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ON THE INFLUENCE OF NOISE IN CHAOS CONTROL METHODS

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Abstract: *Chaos may be exploited in order to design dynamical systems that may quickly react to some new situation, changing conditions and their response. In this regard, the idea that chaotic behavior may be controlled by small perturbations allows this kind of behavior to be desirable in different applications. Chaos control may be understood as a two stage technique: the first one is known as the learning stage where the unstable periodic orbits (UPOs) embedded in chaotic attractor are identified and system characteristics are evaluated; after that, the control stage stabilizes desired UPOs. This paper presents an overview of chaos control methods classified as follows: OGY methods – that includes discrete and semi-continuous approaches; multiparameter methods – that also includes discrete and semi-continuous approaches; and time-delayed feedback methods that are continuous approaches. These methods are employed in order to stabilize some desired UPOs establishing a comparative analysis of all methods. Essentially, a control rule is of concern and each controller needs to follow this rule in the presence of observed noise. The main goal is to establish a comparative analysis of chaos control methods focusing on the control procedures robustness.*

Keywords: *Chaos, Control, Nonlinear dynamics, Nonlinear pendulum, Noise.*

1. INTRODUCTION

Nonlinearities are responsible for a great variety of possibilities in natural systems. Chaos is one of these possibilities being related to an intrinsic richness. A geometrical form to understand chaos is related to a transformation known as Smale horseshoe that establishes a sequence of contraction-expansion-folding which causes the existence of an infinity number of unstable periodic orbits (UPOs) embedded in a chaotic attractor. This set of UPO constitutes the essential structure of chaos. Besides, chaotic behavior has other important aspects as sensitive dependence to initial conditions and ergodicity. The idea that chaotic behavior may be controlled by small perturbations applied in some system parameters allows this kind of behavior to be desirable in different applications.

In brief, chaos control methods may be classified as discrete and continuous methods. Semi-continuous method is a class of discrete method that lies between discrete and continuous method. The pioneer work of Ott *et al.* (1990) introduced the basic idea of chaos control proposing the discrete OGY method. Afterwards, Hübinger *et al.* (1994) proposed a variation of the OGY technique considering semi-continuous actuations in order to improve the original method capacity to stabilize unstable orbits. Pyragas (1992) proposed a continuous method that stabilizes UPOs by a feedback perturbation proportional to the difference between the present and a delayed state of the system.

This article deals with a comparative analysis of chaos control methods that are classified as follows: OGY methods – that includes discrete and semi-continuous approaches (Ott *et al.*, 1990; Hübinger *et al.*, 1994); multiparameter methods – that also includes discrete and semi-continuous approaches (De Paula *et al.*, 2008, 2009a); and time-delayed feedback methods that are continuous approaches (Pyragas, 1992; Socolar *et al.*, 1994).

Many research efforts were presented in literature in order to improve the originals chaos control techniques and there are numerous review papers concerning these procedures. Andrievskii & Fradkov (2003), Andrievskii & Fradkov (2004), Fradkov *et al.* (2006) and Savi *et al.* (2006) discussed several methods for controlling chaotic systems.

Despite the numerous review papers concerning the chaos control, there is a lack of reports that present a comparative analysis of the control strategies that is the main goal of this contribution. Moreover, noise contamination is unavoidable in experimental data acquisition and, therefore, it is important to evaluate its effect on control strategies. In this paper, the capability of the chaos control methods to stabilize desired UPOs in the presence of observed noise is analyzed, presenting a comparative analysis of methods performance focusing on control procedures robustness. A specific dynamical system is of concern and all signals are generated by numerical integration of a mathematical model, using experimentally identified parameters. In order to consider a system with high instability, a nonlinear pendulum treated in other references is considered (De Paula & Savi, 2009a,b; Pereira-Pinto *et al.*, 2004). Results show the performance of each method to stabilize desired orbits exploring some limitations of its application considering noisy time series. Basically, it is considered an observed noise, simulating noise in experimental data due to instrumentation apparatus and, therefore, noise does not have influence in system dynamics.

The paper is organized as follows. Initially, a brief introduction of chaos control methods is presented. Afterwards, a comparative study is carried out by defining a control rule that should be followed by each controller considering noisy time series.

2. CHAOS CONTROL METHODS

The control of chaos can be treated as a two-stage process. The first stage is called learning stage where it is performed the identification of UPOs and system parameters necessary for control purposes. A good alternative for the UPO identification is the close return method (Auerbach *et al.*, 1987). This identification is not related to the knowledge of the system dynamics details. The estimation of system parameters is done in different ways for discrete, semi-continuous and continuous methods. After the learning stage, the second stage starts promoting the UPO stabilization.

This section considers an overview of the chaos control methods, classified as follows: OGY methods – that includes discrete and semi-continuous approaches (Ott *et al.*, 1990; Hübinger *et al.*, 1994); multiparameter methods – that also includes discrete and semi-continuous approaches (De Paula & Savi, 2008, 2009a); and time-delayed feedback methods that are continuous approaches (Pyragas, 1992; Socolar *et al.*, 1994).

