(1 - \delta), \quad \bar{a}_7 = \bar{\delta}\Delta t
\end{align*}
\]

with \( u_{g,0} = 0, w_0 = 0, \bar{w}_0 = 0, \bar{w}_0 = 0, \delta = 0.5 \) and \( \bar{\delta} = 0.25 \).

**Algorithm used**

The method used to construct the solution of Eq. 8 and Eq. 9 consists in finding \( u_{g,i} \) as the limit of the sequence \( \{u_{g,i}\} \), \( \alpha \geq 0 \), when \( \alpha \) tends to infinity such that, for all \( \alpha \geq 1, i \geq 1, \)

\[
\phi_1(w_i^{\alpha-1})u_{g,i}^{\alpha} + \phi_2(w_i^{\alpha-1})u_{g,i}^{\alpha} + \phi_3(w_i^{\alpha-1})\frac{1}{\Delta t}(u_{g,i}^{\alpha} - u_{g,i-1}) + \frac{1}{\bar{\ell}_i} \Delta t \sum_{k=0}^{i-1} (u_{g,i}^{\alpha} - w_{1,k}) - p_3 = 0
\]
with \( w_i^0 = w_{i-1} \), \( u_{g0} = u_{g_{i-1}} \).

In Eq. 12, \( w_i^{\alpha-1} \) is the limit of the sequence \( \{w_i^{\alpha-1, \beta}\} \), \( \beta \geq 0 \), when \( \beta \) tends to infinity; such that for all \( \beta \geq 1, \alpha > 1, i \geq 1, \)

\[
[A]w_i^{\alpha, \beta-1} = z_i - h \left( w_i^{\alpha, \beta-1} \frac{w_i^{\alpha-1, \beta-1} - w_{i-1}^{\alpha-1, \beta-1}}{\Delta t}, u_{g0}^{\alpha-1} \right)
\]

with \( w_i^{\alpha, 0} = w_i^{\alpha-1} \).

In other words, our strategy consists in considering two loops (outer \( \alpha \) and inner \( \beta \)). For each iteration \( i \), we find \( u_g \) and we use this value for finding \( w \), but with this value of \( w \) found, we must return to correct \( u_g \) and we repeat this process up to the tolerances are reached.

As an example, we simulated a vowel /a/, considering two tubes for the vocal tract and the same data used by Ishizaka and Flanagan (1972): \( d_1 = 0.25 \text{ cm}, d_2 = 0.05 \text{ cm}, a_{00} = 0.05 \text{ cm}^2, p_s = 8000 \text{ Pa}, m_1 = 0.125 \text{ g}, m_2 = 0.025 \text{ g}, k_1 = 80,000 \text{ dyn/cm}, k_2 = 8,000 \text{ dyn/cm}, k_c = 25,000 \text{ dyn/cm}, \xi_1 = 0.1, \xi_2 = 0.6, \eta_1 = \eta_2 = 100, \eta_{h1} = \eta_{h2} = 500 \). The graphs obtained are showed in the Fig. 2. These results are coherent with those presented by Ishizaka and Flanagan (1972).

![Graphs showing displacements and radiated pressure](image)

Figure 2 – Displacements of the two masses \((x_1 \text{ and } x_2)\); glottal volume velocity \((u_g)\); radiated pressure \((p_r)\).

Production of a vowel /a/.

**INVERSE DYNAMICS PROBLEM (DETERMINISTIC)**

Most speech simulation models are based on the assumption of one-dimensional wave propagation, as the IF72 model. This means that the tubular vocal tract shape can be approximated as a finite number of cylindrical elements that are stacked consecutively from the larynx to the mouth. A particular vocal tract shape can be imposed on a model by specifying the cross-sectional area of each cylindrical element as a function of the distance from the glottis. For modelling purposes, any vocal tract shape can be defined by its unique *area function*. Hence, a necessary component for the simulation of natural sounding speech is an inventory of vocal tract area functions that correspond to the vowels (and consonants) used to produce human speech. The success of speech simulators has been limited, in part, by the lack of a body of morphological information about the vocal tract shape on which to base these area functions and many efforts have been made to find the description of the vocal tract configurations for vowels relative to their acoustic output (Fant, 1960; Adachi and Yamada, 1999; Titze and Story, 1996; Takemoto and Honda, 2006). Here, we will discuss a method for finding the parameters of the vocal tract, from a given voice signal.

