

## PRELIMINARY PERFORMANCE PARAMETERS OF THE HYPERSONIC VEHICLE 14-X B

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**Abstract.** High-speed airbreathing propulsion system based on the supersonic combustion (scramjet) is being designed to apply to the 14-X B Hypersonic Aerospace Vehicle, VHA 14-X B. The Brazilian VHA 14-X B is a technological demonstrator of the scramjet to fly at Earth's atmosphere at 30km altitude at Mach number 7, designed at the Prof. Henry T. Nagamatsu Laboratory of Aerothermodynamics and Hypersonics, at the Institute for Advanced Studies. Basically, scramjet is an aeronautical engine, without moving parts, therefore it is necessary another propulsion system to accelerate the scramjet to the operation conditions. Rocket engines are a low-cost solution to launch scramjet integrated vehicle to fly to the test conditions. The Brazilian two-stage rocket engines (S31 and S30) are able to boost at ballistic trajectory the VHA 14-X B to the predetermined conditions of the scramjet operation, 30km altitude at Mach number 7. In this work, a preliminary performance parameter calculations is done considering equilibrium air model in the compression and expansion components and frozen or equilibrium air model at the combustion chamber, modeled as one-dimensional flow with heat addition (Rayleigh flow) without addition of mass. Two methodologies are used for these preliminary performance calculations: considering variable specific heat ratio along 14-X B and using the Stream Thrust Analysis Method. The results are compared and the importance of a suitable model for performance parameter calculations is discussed.

**Keywords:** VHA 14-X B, hypersonic airbreathing propulsion, scramjet, stream thrust analysis

### 1. THE BRAZILIAN 14-X AND 14-X B HYPERSONIC AEROSPACE VEHICLES

The Brazilian 14-X B Hypersonic Aerospace Vehicle, VHA 14-X B (Fig. 1), is being designed at the Prof. Henry T. Nagamatsu Laboratory of Aerothermodynamics and Hypersonics, at the Institute for Advanced Studies (IEAv) since March 2012, as an option to demonstrate the hypersonic airbreathing propulsion system based on supersonic combustion (scramjet) at free flight at 30km altitude at Mach number 7, using Brazilian two-stage rocket engines (S31 and S30), which are able to boost the VHA 14-X B to the predetermined conditions of the scramjet operation.

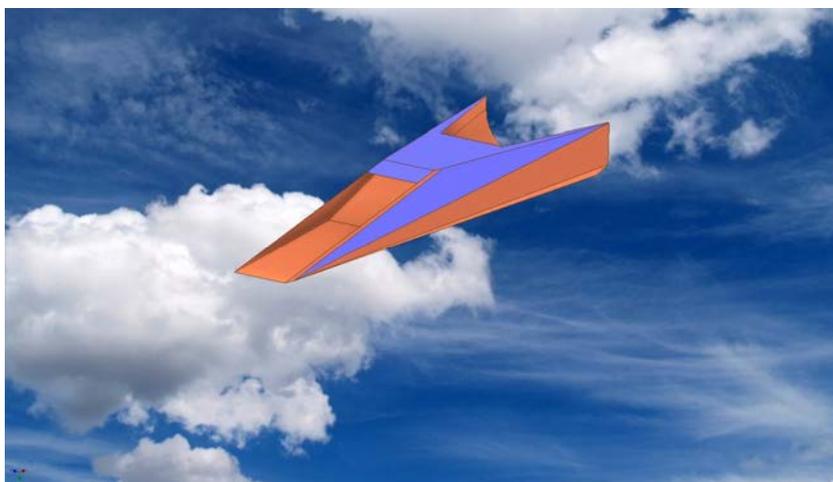


Figure 1: VHA 14-X B

The VHA 14-X B is a version of the VHA 14-X vehicle (Fig. 2), which the last one is designed to demonstrate, in free flight at 30km altitude at Mach number 10, using: i) "waverider" technology to provide lift to the aerospace

vehicle, and ii) "scramjet" technology to provide hypersonic airbreathing propulsion system based on supersonic combustion (Ricco et al., 2011; Toro et al., 2012).



Figure 2: VHA 14-X.