2.1 OGY Method

The OGY method (Ott *et al.*, 1990) is described by considering a discrete system of the form of a map $\xi^{n+1} = F(\xi^n, p^n)$, where $p \in \Re$ is an accessible parameter for control. This is equivalent to a parameter dependent map associated with a general surface, usually a Poincaré section. Let $\xi_C^{n+1} = F(\xi_C^n, p_0)$ denote the unstable fixed point on this section corresponding to an unstable periodic orbit in the chaotic attractor that one wants to stabilize. Basically, the control idea is to monitor the system dynamics until the neighborhood of this point is reached. When this happens, a proper small change in the parameter p causes the next state ξ^{n+1} to fall into the stable direction of the fixed point. In order to find the proper variation in the control parameter, δp, it is considered a linearized version of the dynamical system in the neighborhood of the equilibrium point given by Eq.(1). The linearization has a homeomorphism with the nonlinear problem that is assured by the Hartman-Grobman theorem (Savi, 2006).

$$\delta\xi^{n+1} = J^n \delta\xi^n + w^n \delta p^n \tag{1}$$

where $\delta\xi^n = \xi^n - \xi_C^n$, $\delta\xi^{n+1} = \xi^{n+1} - \xi_C^{n+1}$, $\delta p^n = p^n - p_0$, $J^n = D_{\xi^n} F(\xi^n, p^n)\Big|_{\xi^n = \xi_C^n, p = p_0}$ and $w^n = D_{p^n} F(\xi^n, p^n)\Big|_{\xi^n = \xi_C^n, p^n = p_0}$.

Hübinger et al. (1994) verified that the linear mapping J^n deforms a sphere around ξ_C^n into an ellipsoid around ξ_C^{n+1}. Therefore, a singular value decomposition (SVD) can be employed in order to determine the unstable and stable directions, v_u^n and v_s^n, in Σ_n which are mapped onto the largest, $\sigma_u^n u_u^n$, and shortest, $\sigma_s^n u_s^n$, semi-axis of the ellipsoid in Σ_{n+1}, respectively. Here, σ_u^n and σ_s^n are the singular values of J^n.

$$J^n = U^n W^n (V^n)^T = \begin{Bmatrix} u_u^n & u_s^n \end{Bmatrix} \begin{bmatrix} \sigma_u^n & 0 \\ 0 & \sigma_s^n \end{bmatrix} \begin{Bmatrix} v_u^n & v_s^n \end{Bmatrix}^T \tag{2}$$

Korte et al. (1995) established the control target as being the adjustment of δp^n such that the direction v_s^{n+1} on the map n+1 is obtained, resulting in a maximal shrinking on map n+2. Therefore, it demands $\delta\xi^{n+1} = \alpha v_s^{n+1}$, where $\alpha \in \Re$. Hence:

$$J^n \delta\xi^n + w^n \delta p^n = \alpha v_s^{n+1} \tag{3}$$

that is a relation from which α and δp^n can be conveniently chosen.

The OGY method can be employed even in situations where a mathematical model is not available. Under this situation, all parameters can be extracted from time series analysis. The Jacobian J^n and the sensitivity vector w^n can be estimated from a time series using a least-square fit method (Auerbach *et al.*, 1987; Otani & Jones, 1997).

Otani & Jones (1997) presented some important aspects of the OGY method. As positive points, they mentioned the use of small perturbations for stabilization, the flexibility due to chaos, independence from equations of motion, high computational efficiency, and robustness due to parameter uncertainties. As drawbacks, the authors mentioned the difficulty to stabilize either orbits with high periodicity or systems with high instability, and the necessity to wait the system to visit the neighborhood of some UPO.

An alternative to deal with some of the OGY drawbacks is the use of as many control stations as it is necessary to stabilize some orbits. This is the essential point related to semi-continuous method.

2.1.1 Semi-continuous Method

The semi-continuous method (SC) lies between the continuous and the discrete time control because one can introduce as many intermediate Poincaré sections, viewed as control stations, as it is necessary to achieve stabilization of a desired UPO (Hübinger *et al.*, 1994). Therefore, the SC method is based on measuring transition maps of the system. These maps relate the state of the system in one Poincaré section to the next.

In order to use N control stations per forcing period T, one introduces N equally spaced successive Poincaré sections $\Sigma_n, n = 0,...,(N-1)$. Let $\xi_C^n \in \Sigma_n$ be the intersections of the UPO with Σ_n and F be the mapping from one control station Σ_n to the next one Σ_{n+1}.

2.2 Multiparameter Method

The multiparameter chaos control method (MP) is based on the OGY approach and considers N_p different control parameters, p_i ($i = 1,..., N_p$). Moreover, only one of these control parameters actuates in each control station (De Paula & Savi, 2008, 2009a). Under this assumption, the map F that establishes the relation of the system behavior between the control stations Σ_n and Σ_{n+1}, depends on all control parameters. Although only one parameter actuates in each section, it is considered the influence of all control parameters based on their positions in station Σ_n. On this basis,

$$\xi^{n+1} = F(\xi^n, P^n) \tag{4}$$

where P^n is a vector with all control parameters. By using a first order Taylor expansion, one obtains the linear behavior of the map F in the neighborhood of the control point ξ_C^n and around the control parameter reference position, P_0, is defined by.