We also introduced a factor \( q \), as done by Ishizaka and Flanagan, called *tension parameter*, that controls the fundamental frequency \( f_0 \) of the vocal folds oscillation, because vocal fold abduction and tension should be the main factor used by speakers to control phonation (Koenig, 2000; McGowan et al., 1995). We write \( k_1 = q^2 \hat{k}_1, k_2 = q^2 \hat{k}_2, k_c = q^2 \hat{k}_c \) and we control the parameter \( q \), where \( \hat{k}_1, \hat{k}_2 \) and \( \hat{k}_c \) are values fixed (we used the same values that Ishizaka and Flanagan (1972)).

One of the objectives of this paper is to identify the values of \( q \) (*tension parameter*), \( \ell_1, \ell_2, a_1 \) and \( a_2 \) (geometrical
Uncertainties related to mechanical models for voice production

parameters of the vocal tract) corresponding to the recorded signal, by solving an optimization problem in terms of $q, \ell, a_{1}$, $a_{2}$ in a mean-square sense. This identifies the appropriate parameters to reconstruct a recorded signal.

This optimization problem consists in: (1) finding the value of $q$ that minimizes the function $q \mapsto |f_{0}^{\exp} - f_{0}^{\mod}(q)|^{2}$, where $f_{0}^{\exp}$ is the fundamental frequency of the recorded signal and $f_{0}(q)$ is the fundamental frequency of the signal generated by the model, for a given $q$; (2) finding the values of $\ell, a_{1}, a_{2}$ that minimizes the function $(\ell, a_{1}, a_{2}) \mapsto ||\mathbf{c}^{\exp}(\ell, a_{1}, a_{2}) - \mathbf{c}^{\exp}||^{2}$.

The components of the vector $\mathbf{c}^{\mod}$ are the Fourier coefficients obtained from an generated by the model $t \mapsto p_{r}^{\mod}(t)$, where $p_{r}^{\mod}(t) = u_{r}(t) r_{r}$ and the components of the vector $\mathbf{c}^{\exp}$ are the coefficients of a pseudo Fourier expansion of the quasi-periodic recorded signal $t \mapsto p_{r}^{\reg}(t)$; $p_{r}^{\reg}(t)$ is the radiated pressure obtained from a real voice at time $t$.

We write $\mathbf{c}^{\exp} = (\mathbf{c}_{-\nu}^{\exp}, \ldots, \mathbf{c}_{\nu}^{\exp})$ and $\mathbf{c}^{\mod} = (\mathbf{c}_{-\nu}^{\mod}, \ldots, \mathbf{c}_{\nu}^{\mod})$, where
\begin{align}
\mathbf{c}_{n}^{\exp} &= \frac{1}{N_{\exp}} \sum_{m=0}^{N_{\exp}-1} c_{n,m}^{\exp}, \quad -\nu \leq n \leq \nu, \\
\mathbf{c}_{n}^{\mod} &= \frac{1}{T_{\mod}} \sum_{k=0}^{N_{\mod}-1} p_{r}^{\mod}(k\Delta t) \exp(-jn2\pi f_{0}^{\exp} k\Delta t), \quad T_{\exp} = \frac{1}{f_{0}^{\exp}}, \quad N_{\exp} = \frac{T_{\exp}}{\Delta t}, \\
\mathbf{c}_{n}^{\reg} &= \frac{1}{T_{\reg}} \sum_{k=0}^{N_{\reg}-1} p_{r}^{\reg}(k\Delta t) \exp(-jn2\pi f_{0}^{\reg} k\Delta t), \quad -\nu \leq n \leq \nu, \quad T_{\reg} = \frac{1}{f_{0}^{\reg}}, \quad N_{\reg} = \frac{T_{\reg}}{\Delta t}.
\end{align}

We will consider a signal, collected from a brazilian person, speaking a (sustained) vowel /a/. The data were read into MATLAB software, which was used for all further processing.

Figure 3 shows the signal recorded, in the time domain and in the frequency domain.

![Figure 3 – Signal recorded. (a) signal in the time domain; (b) signal in the frequency domain.](image)

Figure 4 shows the graph of the recorded signal, $t \mapsto p_{r}^{\exp}(t)$, and the graph of the signal $t \mapsto p_{r}^{\reg}(t)$, given by

\[ p_{r}^{\reg}(t) = \sum_{n=-\nu}^{\nu} \mathbf{c}_{n}^{\exp} \exp(jn2\pi f_{0}^{\exp} t) \]

using $\nu = 30$ pseudo Fourier coefficients.

We can observe that the regenerated signal is well reproduced.

