MODELING AND SIMULATION OF A POSITION CONTROL OF A MULTI-LINK FLEXIBLE STRUCTURE

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Abstract

The objective of this work is to describe the positional control of an unconstrained multi-link flexible structure. The experimental apparatus was designed to be representative of a flexible space structure such as a satellite with multiple flexible appendages. In this work we describe the analytical modeling and the simulation of a position control using a Linear Quadratic Regulator.

Keywords : Flexible structures, Modal analysis, Identification, Control of structures.

1. INTRODUCTION

This paper presents the analytical modeling of a multibody flexible structure and the simulation of its position control using LQR design, with a reduced-order estimator. The experimental setup, show in the figure 1, was assembled at ITA Dynamics Laboratory with the aim to investigate the dynamics and the position control of flexible structures representative of aerospace structures such as a satellite with flexible appendages. The experimental setup is composed of two flexible aluminum beams coupled to a central rigid hub. The hub is mounted on a steel disc supported on a gas bearing table, in an attempt to minimize the static friction and to simulate the structure's slew motion in space conditions. The steel disc is linked to a brushless DC motor which gives the necessary excitation to the structure. The direct-drive torque actuation avoids the introduction of spurious non-linear effects such as dry friction and backlash in the gear transmission system.

The instrumentation and measurement subsystems consist of collocated and non-collocated sensors and their respective signal conditioning systems. An accelerometer is used to measure the vibrations of the beam tip. A full strain-gage bridge is used to measure the elastic deformation at a known position of the beam. The collocated sensors consist of a tachometer and a potentiometer both fixed to the motor axis. A schematic view of the experimental set up is shown in figure 1.

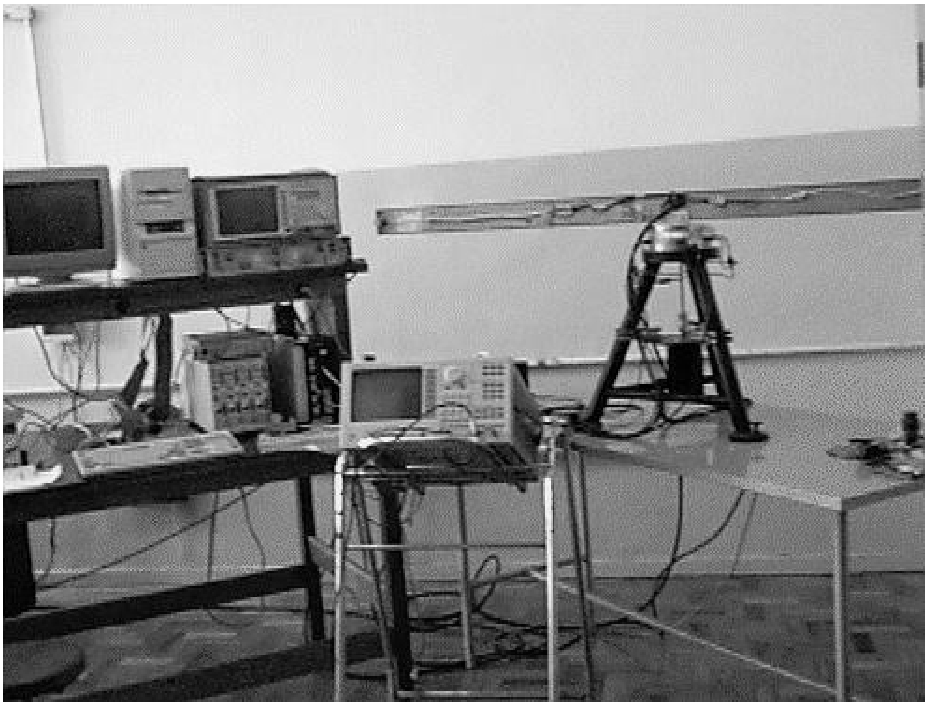

Figure 1- Experimental Setup

2. THE ANALYTICAL MODEL

The generalized Lagrangian approach is used to derive the analytical model of the unconstrained multi-link flexible structure. The unconstrained characteristic results from the natural motion without external influences, i.e, all the structure is allowed to vibrate and its solution involves both the inertia of the rigid and the flexible parts (Barbieri & Özgüner, 1988). In this study we assume that the elastic deformation of the beams are symmetric with respect to the hub, consequently it is necessary to model only the elastic displacement of one of the arms (Junkins and Kim, 1993). The position of a generic point on the beam is written on a local body fixed coordinate system, as shown in the figure 2.