$$\delta\xi^{n+1} = J^n \delta\xi^n + W^n \delta P^n \tag{5}$$

where $\delta\xi^{n+1} = \xi^{n+1} - \xi_C^{n+1}$, $\delta\xi^n = \xi^n - \xi_C^n$, $\delta P^n = P^n - P_0$ is related to the control actuation, $J^n = D_{\xi^n} F(\xi^n, P^n)\Big|_{\xi^n = \xi_C^n, P^n = P_0}$ is the Jacobian matrix and $W^n = D_{P^n} F(\xi^n, P^n)\Big|_{\xi^n = \xi_C^n, P^n = P_0}$ is the sensitivity matrix which each column is related to a control parameter. In order to evaluate the influence of all parameters actuation, it is assumed that the system response to all parameters perturbation is given by a linear combination of the system responses when each parameter actuates isolated and the others are fixed at their reference value. Therefore,

$$\delta P^n = B^n \delta p^n \tag{6}$$

where B^n is defined as a $[N_p \times N_p]$ diagonal matrix formed by the weighting parameters, *i.e.*, $diag(B^n)_i = \beta_i^n$. This can be understood considering that each parameter influence is related to a vector with components $q_i = W_i^n \delta p_i^n = W_i^n (p_i^n - p_{0i})$, and the general perturbation is given by:

$$q = \beta_1 q_1 + \beta_2 q_2 + \ldots + \beta_{N_p} q_{N_p} = W^n B^n \delta p^n \tag{7}$$

Moreover, by assuming that only one parameter actuates in each control station it is possible to define active parameters, represented by subscript a, $\delta P_a^n = B_a^n \delta p_a^n$ (actuate in station Σ_n), and passive parameters, represented by subscript p, $\delta P_p^n = B_p^n \delta p_p^n$ (do not actuate in station Σ_n). At this point, it is assumed a weighting matrix for active parameter, B_a^n, and other for passive parameters, B_p^n. Therefore,

$$\delta\xi^{n+1} = J^n \delta\xi^n + W^n \delta P_a^n + W^n \delta P_p^n \tag{8}$$

Now, it is necessary to align the vector $\delta\xi^{n+1}$ with the stable direction v_s^{n+1}:

$$\delta\xi^{n+1} = \alpha v_s^{n+1} \tag{9}$$

where $\alpha \in \Re$ needs to be satisfied as follows:

$$J^n \delta\xi^n + W^n \delta P_a^n + W^n \delta P_p^n = \alpha v_s^{n+1} \tag{10}$$

Therefore, once the unknown variables are α and the non-vanishing term of the vector δP_a^n, one obtains the following system:

$$\begin{bmatrix} \delta P_{ai}^n \\ \alpha \end{bmatrix} = -[W_i^n \quad -v_s^{n+1}]^{-1} [J^n \quad W^n] \begin{bmatrix} \delta\xi^n \\ \delta P_p^n \end{bmatrix} \tag{11}$$

where δP_{ai}^n is related to the non-vanishing element of the vector δP_a^n, that consists in the active parameter in Σ_n, and W_i^n correspond to the sensitivity matrix column related to this active parameter. The solution of this system furnishes the necessary values for the system stabilization: α and δP_{ai}^n. The real perturbation is given by $\delta p_{ai}^n = \delta P_{ai}^n / \beta_{ai}^n$.

A particular case of this control procedure has uncoupled control parameters meaning that each parameter returns to the reference value when it becomes passive. Moreover, since there is only one active parameter in each control station, the system response to parameter perturbation is the same as when it actuates alone. Under this assumption, passive influence vanishes and active vector is weighted by 1, which is represented by:

$$B_p^n = 0 \text{ and } B_a^n = I \tag{12}$$

where I is the identity matrix.

Therefore, the map F is just a function of the active parameters, $\xi^{n+1} = F(\xi^n, P_a^n)$, and the linear behavior of the map F in the neighborhood of the control point ξ_C^n and around the control parameter reference positions, P_0, is now defined by:

$$\delta\xi^{n+1} = J^n \delta\xi^n + W^n \delta P_a^n \tag{13}$$

where the sensitivity matrix W^n is the same of the previous case. Moreover, since $B_a^n = I$, it follows that $\delta P_a^n = \delta p_a^n$, thus the value of δP_a^n corresponds to the real perturbation necessary to stabilize the system. In order to align the vector $\delta\xi^{n+1}$ with the stable direction, the following system is obtained:

$$\begin{bmatrix} \delta P_{ai}^n \\ \alpha \end{bmatrix} = -[W_i^n \quad -v_s^{n+1}]^{-1} J^n \delta\xi^n \tag{14}$$

The difference between the multiparameter method (MP) (De Paula & Savi, 2008) and the semi-continuous multiparameter method (SC-MP) (De Paula & Savi, 2009a) is that the first considers only one control station per forcing period while the other considers as many control stations as necessary to stabilize the system per forcing period. Therefore, the SC-MP is the general case that can represent the MP when only one control station per period is of concern. In the same way, the OGY can be seen as a particular case when only one control station and only one control parameter are considered.

2.3. Time-delayed Feedback Methods

Continuous methods for chaos control were first proposed by Pyragas (1992) and are based on continuous-time perturbations to perform chaos control. This control technique deals with a dynamical system modeled by a set of ordinary nonlinear differential equations as follows:

$$\dot{x}(t) = Q(x,t) + B(t) \tag{15}$$

where $x(t) \in R^n$ is the state variable vector, $Q(x,t) \in R^n$ defines the system dynamics, while $B(t) \in R^n$ is associated with the control action.