**RANDOMIZATION OF THE DIRECT PROBLEM**

Up to now, we have discussed the deterministic problem, direct and inverse. However, our principal goal is to analyse the system when uncertainties are present.
We collected three signals, from two different people ($P_1$ and $P_2$), each one spoke the sustained vowel /a/. Then, for each signal, we solved the inverse problem discussed in the previous section. We found the values showed in the Tab. 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$q$</th>
<th>$a_1$ (cm$^2$)</th>
<th>$a_2$ (cm$^2$)</th>
<th>$\ell_1$ (cm)</th>
<th>$\ell_2$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$ Signal 1</td>
<td>0.6300</td>
<td>1.5975</td>
<td>5.9258</td>
<td>9.9780</td>
<td>9.4872</td>
</tr>
<tr>
<td>Signal 2</td>
<td>0.6000</td>
<td>1.8052</td>
<td>5.8557</td>
<td>8.6928</td>
<td>8.9612</td>
</tr>
<tr>
<td>Signal 3</td>
<td>0.6150</td>
<td>1.7948</td>
<td>5.9964</td>
<td>8.2503</td>
<td>9.4466</td>
</tr>
<tr>
<td>$P_2$ Signal 1</td>
<td>1.25</td>
<td>1.66</td>
<td>6.1871</td>
<td>8.6303</td>
<td>9.2965</td>
</tr>
<tr>
<td>Signal 2</td>
<td>1.25</td>
<td>1.6576</td>
<td>6.0046</td>
<td>9.1714</td>
<td>9.1696</td>
</tr>
<tr>
<td>Signal 3</td>
<td>1.27</td>
<td>1.7900</td>
<td>5.1090</td>
<td>9.2100</td>
<td>9.1600</td>
</tr>
</tbody>
</table>

As we can see, the values for the parameter $q$ and for the parameters of the vocal are different, for each signal produced, as we expected.

We consider, then, the random variables $Q$, $A_1$, $A_2$, $L_1$ and $L_2$ related to the variables $q$, $a_1$, $a_2$, $\ell_1$ and $\ell_2$ and we are going to study the results obtained from the system when these quantities vary.

### Construction of the probabilistic model for the random parameters

To take into account the uncertainties, a probabilist model is constructed (Soize, 2000), which consists in modeling the parameters $q$, $a_1$, $a_2$, $\ell_1$ and $\ell_2$ as random variables $Q$, $A_1$, $A_2$, $L_1$ and $L_2$, respectively.

The probabilistic model of each random variable ($Q$, $A_1$, $A_2$, $L_1$ and $L_2$) is constructed taking into account the following available informations:

1. $E\{Q\} = q$, $E\{A_1\} = a_1$, $E\{A_2\} = a_2$, $E\{L_1\} = \ell_1$, $E\{L_2\} = \ell_2$
2. $E\{Q^2\} < +\infty$, $E\{A_1^2\} < +\infty$, $E\{A_2^2\} < +\infty$, $E\{L_1^2\} < +\infty$, $E\{L_2^2\} < +\infty$

where $E\{\cdot\}$ denotes the mathematical expectation operator. The maximum entropy principle (Shannon, 1948) yields the following probability distribution

$$
px(x) = 1_{[0,\infty]}(x) \frac{1}{m_X} \left( \frac{1}{\delta^2} \right)^{\frac{1}{2\sigma}} \frac{1}{\Gamma(1/\delta^2)} \left( \frac{x}{m_X} \right)^{\frac{1}{2\sigma}-1} \exp\left( -\frac{x}{\delta^2 m_X} \right)
$$

where $X$ represents $Q$, $L_1$, $L_2$, $A_1$, $A_2$; $\delta = \frac{\sigma}{m_X}$ is a dispersion coefficient such that $0 \leq \delta \leq 1/\sqrt{2}$, $\sigma_X$ is the standard deviation of $X$; $m_X$ is the mean value of $X$ and $\alpha \mapsto \Gamma(\alpha)$ is the Gamma function defined by $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$. 

![Figure 4 – Signal recorded: section of the original signal; section of the regenerated signal.](image-url)
Confidence region for the random frequencies

We construct the confidence region associated with a probability level $P_c$ (we considered $P_c = 0.95$) for each one of the frequencies (we considered the three first significant frequencies in the spectrum of the signals). The confidence region is constructed using the quantiles (Serfling, 1980). Let $F_X(\xi)(x)$ be the distribution function of random variable $X(\xi)$. For $0 < p < 1$, the $p$th quantile (denoted by $qu(p)$) of $F_X$ is defined as

$$ qu(p) = \inf \{ x : F_X(\xi)(x) \geq p \} . $$

Then, the upper envelope $x^+(\xi)$ and the lower envelope $x^-(\xi)$ of the confidence region are defined by

$$ x^+(\xi) = qu((1 + P_c)/2) \quad \text{and} \quad x^-(\xi) = qu((1 - P_c)/2) $$