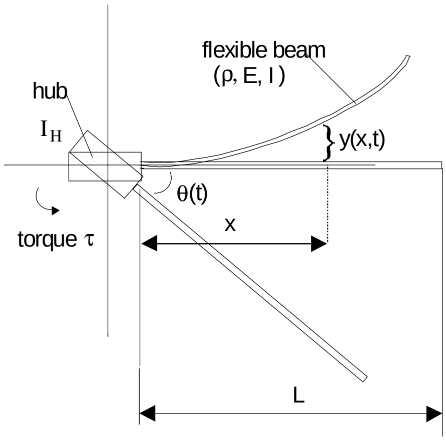

Figure 2. Coordinate system

The kinetic energy of the system is the sum of the kinetic energy of the hub, the arms and the tip mass, where the latter is considered as a boundary element.

$$T = T_{hub} + T_{beam} + T_{boundary} \tag{1}$$

with

$$T_{hub} = \frac{1}{2} I_{hub} \dot{\theta}^2 \tag{2}$$

$$T_{beam} = \int_o^L \rho \, \underline{\dot{R}}^2 dx \tag{3}$$

$$T_{boundary} = \frac{1}{2} m_t \, \underline{\dot{R}}^2 (L) \tag{4}$$

I_{hub} is the hub inertia, ρ is the linear mass density of the beam, L is the appendages length and m_t is the mass of the accelerometer located at the tip of the beam, and $\underline{R}$ is the position vector.

The elastic potential energy of the beam does not take into account the shear deformation and the rotary inertia of the beam, and is given by the following expression:

$$V = \int_0^L EI \left[\frac{\partial^2 y(x,t)}{\partial x^2} \right]^2 dx \tag{5}$$

The Lagrangian of the system, is written as the total kinetic energy minus the potential energy of the structures, $L = T - V$, while the non-conservative work done by the applied torque is given by:

$$\delta W_{nc} = \tau \delta \theta \tag{6}$$

From previous work, Góes et al. (1998) and Negrão (1998 and 1999), it follows that the equations of motion can be written in the following matrix equation, where it was considered only the first three modes of the distributed system:

$$M\ddot{q} + Kq = F \tag{7}$$

$$M = \begin{bmatrix} I_T & 0 & 0 & 0 \\ I_j & 1 & 0 & 0 \\ I_j & 0 & 1 & 0 \\ I_j & 0 & 0 & 1 \end{bmatrix} \text{ and } K = \begin{bmatrix} 0 & 0 \\ 0 & diag[\omega_1^2 \ldots \omega_3^2] \end{bmatrix} \tag{8}$$

where:

$$I_T = I_{Hub} + I_{beam} + m_t l^2 \quad ; \quad I_j = \frac{\left(-\left(I_{Hub} + I_{beam} + m_t l^2\right)\theta_j\right)}{\left(\int_0^l \rho \phi_j^2 dx + m_t \phi_j^2(l) + I_{Hub} \theta_j^2\right)} \quad ; \quad j = 1,2,3 \tag{9}$$

$$q = [\Theta \quad \eta_1 \quad \eta_2 \quad \eta_3]^T \; ; \quad F = [\tau_m \quad \phi_1'(0)\tau_m \quad \phi_2'(0)\tau_m \quad \phi_3'(0)\tau_m]^T \tag{10}$$

$$I_{beam} \equiv \int_0^L \rho \, x^2 \, dx \quad ; \qquad \theta_i = -\frac{\rho \int_0^L (x)\phi_i(x) dx + m_t(L)\phi_i(L)}{I_{hub}} \quad ; \qquad i,j=1,2,... \tag{11}$$

and $\phi_j(x)$ are the eigenfunctions of the hub-beam system.

Now it is simple to get the state-space representation of the system in the form:

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{B}u \tag{12}$$

where the $\underline{\underline{A}}$ e $\underline{B}$ matrices are:

$$\underline{\underline{A}} = \begin{bmatrix} 0 & \underline{\underline{I}} \\ \underline{\underline{M}}^{-1}\underline{\underline{K}} & 0 \end{bmatrix} \quad ; \quad \underline{B} = \begin{bmatrix} 0 \\ \underline{\underline{M}}^{-1}\underline{F} \end{bmatrix} \tag{13}$$