Hypersonic airbreathing propulsion, that uses supersonic combustion ramjet (scramjet) technology (Fig. 3), offers substantial advantages to improve performance of aerospace vehicle that flies at hypersonic speeds through the Earth's atmosphere, by reducing onboard fuel. As a matter of fact, at hypersonic speeds, a typical value for the specific impulse of a  $H_2-O_2$  rocket engine is 400s to 500s, while the specific impulse of a  $H_2$  fueled scramjet is 2000s to 3000s.

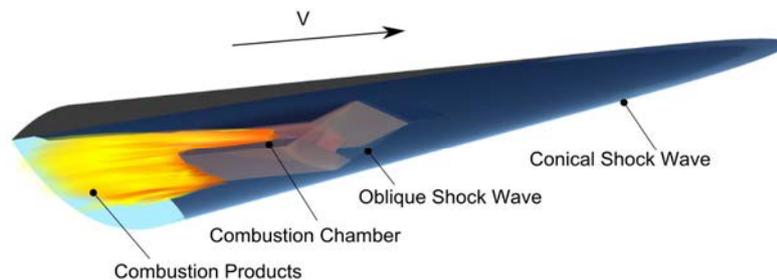


Figure 3: waverider and scramjet hypersonic airbreathing propulsion concepts.

In fact, the use of atmospheric air as oxidizer allows airbreathing propulsion vehicles to substantially increase payload weight. Basically, scramjet is a fully integrated airbreathing aeronautical engine that uses the inlet oblique (VHA 14-X B)/conical (VHA 14-X) shock waves generated during the hypersonic flight, to provide compression and deceleration of freestream atmospheric air at the inlet of the scramjet. Fuel, at least sonic speed, may be injected into the supersonic airflow just downstream of the inlet or at the beginning of the combustion chamber. Right after, both oxygen from the atmosphere and on-board hydrogen fuel are mixed. The combination of the high energies of the fuel and of the oncoming hypersonic airflow starts the combustion at supersonic speed. Finally, the divergent exhaust nozzle at the afterbody vehicle accelerates the exhaust gases, generating thrust.

Therefore, the VHA 14-X B (as well as VHA 14-X) is operational only on hypersonic speeds (higher than Mach number 4), and a hypersonic accelerator vehicle is needed to take to the optimum designed conditions of the scramjet operation of the VHA 14-X B at the 30km altitude at Mach number 7.

As a low-cost Brazilian solution to launch scramjet integrated vehicle to fly to the test conditions (30km altitude at Mach number 7) is to use Brazilian rocket engines based on solid propulsion, in ballistic trajectory. Such approach may provide an affordable path for maturing of the Brazilian hypersonic airbreathing components and systems in flight.

The Brazilian hypersonic accelerator vehicle (Fig. 4), which may be composed by two-stage solid rocket engines (S31 and S30), unguided, rail launched, is able to accelerate the VHA 14-X B to the predetermined conditions of the scramjet operation, 30km altitude at Mach number 7 (approximately 2100 m/s) from a Brazilian Launch Center.



Figure 4: Hypersonic Accelerator Vehicle and VHA 14-X.

It is planning for the flight test (Ricco et al., 2011; Toro et al., 2012) the VHA 14-X B will separate, at 30km altitude, from the 2<sup>nd</sup> stage rocket engine of the Hypersonic Accelerator Vehicle (Fig. 5). The scramjet will be

operational for about 4-5 seconds in upward flight of the VHA 14-X B. After scramjet engine demonstration is completed, the VHA 14-X B will follow the ballistic flight. After reaching the apogee, the VHA 14-X B will follow the descending flight to splash into the Atlantic Ocean. Both Hypersonic Accelerator Vehicle and VHA 14-X B will not be recovered.



Figure 5: Hypersonic Accelerator Vehicle and VHA 14-X B in ballistic trajectory.

The Alcântara Launch Center (CLA) is a satellite launching base of the Brazilian Space Agency (AEB), located at the Latitude  $2^{\circ} 18' S$  Longitude  $44^{\circ} 22' W$ , on Brazil's northern Atlantic coast, outside São Luis city (capital of Maranhão State). The CLA is operated by the Department of Aerospace Science and Technology (DCTA). CLA is the one of the world's closest launching base to the equator line, which gives the launch site a significant advantage in launching geosynchronous satellites, an attribute shared only by the Guiana Space Center (utilized by France), and its position nearer the equator offers an advantage over Cape Canaveral (USA).