Socolar *et al.* (1994) proposed a control law named as the extended time-delayed feedback control (ETDF) considering the information of time-delayed states of the system represented by the following equations:

$$\begin{gathered} B(t) = K\left[(1-R)S_\tau - x\right] \\ S_\tau = \sum_{m=1}^{\infty} R^{m-1} x_{m\tau} \end{gathered} \tag{16}$$

where $K \in R^{n \times n}$ is the feedback gain matrix, $0 \le R < 1$, $S_\tau = S(t-\tau)$ and $x_{m\tau} = x(t - m\tau)$. The UPO stabilization can be achieved by a proper choice of *R* and *K*. Note that for any *R* and *K*, perturbation of Eq.(16) vanishes when the system is on the UPO since $x(t - m\tau) = x(t)$ for all *m* if $\tau = T_i$, where T_i is the periodicity of the *i*th UPO.

The controlled dynamical system consists of a set of delay differential equations (DDEs). The solution of this system is done by establishing an initial function $x_0 = x_0(t)$ over the interval $(-m\tau, 0)$. This function can be estimated by a Taylor series expansion as proposed by Cunningham (1954):

$$x_{m\tau} = x - m\tau\,\dot{x} \tag{17}$$

Under this assumption, the following system is obtained:

$$\dot{x} = Q(x,t) + K\left[(1-R)S_\tau - x\right]$$

$$\text{where} \begin{cases} S_\tau = \sum_{m=1}^{\infty} R^{m-1}\left[x - m\tau\,\dot{x}\right], & \text{for } (t - m\tau) < 0 \\ S_\tau = \sum_{m=1}^{\infty} R^{m-1} x_{m\tau}, & \text{for } (t - m\tau) \ge 0 \end{cases} \tag{18}$$

Note that DDEs contain derivatives that depend on the solution at delayed time instants. Therefore, besides the special treatment that must be given for $(t - m\tau) < 0$, it is necessary to deal with time-delayed states while integrating the system. A fourth-order Runge-Kutta method with linear interpolation on the delayed variables is employed in this work for the numerical integration of the controlled dynamical system (Mensour & Longtin, 1997).

It should be pointed out that when *R* = 0, the ETDF turns into the original time-delayed feedback control method (TDF) proposed by Pyragas (1992) where the control law is based on a feedback of the difference between the current and a delayed state given by:

$$B(t) = K[x_\tau - x] \tag{19}$$

where τ is the time delay, $x = x(t)$ and $x_\tau = x(t - \tau)$.

An important difference between continuous and discrete methods is that in continuous methods it is not necessary to wait the system to visit the neighborhood of the desired orbit. Another particular characteristic related to the learning stage is that, besides the UPO identification common to all control methods, it is necessary to establish proper values of the control parameters, *K* and *R*, for each desired orbit. This choice is done by analyzing Lyapunov exponents of the UPO, establishing negative values of the largest Lyapunov exponent. After this first stage, the control stage is performed, where the desired UPOs are stabilized. De Paula & Savi (2009b) discussed a proper procedure to evaluate the largest Lyapunov exponents necessary for the controller parameters.

3. COMPARATIVE ANALYSIS

As an application of the general chaos control methods, a system with high instability characteristic is of concern. A nonlinear pendulum actuated by two different control parameters is considered. The motivation of the proposed pendulum is an experimental set up discussed in De Paula *et al.* (2006) that proposed a mathematical model to describe the pendulum dynamical behavior. Basically, the pendulum consists of an aluminum disc with a lumped mass. An electric motor harmonically excites the pendulum via a string-spring device, which provides torsional stiffness to the system.

The mathematical model for the pendulum dynamics describes the time evolution of the angular position, ϕ, assuming that ϖ is the forcing frequency, *I* is the total inertia of rotating parts, *k* is the spring stiffness, ζ represents the viscous damping coefficient and μ the dry friction coefficient, *m* is the lumped mass, *a* defines the position of the guide of the string with respect to the motor, *b* is the length of the excitation arm of the motor, *D* is the diameter of the metallic disc and *d* is the diameter of the driving pulley. The equation of motion is given by (De Paula *et al.*, 2006):

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{kd^2}{2I} & -\dfrac{\zeta}{I} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} 0 \\ \dfrac{kd}{2I}(\Delta f(t) - \Delta l_1) - \dfrac{mgD\,\mathrm{sen}(x_1)}{2I} - \dfrac{2\mu}{\pi I}\arctan(q x_2) \end{bmatrix} \tag{20}$$

where $\Delta f(t) = \sqrt{a^2 + b^2 + \Delta l_2^{\,2} - 2ab\cos(\varpi t) - 2b\Delta l_2 \sin(\varpi t)} - (a - b)$ and Δl_1 and Δl_2 correspond to actuations.

Numerical simulations of the pendulum dynamics are in close agreement with experimental data by assuming parameters used in De Paula *et al.* (2006): $a = 1.6\times10^{-1}$ m; $b = 6.0\times10^{-2}$ m; $d = 4.8\times10^{-2}$ m; $D = 9.5\times10^{-2}$ m; $m = 1.47\times10^{-2}$ kg; $I = 1.738\times10^{-4}$ kg m^2; $k = 2.47$ N/m; $\zeta = 2.368\times10^{-5}\ \mathrm{kg\,m^2 s^{-1}}$; $\mu = 1.272\times10^{-4}\ \mathrm{N\,m}$; $\omega = 5.61\,\mathrm{rad/s}$.