We define the observation matrix, $\underline{\underline{C}}$, that describes the measured signals in terms of the state variables. This matrix is obtained from the model of the available sensors. The accelerometer is located at the free tip of the beam and its signal is conditioned by a pre-amplifier and a double integrator filter with a global coefficient of sensitivity given by G_a, expressed in [V/cm] units. Thus, one can write:

$$e_{ac} = Ga(L\theta + y(L,t)) \tag{14}$$

Rewriting the integrated accelerometer equation, as in (Negrão, 1998) it follows that:

$$e_{ac} = Ga[L\ \ \phi_1(L)\ \ \phi_2(L)\ \ \phi_3(L)\ \ 0\ \ 0\ \ 0\ \ 0][\theta(t)\ \ \eta_1(t)\ \ \eta_2(t)\ \ \eta_3(t)\ \ \dot{\theta}(t)\ \ \dot{\eta}_1(t)\ \ \dot{\eta}_2(t)\ \ \dot{\eta}_3(t)]^T \tag{15}$$

The potentiometer provides a voltage proportional to the angular position of the hub, $e_p = Gp\,\theta(t)$. The full strain-gage bridge gives a signal proportional to the axial strain of the beam (ε_s), which can be related with the elastic deformation y(x, t), at the point were it is located by following equation,

$$\varepsilon_s\big|_x = \left[\frac{e}{2}\right]\left(\frac{\partial^2 y}{dx^2}\right)\Bigg|_x \tag{16}$$

where, *e* is the thickness of the beam. The strain-gage sensor output is rewritten as:

$$\varepsilon_s = \left[\frac{e}{2}\right]\left[0\ \ \frac{d^2\phi_1(x_1)}{dx^2}\ \ \frac{d^2\phi_2(x_1)}{dx^2}\ \ \frac{d^2\phi_3(x_1)}{dx^2}\ \ 0\ \ 0\ \ 0\ \ 0\right][\theta(t)\ \ \eta_1(t)\ \ \eta_2(t)\ \ \eta_3(t)\ \ \dot{\theta}(t)\ \ \dot{\eta}_1(t)\ \ \dot{\eta}_2(t)\ \ \dot{\eta}_3(t)]^T \tag{17}$$

where, x_1 is the position where the sensor is located on the beam. The tachometer gives a signal proportional to the angular velocity of the hub, $e_t = \dot{\theta}(t)$, which combined with the other sensor equations, gives the observation vector , $\underline{y} = \underline{\underline{C}}.\underline{x}$, here

$$\underline{y} = [e_{ac}\ \ e_p\ \ e_s\ \ e_t]^T \tag{18}$$

and,

$$\underline{\underline{C}} = \begin{bmatrix} Ga\,L & Ga\,\phi_1(L) & Ga\,\phi_2(L) & Ga\,\phi_3(L) & 0 & 0 & 0 & 0 \\ Gp & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{e}{2}\left[\frac{d^2\phi_1(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_2(x_1)}{dx^2}\right] & \frac{e}{2}\left[\frac{d^2\phi_3(x_1)}{dx^2}\right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

3. THE ANALYTICAL TRANSFER FUNCTIONS

To obtain the analytical transfer functions, for the unconstrained multi-link flexible system, we used the physical parameters listed in table 1,.

Table 1. Model parameter of the unconstrained flexible beams

Aluminum density	ρ	$2.7950\ 10^{3}$	kg/m³
Aluminum Young's modulus	E	$6.8900\ 10^{10}$	N/m²
Beams width	E_b	$4.1200\ 10^{-3}$	m
Beams height	H_b	$8.0780\ 10^{-2}$	m
Beams length	L	$9.7150\ 10^{-1}$	m
Beams cross-section area	A	$3.3281\ 10^{-4}$	m^2
Beams moment of inertia	I	$4.7070\ 10^{-10}$	m^4
Beams mass moment of inertia	I_b	$2.8430\ 10^{-1}$	kg m^2
Hub mass moment of inertia	I_{hub}	$7.6749\ 10^{-1}$	kg m^2
Hub radius	r	$9.0000\ 10^{-2}$	m

Applying the Laplace transform into eq. (12), assuming zero initial conditions, and using the model parameters listed in table 1, we can obtain the analytical transfer functions for each one of the sensors in the form bellow.

$$\underline{Y}(s) = \underline{\underline{C}}.(s\underline{\underline{I}} - \underline{\underline{A}})^{-1}.\underline{B}.U(s) \tag{20}$$

4. POSITION CONTROL

Position control of mechanical systems with structural flexibility has been an important research topic in recent years. Here, we show the simulation results of a position control using the LQR design. Considering that the system is described by:

$$\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\underline{x} + \underline{B}u \\ y &= \underline{\underline{C}}\underline{x} \end{aligned} \tag{21}$$

together with a functional,

$$J = \frac{1}{2}\int_0^T (\underline{x}'\underline{\underline{Q}}\underline{x} + \underline{u}'\underline{\underline{R}}\underline{u})dt$$

the solution of the LQR problem is to minimize, *J,* with respect to the control input, *u(t),* where *J* represents the weighted sum of energy of the state and control; and Q and R represent their respective weights on the different states and control channels. The problem is solved by an algebraic Riccati equation :

$$A'P + PA + Q - PBR^{-1}B'P = 0 \tag{22}$$

and, the optimal control law is given by:

$$u = -k\underline{x}, \tag{23}$$

where, $k = R^{-1}B'P$ (24)