However, for the first flight test, the captive mode will be used instead the autonomous (Fig. 5) flight test technique. From the system design standpoint, captive flight has numerous advantages due to its simplicity whereby vehicle integration can be performed with few modifications in the booster aerodynamics and control systems. Moreover, deploying a hypersonic vehicle definitely increases the system's complexity, introducing the need of an entirely new dynamic analysis for the test vehicle, increasing the risk of failure.

The VHA 14-X B (Fig. 1) consists of a two-dimensional configuration, with a constant cross-section (Fig. 6), where the upper flat surface, with zero angle of attack, is aligned with the freestream Mach number 7 hypersonic airflow. The lower surface, taken from the VHA 14-X waverider (Fig. 2) external configuration (Rolim, 2009; Rolim et al., 2009; Rolim et al., 2011; Costa, 2011; Costa et al., 2012, Costa et al., 2013) consists of a frontal surface with a leading edge angle ( $5.5^{\circ}$ ), compression ramp angle of  $14.5^{\circ}$  (related to the angle of the leading edge), the internal expansion chamber combustion angle of ( $4.27^{\circ}$ ) and external expansion angle of  $10.73^{\circ}$  (related to the angle of internal expansion). The cross-section height is 165 mm. The combustor chamber 121.6 mm long with constant area, following by 343.4 mm long with  $4.27^{\circ}$  (to accommodate the boundary layer and expansion due  $H_2$  and  $O_2$  combustion) was defined by research of the Hyslop (1998) and Kasal et al. (2002), respectively. The constant area combustion chamber is 12.29 mm high (to accommodate the airflow captured by the VHA 14-X B frontal area).

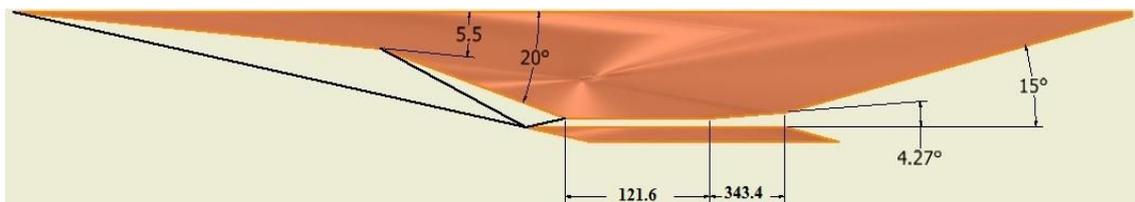


Figure 6: Cross-section of the V14-X B.

## 2. METHODOLOGY

### 2.1 Model and General Considerations

First it is necessary to establish a terminology to be used in the next calculations. Following Heiser and Pratt (1994) the VHA 14-X B may be divided in three (Fig. 7) main components: external and internal compression section (inlet), combustion chamber (combustor) and internal and external expansion section (outlet). Also, the hypersonic vehicle with

airframe-integrated scramjet engine lower surface may be divided by several stations (Fig. 7). Note that auxiliaries stations “a” (after first shock wave) and “b” (after constant section chamber) were defined for the present case.

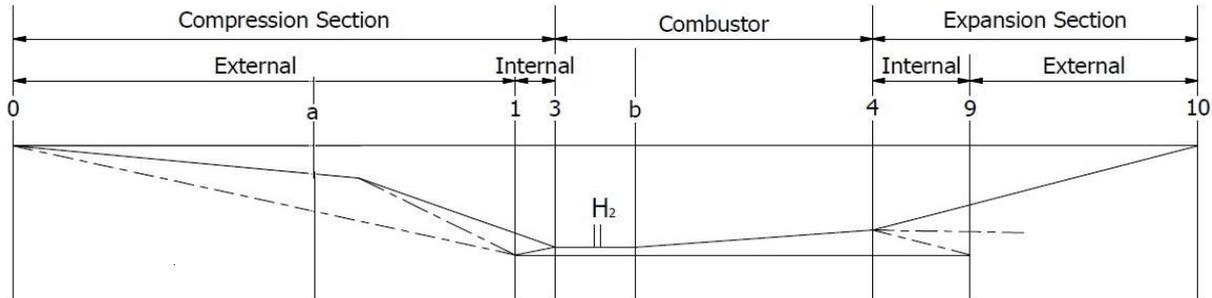


Figure 7: Hypersonic vehicle with airframe-integrated scramjet engine stations and reference terminology.