Position and velocity time series are obtained from numerical integration of the mathematical model with $\omega = 5.61\,\mathrm{rad/s}$, a frequency related to chaotic behavior. UPOs embedded in chaotic attractor are identified by using the close return method (Auerbach *et al.*, 1987). This identification consists in the first step of the learning stage being common to all control methods.

This section establishes a comparative analysis of chaos control methods that, in principle, are capable to perform UPO stabilization of the nonlinear pendulum. Due to system instability, the discrete OGY and OGY-MP methods don't present good performance in stabilizing UPOs of the system. De Paula & Savi (2008) showed some situations where the OGY-MP method presents better performance than the discrete original OGY method. In this regard, the comparative analysis deals with only four different controllers: semi-continuous (SC), semi-continuous multiparameter (SC-MP) (coupled and uncoupled approaches) and extended time-delayed feedback method (ETDF).

Let us start discussing some aspects of the learning stage. Concerning discrete and semi-continuous techniques, besides the UPO identification, it is necessary to evaluate the local dynamics expressed by the Jacobian matrix and the sensitivity vector of the transition maps in a neighborhood of the fixed points. The least−square fit method (Auerbach *et al.*, 1987; Otani & Jones, 1997) is employed to estimate Jacobian matrix. After that, the SVD technique is employed for determining stable and unstable directions near the next fixed point. The sensitivity vectors are evaluated allowing the trajectories to come close to a fixed point and then perturbing the parameters by the maximum permissible value. Multiparameter methods need to define the sensitivity matrix where each column is evaluated by the same way of the sensitivity vector of the single-parameter method. The MP coupled approach needs to define proper values for parameters β_a and β_p. A brute-force approach is an alternative to establish values for these parameters by increasing controller efficacy as described in De Paula & Savi (2009a). This approach considers β_a=2.5 and β_p=1.5. The uncoupled approach avoids this evaluation since β_a=1 and β_p=0. Concerning the continuous methods, the learning stage involves the determination of control parameters, *R* and *K*, which is done by evaluating the largest Lyapunov exponent of the desired UPO. The idea is to find controller parameters that are related to negative values of the maximum Lyapunov exponent, which means that the UPO becomes stable (De Paula & Savi, 2009b).

After the learning stage, the stabilization stage is initiated. Discrete methods need to wait the system to visit the neighborhood of the control orbit, when the control procedure is turned on. Single-parameter methods are employed by considering the isolated perturbation performed by the parameters Δl_1 or Δl_2. Multiparameter methods assume that the first control parameter actuates in odd stations while the second actuates in even stations. Continuous methods, on the other hand, use the first control parameter, Δl_1, to actuate over the system. Under this assumption, *K* is a scalar.

3.1. Control Methods Performance Considering Noisy Signals

Noise contamination is unavoidable in experimental data acquisition. Therefore, it is important to evaluate its effect on chaos control procedures. This section evaluates noise sensitivity of the chaos control techniques considered in the comparative analysis: SC, SC-MP, coupled and uncoupled approaches, and ETDF. In order to simulate noisy data sets, a white Gaussian noise is introduced in the signal, comparing results of control procedures with an ideal time series, free of noise. In general, noise can be expressed as follows,

$$\begin{cases} \dot{x} = Q(x,t) + \mu_d \\ \dot{y} = P(x,t) + \mu_o \end{cases} \tag{21}$$

where x represents state variables, y represents the observed response and $Q(x,t)$ and $P(x,t)$ are nonlinear functions. μ_d and μ_o are, respectively, dynamical and observed noises. Notice that μ_d has influence on system dynamics in contrast with μ_o. In this work, it is considered only an observed noise, simulating noise in experimental data due to instrumentation apparatus and, therefore, noise does not have influence in system dynamics.

The noise level can be expressed by the standard deviation, σ, of the system probability Gaussian distribution, that is parameterized by the standard deviation of the clean signal, σ_{signal} , as follows:

$$\eta(\%) = \frac{\sigma}{\sigma_{\text{signal}}} \times 100 \tag{22}$$

At this point, the comparative analysis is of concern. A control rule is assumed in order to analyze the control methods performance in the presence of observed noisy. This control rule is defined in order to choose orbits that can be stabilized by all control methods for an ideal signal: a period-6 orbit during the first 500 periods, a period-2 from period 500 to 1000, a period-3 from 1000 to 1500 and, finally a period-1, from period 1500 to 2000. Figure 1 presents these four UPOs in one of the control stations (CS) considered by SC methods, while Figure 2 shows the UPOs phase space.

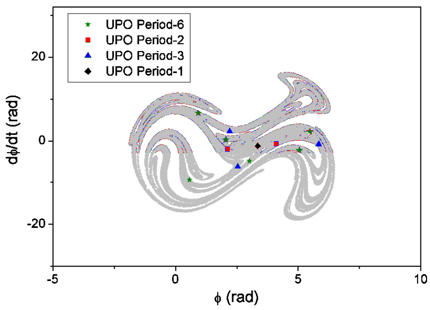

Figure 1 – UPOs of the second control rule at control station #1.