The implementation of the state feedback law requires that the state vector, $\underline{x}$, is available for measurement and feedback, which is not the case here. In this case a reduced-order observer was used to estimate the modal coordinates of the system. As given in Chen(1984), the reduced-order observer is:

$$\dot{w} = Fw + Hy + Gu$$
$$\hat{x} = Mw + Ny \qquad (25)$$

where,

$$F = A_{22} - LA_{12}\ ; \quad H = FL + A_{21} - LA_{11}\ ; \quad G = B_2 - LB_1\ ; \quad N = P + ML\ ; \qquad (26)$$

and,

L is the observer gain;

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \begin{bmatrix} CAP & CAM \\ TAP & TAM \end{bmatrix}; \qquad (27)$$

N,*M*,*P*,*T* are matrices transformations with properties as defined in Chen(1984);

The control law with an external reference, *r*, is then written as:

$$u = -k\underline{\hat{x}} + r\ ; \qquad (28)$$

The closed loop transfer function can be obtained directly by combining the closed loop system and observer equations using the external reference

$$\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A - BkNC & -BkM \\ HC - GkNC & F\_GkM \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} B \\ G \end{bmatrix} r\ ; \qquad (29)$$

The scheme of the LQR design using a reduced-order observer is shown in figure 3.

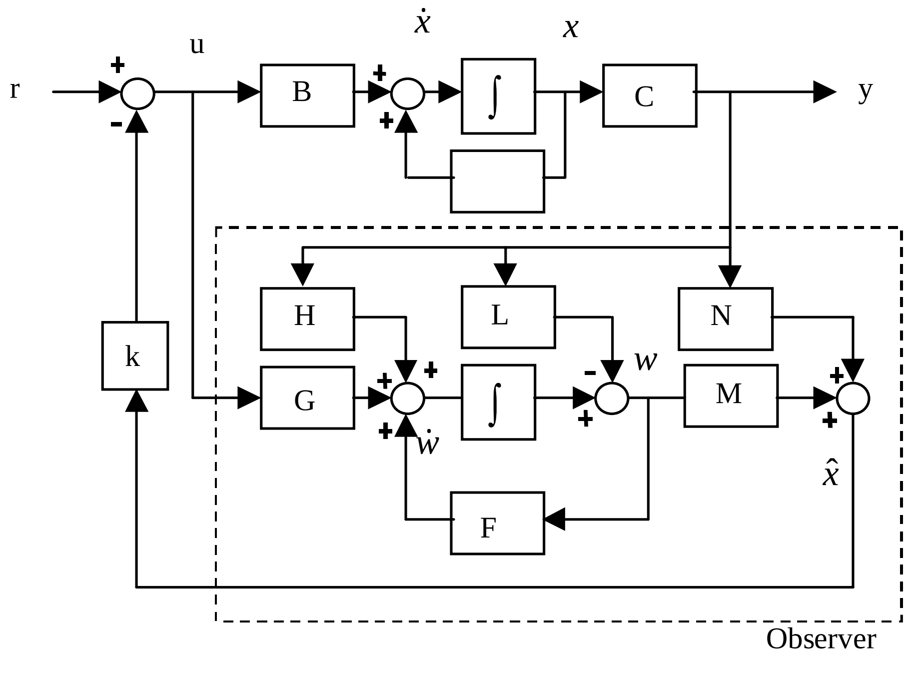

Figure 3. LQR Control Scheme

The eigenvalues of the estimator are chosen arbitrarily, and table 2 shows the chosen numerical eigenvalues. The gain L is determined such that the eigenvalues of, $A_{22} - LA_{12}$, are the eigenvalues of the estimator.