For the performance parameter calculations the thermodynamic properties along full VHA 14-X B lower surface is first determined considering one-dimensional flow with no boundary layer effects, combustor chamber burning  $H_2$  and  $O_2$  (modeled as heat addition without mass addition in constant area chamber) and VHA 14-X B flying at Mach number 7 at 30km geometric altitude (U.S. standard Atmosphere, 1976).

The standard atmospheric properties at 30km geometric altitude are given as  $p = 1197(Pa)$ ,  $T = 226.5(K)$ ,  $\rho = 0.01841(kg/m^3)$ ,  $a = 301,7(m/s)$ , where  $a$  is sound velocity (U.S. standard Atmosphere, 1976).

Note that in the on design of the VHA 14-X B, the incident shock waves generated at the  $5.5^\circ$  attached leading-edge deflection angle and at the  $14.5^\circ$  deflection angle (following the leading-edge deflection) hit the cowl leading-edge. Also, the reflected shock wave generated at the cowl leading-edge hits the entrance of the combustor station (Fig. 6). Yet, the flow from the external and internal compression section are entirely deflected to the combustor entrance (Fig. 6) at supersonic speed.

## 2.2 Performance Parameters Definitions

According with Heiser and Pratt (1994) the major performance parameters for an airbreathing engine vehicle are:

- a) Fuel/Air Ratio ( $f$ ): fuel mass flow rate  $\dot{m}_f$  by entry mass air flow rate  $\dot{m}_o$

$$f = \frac{\dot{m}_f}{\dot{m}_o} \quad (1)$$

where, the ideal value for  $f$  corresponds to a stoichiometric reaction Fuel in excess is waste of energy and oxygen excess will not be useful and does not represent an advantage. The stoichiometric ratio  $f_{st}$  of  $H_2$  and  $O_2$  can be calculated from the basic principles of chemical reactions, where for  $H_2$

$$f_{st} = \frac{36x + 3y}{103(4x + y)} \quad (2)$$

where for  $H_2$ ,  $x = 0$  and  $y = 0$ , then  $f_{st} = 0.0291$ .

- b) Specific Thrust (ST): uninstalled thrust  $F$  by entry air mass flow rate  $\dot{m}_o$

$$ST = \frac{F}{\dot{m}_o} \quad (3)$$

Writing the uninstalled force  $F$  in terms of velocities at the stations 0 and 10 (Fig. 7),  $V_0$  and  $V_{10}$ , pressures,  $p_0$  and  $p_{10}$  at the stations 0 and 10, entry air mass flow rate,  $\dot{m}_0$  and  $\dot{m}_{10}$  at the stations 0 and 10, and area  $A_{10}$  at the stations 10.

$$F = \dot{m}_{10} V_{10} - \dot{m}_0 V_0 + (p_{10} - p_0) A_{10} \quad (4)$$

By mass conservation law,  $\dot{m}_{10} = \dot{m}_0$  and, consequently,  $A_{10} = \frac{\dot{m}_{10}}{\rho_{10} V_{10}}$ , then:

$$\frac{F}{\dot{m}_0} = V_{10} - V_0 + (p_{10} - p_0) \left( \frac{V_{10}}{\rho_{10}} \right)^{-1} \quad (5)$$

where  $\rho_{10}$  is the density (specific mass) at the stations 10.

c) Specific Impulse ( $I_{sp}$ ): uninstalled thrust by fuel weight flow rate

$$I_{sp} = \frac{F}{g_o \dot{m}_f} = V_{10} - V_0 + (p_{10} - p_0) \left( \frac{V_{10}}{\rho_{10} f g_o} \right)^{-1} \quad (6)$$

where  $g_o$  is the standard sea level value of the acceleration of gravity.

d) Specific Fuel Consumption ( $S$ ): fuel mass flow rate by uninstalled thrust

$$S = \frac{\dot{m}_f}{F} = \frac{f}{V_{10} - V_0 + (p_{10} - p_0) \left( \frac{V_{10}}{\rho_{10}} \right)^{-1}} \quad (7)$$

e) Overall Efficiency ( $\eta_o$ ): thrust power by chemical energy rate

$$\eta_o = \frac{F V_0}{\eta_b \dot{m}_f h_{PR}} = V_0 \left[ V_{10} - V_0 + (p_{10} - p_0) \left( \frac{V_{10}}{\rho_{10} \eta_b f h_{PR}} \right)^{-1} \right] \quad (8)$$

where  $\eta_b$  is the combustion efficiency and  $h_{PR}$  is the heat of reaction of the fuel. For the present work  $\eta_o = 0.9$  and for  $H_2$   $h_{PR} = 119954 [kJ/kg]$ .