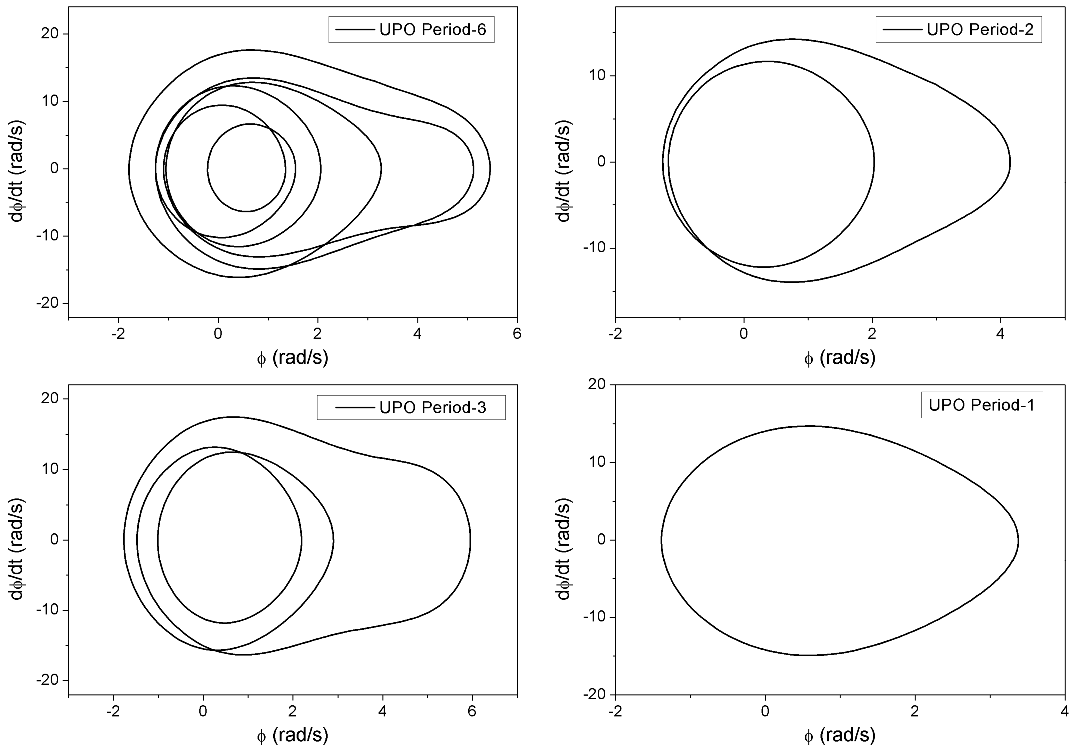

Figure 2 – Control rule UPOs.

By considering signals without noise, $\eta = 0\%$, the four control strategies considered in the comparative analysis are effective to stabilize all UPOs of the control rule. In SC and SC-MP methods evaluation it is considered four control stations per forcing period and maximum perturbation of $|\Delta l_{1\max}| = 15\,\text{mm}$ and $|\Delta l_{2\max}| = 25\,\text{mm}$. For all methods evaluation, reference positions of control parameters are $\Delta l_{10} = \Delta l_{20} = 0\,\text{mm}$.

Initially, it is considered a noisy signal with 1% of amplitude. Figure 3 shows the desired trajectory, imposed by the control rule, and the system time evolution at control station #1 when the SC is employed considering the isolated actuation performed by the parameters Δl_1 and Δl_2. Figure 4 presents the same pictures for the SC-MP, coupled and uncoupled approaches, while Figure 5 presents results for the ETDF. Note that for $\eta = 1\%$, the SC with first control parameter stabilizes all UPOs of the control rule, however, sometimes system trajectory escapes from the desired orbit, returning back later. By using the second control parameter, only two of the orbits are successfully stabilized. By using the SC-MP coupled approach, the second orbit of the control rule is not satisfactory stabilized. The uncoupled approach of the SC-MP and the ETDF successfully stabilizes all orbits. It should be highlighted, however, that the ETDF stabilizes a different UPO for the first orbit. The same difference happens when no noise is considered as observed by De Paula & Savi (2009b).

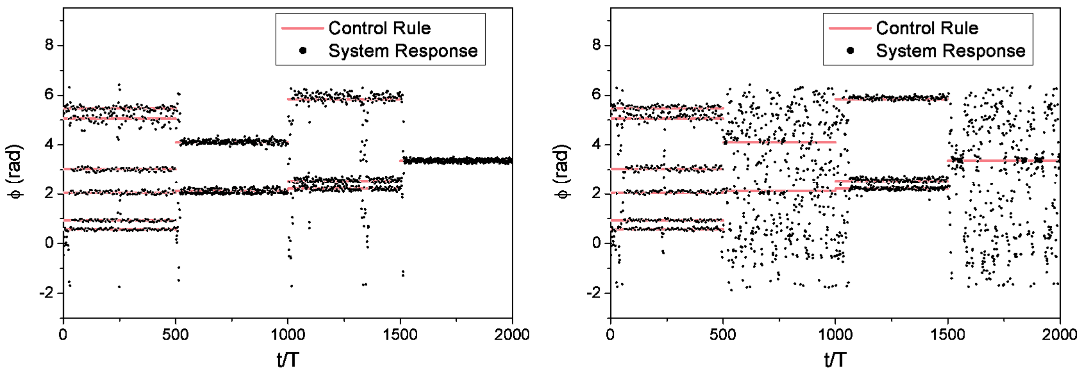

Figure 3 - System controlled using SC at CS #1 with $\eta = 1\%$: (a) Parameter Δl_1; (b) Parameter Δl_2.