Let $x_1(\xi) = X(\xi; \theta_1), \ldots, x_n(\xi) = X(\xi; \theta_n)$ be the $n$ independent realizations of random variable $X(\xi)$. Let $\tilde{x}_1(\xi) < \ldots < \tilde{x}_n(\xi)$ be the order statistics associated with $x_1(\xi), \ldots, x_n(\xi)$. Therefore, one has the following estimation:

$$ x^+(\xi) \simeq \tilde{x}_{j^+}(\xi) \quad \text{and} \quad x^-(\xi) \simeq \tilde{x}_{j^-}(\xi) $$

in which $fix(z)$ is the integer part of the real number $z$.

First, the geometrical parameters of the vocal tract are modeled as random variables $A_1, A_2, L_1$ and $L_2$, while the tension parameter is fixed at $\ell = 1.25$. Figure 5(a) shows the confidence region, obtained in this case, for the first three frequencies considered. It can be seen that these frequencies are not sensitive to the geometrical parameters.

Secondly, the tension parameter is considered as the random variable $Q$ and the geometrical parameters of the vocal tract are values fixed at $a_1 = 1.7 cm^2$, $a_2 = 5.9 cm^2$, $\ell_1 = 9.0 cm$, $\ell_2 = 9.0 cm$.

The values considered for the realizations above were: $E\{Q\} = 1.25$, $E\{A_1\} = 1.7 cm^2$, $E\{A_2\} = 5.9 cm^2$, $E\{L_1\} = 9.0 cm$, $E\{L_2\} = 9.0 cm$, $\frac{\delta}{E\{Q\}} = 0.4$, $\frac{\delta}{E\{A_1\}} = 0.4$, $\frac{\delta}{E\{A_2\}} = 0.2$, $\frac{\delta}{E\{L_1\}} = 0.1$, $\frac{\delta}{E\{L_2\}} = 0.1$. These two graphs justify the method used for solving the inverse problem, for which two independent optimization problems were set, one for the tension parameter, related to the fundamental frequency, and other for the geometrical parameters of the vocal tract, related to the pseudo Fourier coefficients of the signal.

Figure 6, we show the results for the probability distribution for $Q$, whose analytical expression is given by Eq. 17 (with $X = Q$), and we show also the probability distribution for the fundamental frequency, which were obtained from the model, considering $Q$ as a random variable.

![Figure 5](image1.png)

**Figure 5** – Confidence region for the first three frequencies: $x \rightarrow x^-$ and $\ell \rightarrow x^+$. (a) Geometrical parameters are random variables and $\ell = 1.25$; (b) Tension parameter is random variable and $a_1 = 1.7 cm^2$, $a_2 = 5.9 cm^2$, $\ell_1 = 9.0 cm$, $\ell_2 = 9.0 cm$. 

![Figure 6](image2.png)
for the fundamental frequency, considering the some frequencies involved in the process to the parameter uncertainty. Finally, we constructed the probability distribution considering only the available information and the maximum entropy principle. We also investigated the sensitivity of the parameter that controls the fundamental frequency, and another to identify the geometrical parameters of the vocal tract. The results obtained here support the strategy used and also confirm that the voice production system is stochastic.

CONCLUSIONS

We used an implicit numerical method for solving the system of differential equations related to the voice production problem, modeled by Ishizaka and Flanagan (1972). The results for the direct problem are consistent with those previously presented in the literature. This particular method was chosen because not only it is more efficient in terms of computational cost, but it also allows the solution of the associated inverse dynamics problems; that is, the identification of parameters used in the model, given a recorded signal. The inverse problem was divided in two parts, one to identify the parameter that controls the fundamental frequency, and another to identify the geometrical parameters of the vocal tract. The results obtained here support the strategy used and also confirm that the voice production system is stochastic. In addition, we constructed a probabilistic model to analyze the uncertainties in the parameters involved in the process, considering only the available information and the maximum entropy principle. We also investigated the sensitivity of some parameters involved in the process to the parameter uncertainty. Finally, we constructed the probability distribution for the fundamental frequency, considering the tension parameter as a random variable, using the maximum entropy principle. These results indicate our next step, that will be pursued in a future work: to solve the stochastic inverse problem; that is, to identify the statistics of the parameters involved in the process of voice production.