### 3. CALCULATIONS CONSIDERING VARIABLE SPECIFIC HEAT RATIO ALONG 14-X B

As one may see in Fig. 8, a change in behavior of the air occurs as its temperature increase (Heiser and Pratt, 1994). When the temperature of an initial calorically perfect air is increased the vibrational energy of the molecules are excited and the air becomes thermally perfect. For additional increase of the temperature chemical reactions occur and the properties of the air become a function of both, temperature and pressure. Calculations for gas in non ideal condition considering calorically perfect model with ratio of specific heats  $\gamma = 1.4$  can generated large errors in the calculations with an overestimated temperature. When the time  $t_v$  for vibrational relaxation process and the time  $t_c$  for chemical changes are very short in comparison with the time  $t_f$  for a particle of fluid runs a characteristic length, then the flow is in equilibrium (therefore, equilibrium fluid require thermal and chemical equilibrium) and its composition and

vibrational energy are constants with time. Otherwise, when  $t_v$  and  $t_c$  are larger than  $t_f$ , the changes do not have time to occur and the fluid is said frozen. When the time for the changes accommodation is of the same order of the time for a particle of fluid runs a characteristic length, the fluid is called a non-equilibrium flow, a more complex case.

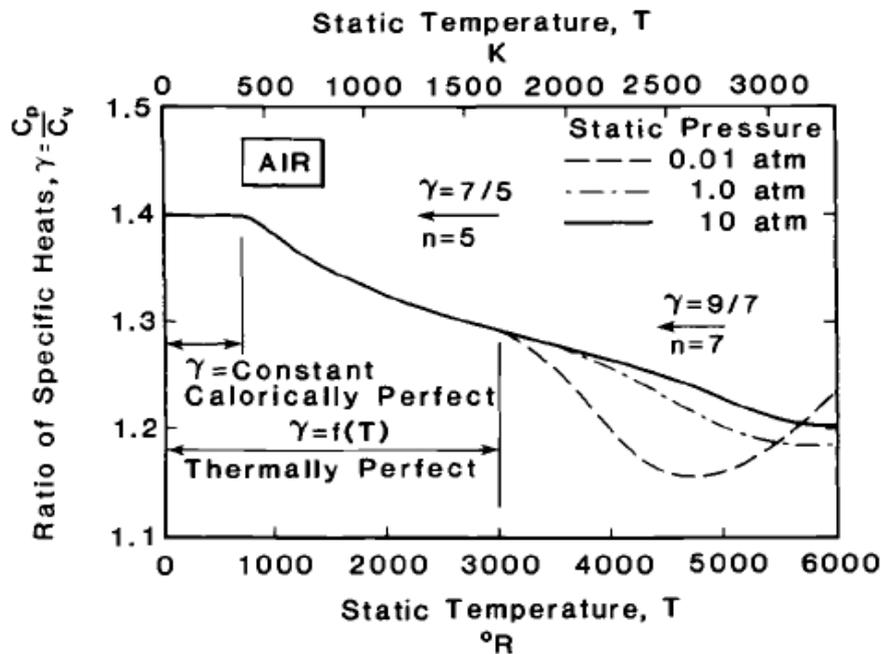


Figure 8: Equilibrium ratio of specific heats of air (Heiser and Pratt, 1994).

### 3.1 Compression and Expansion Components

With the application of equations from statistical thermodynamics in the governing equations relations can be found for temperature, pressure and density. Heiser and Pratt (1994) created a program, Hypersonic Airbreathing Propulsion (HAP), that among other things, allow to find properties equilibrium air.

Then, to account the effects of high temperature that must occur along 14- X B, calculations are performed considering equilibrium air model in the compression and expansion components, using HAP, and modeling the combustor chamber considering equilibrium or frozen flow. The details about the model adopted for combustion chamber are presented in the next subsection.