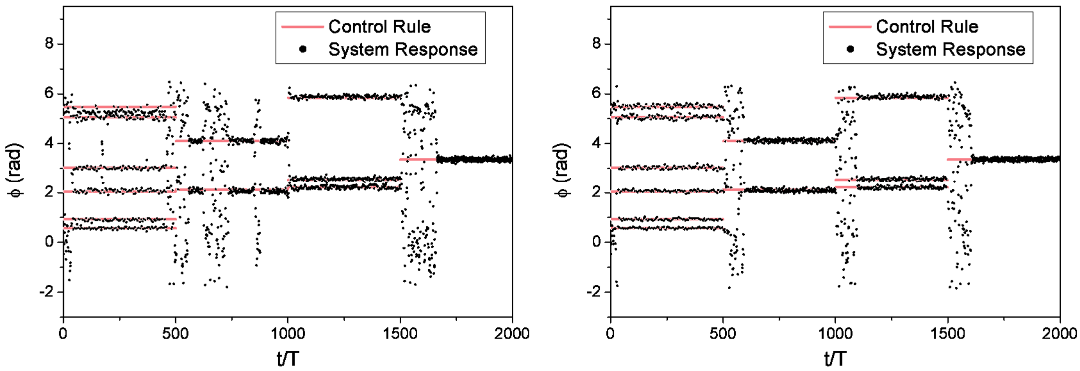

Figure 4 - System controlled using SC-MP at CS #1 with $\eta = 1\%$: (a) Coupled; (b) Uncoupled.

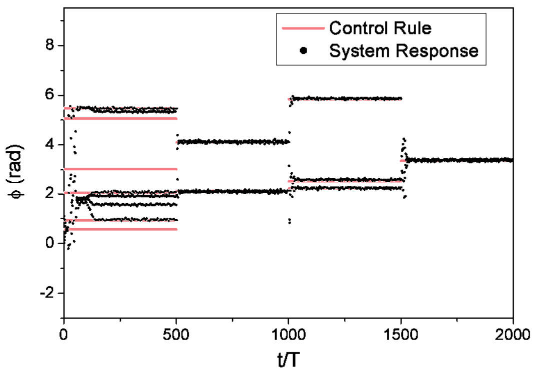

Figure 5 - System controlled using ETDF at control station #1 with $\eta = 1\%$.

A noise level of 2% is now considered. Figure 6 shows the desired trajectory imposed by the control rule and the system time evolution at control station #1 when the SC is employed considering the isolated actuation performed by the parameters Δl_1 and Δl_2. Figure 7 presents the same pictures for the SC-MP, coupled and uncoupled approaches, while Figure 8 presents results of the ETDF. Note that the increase in noise level makes the single-parameter SC to be not able to stabilize some orbits. Although the coupled SC-MP presents better results, it is noticeable that its efficacy decreases with the noise level increase. The uncoupled SC-MP presents better results when compared with the preceding methods and the ETDF successfully stabilize all UPOs of the control rule, except for the fact that the period-6 stabilized orbit is different from the desired one.

Concerning the semi-continuous methods, it should be highlighted that the increase of control stations is a useful procedure in order to avoid the effect of noise, however, the effectiveness of this procedure is limited by the time response of the system (Pereira-Pinto et al., 2004). Figure 9 presents results of the SC method with parameter Δl_1 considering four and six control stations with η=2%.

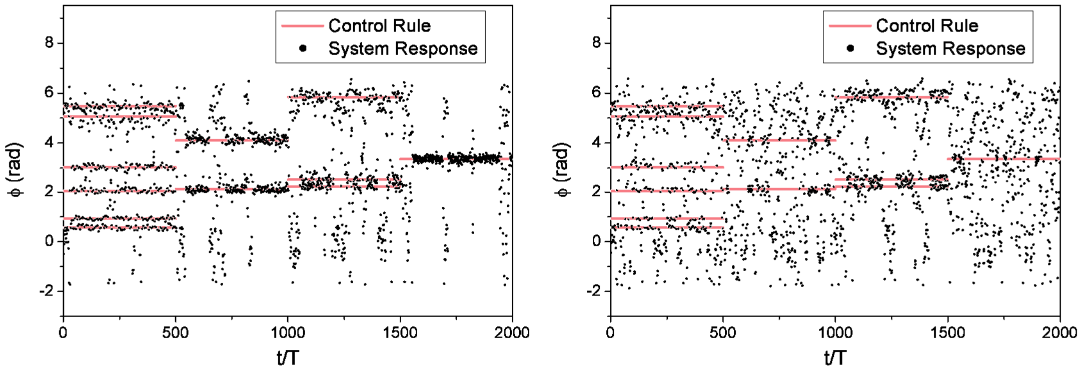

Figure 6 - System controlled using SC at CS #1 with $\eta = 2\%$: (a) Parameter Δl_1 ; (b) Parameter Δl_2 .