APPENDIX

\[
\phi_1(w) = \left( \frac{0.19 \rho}{\sigma_0^2 + 2\ell g w_1} + 2\ell g w_1 \right) + \frac{\rho}{\sigma_0^2 + 2\ell g w_1^2} \left[ 0.5 - \frac{\alpha_0 + 2\ell g w_2}{a_1} \left( 1 - \frac{\alpha_0 + 2\ell g w_2}{a_1} \right) \right] 
\]

\[
\phi_2(w) = (12\mu \ell g \sigma_0^2 + \frac{d_1}{\sigma_0^2 + 2\ell g w_1^2}) + 12\frac{\ell g}{\sigma_0^2 + 2\ell g w_2^2} + \ell g \cdot \phi_3(w) = \left( \frac{\rho d_1}{\sigma_0^2 + 2\ell g w_1^2} + \frac{\rho d_2}{\sigma_0^2 + 2\ell g w_2^2} + \ell g \right)
\]

\[
s_1(w_1) = \begin{cases} 
    k_1 \eta_k w_1^3, & w_1 > -\frac{\alpha_0}{2\ell g} \\
    k_1 \eta_k w_1^3 + 3k_1 \left( w_1 + \frac{\alpha_0}{2\ell g} \right) + \eta_h \left( w_1 + \frac{\alpha_0}{2\ell g} \right)^3, & w_1 \leq -\frac{\alpha_0}{2\ell g} 
\end{cases}
\]

\[
s_2(w_2) = \begin{cases} 
    k_2 \eta_k w_2^3, & w_2 > -\frac{\alpha_0}{2\ell g} \\
    k_2 \eta_k w_2^3 + 3k_2 \left( w_2 + \frac{\alpha_0}{2\ell g} \right) + \eta_h \left( w_2 + \frac{\alpha_0}{2\ell g} \right)^3, & w_2 \leq -\frac{\alpha_0}{2\ell g} 
\end{cases}
\]

\[
t_1(w_1) = \begin{cases} 
    0, w_1 > -\frac{\alpha_0}{2\ell g} \\
    2\ell g \sqrt{m_1 k_1}, w_1 \leq -\frac{\alpha_0}{2\ell g} 
\end{cases}, \quad t_2(w_2) = \begin{cases} 
    0, w_2 > -\frac{\alpha_0}{2\ell g} \\
    2\ell g \sqrt{m_2 k_2}, w_2 \leq -\frac{\alpha_0}{2\ell g} 
\end{cases}
\]

\[
f_1(w_1, u_g, \dot{u}_g) = \begin{cases} 
    \ell g \ell_1 p_{m_1}(w_1, u_g, \dot{u}_g), & w_1 > -\frac{\alpha_0}{2\ell g} \\
    0, & otherwise
\end{cases}
\]

\[
p_{m_1}(w_1, u_g, \dot{u}_g) = p_4 - 1.372 \left( \frac{\mu d_1}{\sigma_0^2 + 2\ell g w_1} \right)^2 - \frac{1}{2} \left( 12\mu \ell g \left( \frac{d_1}{\sigma_0^2 + 2\ell g w_1} \right) + \frac{\rho d_1}{\sigma_0^2 + 2\ell g w_1} \right) \dot{u}_g
\]
Uncertainties related to mechanical models for voice production

\[ f_2(w_1, w_2, u_g, \dot{u}_g) = \begin{cases} \ell_d d_2 p_{m2}(w_1, w_2, u_g, \dot{u}_g), & w_1 > \frac{-a_m}{2\ell_g} \text{ and } w_2 > \frac{-a_m}{2\ell_g} \\ \ell_d d_2 p_1, & w_1 > \frac{-a_m}{2\ell_g} \text{ and } w_2 \leq \frac{-a_m}{2\ell_g} \\ 0, & \text{otherwise} \end{cases} \]

\[ p_{m1}(w_1, w_2, u_g, \dot{u}_g) = p_{m1} - \frac{1}{2} \left\{ 12\mu \ell^2 \left( \frac{d_1}{(a_0 + 2\ell_g w_1)^2} + 12r^2 \frac{d_2}{(a_0 + 2\ell_g w_2)^2} \right) u_g^2 + \left( \frac{\rho d^2}{a_0 + 2\ell_g w_1} + \frac{\rho d^2}{a_0 + 2\ell_g w_2} \right) \dot{u}_g^2 \right\} - \frac{p_g}{2} u_g^2 \left( \frac{1}{(a_0 + 2\ell_g w_1)^2} + \frac{1}{(a_0 + 2\ell_g w_2)^2} \right) \]

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REFERENCES


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