### 3.2 Combustion Component

One-dimensional flow with heat addition, Rayleigh Flow, (Fig. 9) was applied to combustion processes between the entrance (inlet) and the exit (outlet) of the scramjet combustor, where the combustion processes correspond to heat addition without mass addition. The subscripts *in* and *out* are used to identify the upstream (inlet) and the downstream (outlet) conditions. Applying mass, momentum and energy conservation laws (Anderson, 2003) and considering a steady, one-dimensional flow with heat addition and without body forces, the equations 9, 10 and 11 can be obtained.

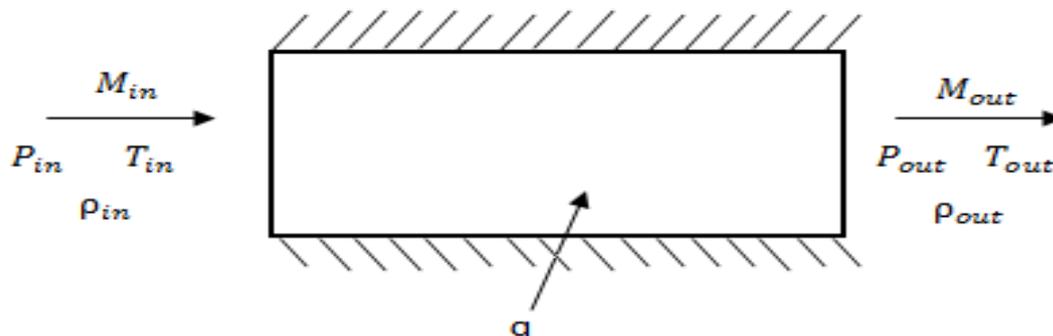


Figure 9: one-dimensional with constant-area heat addition, Rayleigh Flow.

$$\rho_{in} V_{in} = \rho_{out} V_{out} \quad (9)$$

$$p_{in} + \rho_{in} V_{in}^2 = p_{out} + \rho_{out} V_{out}^2 \quad (10)$$

$$h_{in} + \frac{V_{in}^2}{2} + q = h_{out} + \frac{V_{out}^2}{2} \quad (11)$$

where:  $\rho$ ,  $V$ ,  $p$ ,  $h$  and  $q$  are density, velocity, pressure, enthalpy and heat per unit of mass, respectively.

Considering a thermally perfect (air) gas:

$$p = \rho RT \quad (12)$$

$$dh = c_p(T) dT \quad (13)$$

where:  $R$  and  $c_p$  are the perfect gas constant and the heat at constant pressure, respectively.

The addition of heat in the supersonic flow will increase temperature, pressure and decrease Mach number. If more heat that necessary for attaining Mach 1 is added normal shock waves will be generated, reducing the performance of the combustor. Then, an estimative for the maximum amount of heat per unit mass  $q_{max}$  that can be added is necessary.

Initially a estimative for  $q_{max}$  is done by means of the above equations, isolating  $V_{out}$  and finding the value of  $q$  for which  $M=1$  considering frozen flow, modeling the air as a calorically perfect gas with  $R=288.1902$  J/Kg and  $C_p=1146.10$  J/kg.K (values corresponding to the inlet of the combustor chamber, calculated considering equilibrium air model using the program HAP, Heiser and Pratt, 1994).

The results for this initial calculation are:  $q_{max}=1378$  kJ/kg,  $T_{out}=2618$  K and  $P_{out}=314625$  Pa.

If the flow is not frozen, the value  $q_{max}$  must be underestimated because not take into account the endothermic process that occur in the air with high temperature. Then, if the flow is in equilibrium, a more realistic estimative can be done finding a suitable function for  $c_p$  in this conditions.

For equilibrium flow calculations in the combustion chamber values for  $c_p$  (equilibrium) were obtained using HAP for the temperature range  $950 < T < 3000$  [K] (estimated based in the previous calculation for frozen flow) and pressure  $P=201223$  Pa (the average between the inlet, based in equilibrium air model, and outlet, based in estimative with frozen flow). The variation of  $c_p$  with pressure was also evaluated for the range  $p_{in} < p < p_{out}$  for  $T=T_{in}$  and  $T=T_{out}$  and the maximum variation of  $c_p$  with pressure is lower than 1%, therefore negligible. This indicates that air dissociation are not occurring, although the vibrational energy of the gas may be changing due to the heat addition. A polynomial of third degree for equilibrium  $c_p$  as function of  $T$  was obtained for the range of temperature above. The results between the polynomial and the data differ less than 3%.