Table 2. Eigenvalues of the estimator

-14.7575 + 9.8400i
-14.7575 - 9.8400i
-6.3481 + 4.6586i
-6.3481 - 4.6586i

Using a unit step reference signal, the results of the position control using LQR design with reduced-order observer are illustrated in figure 4 to figure7:

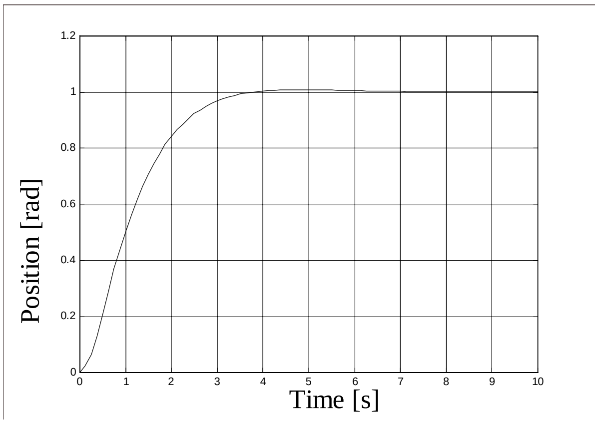

Figure 4. Angular position for a step reference

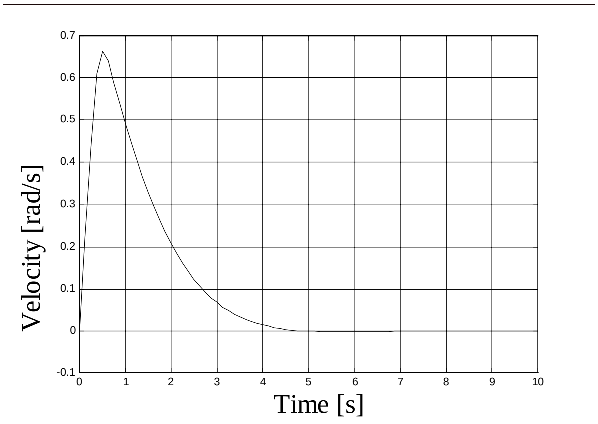

Figure 5. Angular velocity for a step reference

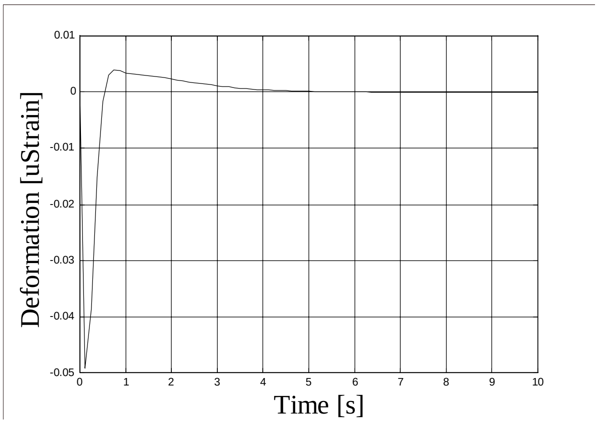

Figure 6. Transversal deformation for a step reference

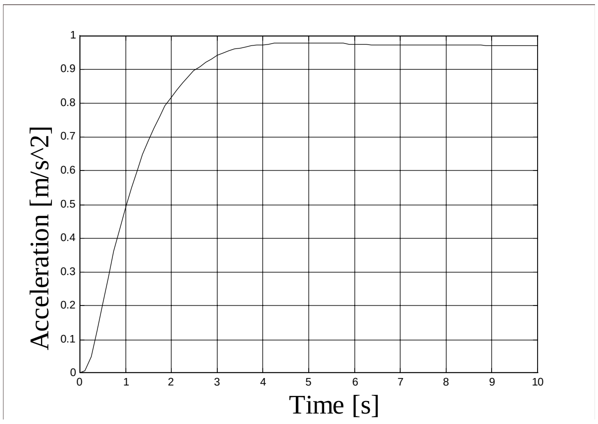

Figure 7. Tip acceleration for a step reference

As one can see in the Figures (4) to (7), the position control is efficient. The final position was reached in 5 seconds. This was the best performance that could be achieved without excitation of the higher vibration modes of the beam. This work is still in progress, and we are implementing an experimental set-up for real-time control, using as platform the program MATLAB/ SIMULINK. We also intend to implement other control strategies including the LQG/LTR, which due to the system inaccuracies, could be proven to be more robust to the unmodelled dynamics and sensor noise.

5. CONCLUSIONS

This paper reports preliminary results of computational simulation for the control of an experimental apparatus with multiple flexible bodies. The model was derived using the Lagrangian approach and its discretization with the Assumed Modes Method. The results of the position control using LQR design, with reduced-order observer, showed that the controller reached the target position in 5 seconds. This work is still in progress using the MATLAB/SIMULINK to implement the real time control. This preliminary result shows that, due to the system inaccuracies, a robust control synthesis like LQG/LTR should be more suitable to control this kind of dynamic system (Soares, Goes and Souza, 1996).

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