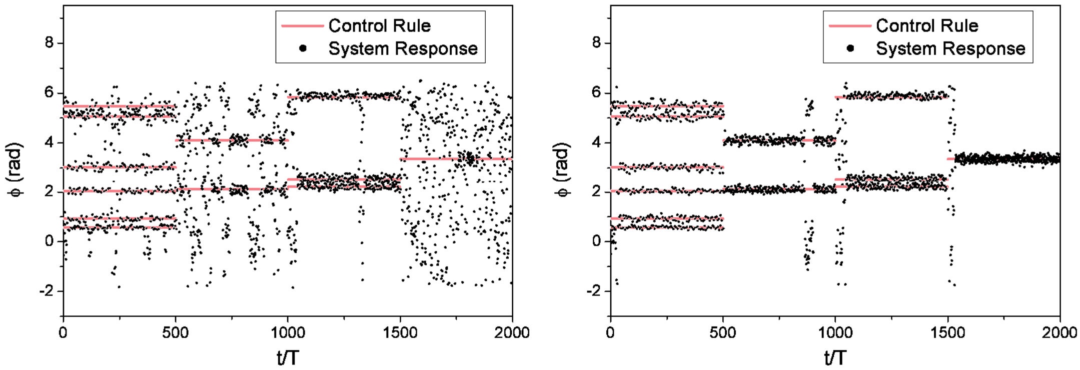

Figure 7 - System controlled using SC-MP at CS #1 with $\eta = 2\%$: (a) Coupled; (b) Uncoupled.

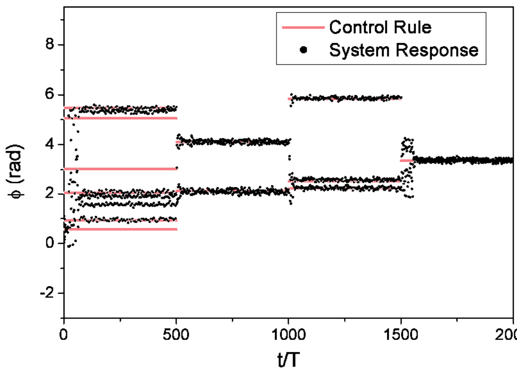

Figure 8 - System controlled using ETDF at control station #1 with $\eta = 2\%$.

Although the increase of control stations can promote a better performance related to orbit stabilization, there are situations where this increase causes the increase of uncertainty that could appear as a consequence of the determination of controller parameters.

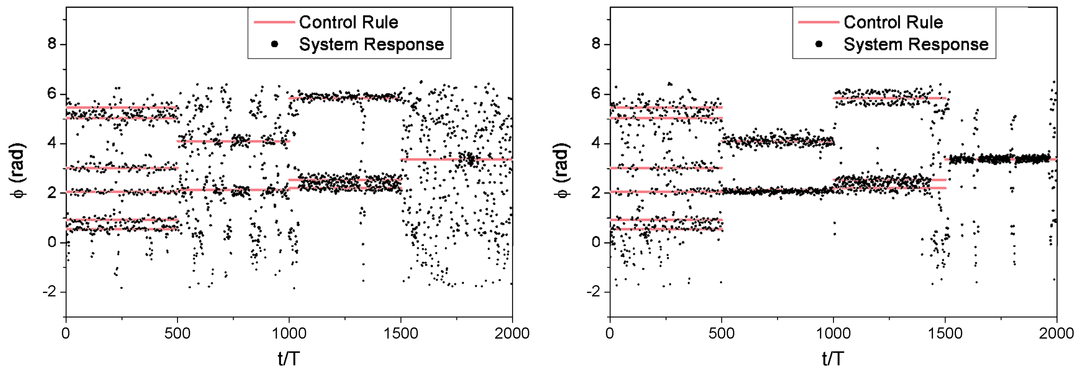

Figure 9 – System controlled using SC at CS #1 with $\eta = 2\%$ and parameter Δl_1: (a) Four CS's; (b) Six CS's.

For noise levels greater than 2% none of the semi-continuous methods presented good results in stabilizing the nonlinear pendulum. The ETDF successfully stabilized orbits of the control rules for noise levels up to $\eta = 5\%$.

4. CONCLUSIONS

This paper presents a comparative analysis of chaos control methods performances. Initially, it is presented an overview of chaos control methods classified as follows: OGY methods – that includes discrete and semi-continuous approaches; multiparameter methods – that also includes discrete and semi-continuous approaches; and time-delayed feedback methods that are continuous approaches. The learning stage is the same for all discrete methods, where system parameters are identified from time series and it is not necessary to know the system dynamics. On the other hand, the learning stage of the continuous methods implies the determination of control parameters from estimating the maximum Lyapunov exponent which imposes the knowledge of the mathematical model. As an application of the general chaos control methods a nonlinear pendulum is of concern. Due to its high instability, the comparative analysis deals with four control strategies: SC; SC-MP, coupled and uncoupled approaches; and ETDF, whose are capable to perform UPOs stabilization of the system. It is considered a control rule where four different UPOs must be stabilized in the presence of observed noise. From this analysis, it is shown that continuous methods present greater robustness being associated with better performances, however, sometimes the ETDF stabilizes orbits different from the desired UPO. Among semi-continuous control strategies, the uncoupled approach of the SC-MP method presents the better performance.

5. ACKNOWLEDGEMENTS

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