The complete results for calculations considering equilibrium air model for expansion and compression components and equilibrium or frozen properties for combustion are presented in section 5. The amount of heat per unit mass  $q$  added in each case is  $q = 0.9q_{max}$ .

It is interesting comment that although the methodology described in this section was named here as "calculations considering variable specific heat ratio along 14-X B", in the case of frozen flow along the combustion chamber, the specific heat ratio is considered constant inside this component (a kind of calorically perfect gas, but considering a different value for the specific heat ratio).

#### 4. STREAM THRUST ANALYSIS

Heiser and Pratt (1994) present three methods to analyze the performance parameters of a hypersonic airbreathing engine: Thermodynamic Closed Cycle Analysis, First Law Analysis and Stream Thrust Analysis. The first and second consider a thermodynamic cycle as a model for the process and as a consequence (for close the cycle)  $p_{10} = p_0$  and  $\dot{m}_f = 0$ . The Stream Thrust Analysis (STA) is not based on a thermodynamic cycle and allow analyzing the case

$p_{10} \neq p_0$  as well as  $\dot{m}_f \neq 0$ , therefore a more complete and realistic method of analysis. These methods may help to design a vehicle with good performance parameters. Stream Thrust Analysis presents closed equations that may facilitate the analysis of tendencies when conditions as altitude and initial Mach  $M_0$  are varied as well as analyzing the influence of the momentum and kinetic energy fluxes contributed by the fuel and how changes in geometry can influence the performance parameters.

In the Stream Thrust Analysis method the gas (air) is treated as calorically perfect in each component (compression, combustion and expansion) however each component has different values for ratio of specific heats  $\gamma$  and  $c_p$ , based in suitable average values. The perfect gas constant  $R$  is considered the same for all components, justified by the fact that molecular weight of air vary negligibly from component to component (in the present work,  $R=287.0596$  J/kg.K, the value corresponding to the standard atmospheric properties at 30 km geometric altitude, U.S. Standard Atmosphere, 1976). The average specific heat ratio for each component was calculated considering frozen flow in compression and expansion component and frozen or equilibrium flow in the combustor chamber. In order to maintain the consistency with the calculations presented in section 4, the fuel mass flow rate  $\dot{m}_f = 0$ . The efficiencies of the compression, combustion and expansion components, inputs for this method, are respectively  $\eta_c = \eta_b = \eta_e = 0.9$  (a typical value used in Heiser and Pratt, 1994). Others important inputs, treated as independent parameters, is the ratios  $\Psi = \frac{T_3}{T_0}$  and

$\frac{p_{10}}{p_0}$ . The ratio  $\Psi$  is only determined by the vehicle geometry and the value adopted for the specific heat ratio, than, if  $\gamma$

is determined, will be a fixed value for the VHA 14-X B. For determined values of  $\gamma$  in each component, the ratio  $\frac{p_{10}}{p_0}$

is dependent of both geometry and amount of heat added per unit of mass that for this case is  $q = 0.9q_{\max}$ . Then, as

$q_{\max}$  assumes different values if frozen or equilibrium flow are considered,  $\frac{p_{10}}{p_0}$  used for equilibrium or frozen

average properties in the combustor is correspondent for this ratio calculated with the methodology presented in section 4.

By means of closed equations this the Stream Thrust Analysis determines the uninstalled thrust, using the Eq. 4 rewrite as below:

$$\frac{F}{\dot{m}_o} = (1 + f)Sa_{10} - Sa_0 - \frac{RT_0}{V_0} \left( \frac{A_{10}}{A_0} - 1 \right) \quad (14)$$

where  $Sa_0 = V \left( 1 + \frac{RT_0}{V} \right)$  is the uninstalled engine specific thrust, known as stream thrust function. Then, calculating

$f$  from  $q_{\max}$ , the others performance analysis may be determined.

The equations used at the present work are presented below. More details about the Stream Thrust Analysis may be found in Heiser and Pratt (1994).

#### 4.1 Compression Component

Independent parameter  $\Psi$ :

$$T_3 = \Psi T_0 \quad (15)$$

Conservation of energy:

$$V_3 = \sqrt{V_0^2 - 2c_{pc} T_0 (\Psi - 1)} \quad (16)$$

where  $c_{pc}$  is the heat at constant pressure of the compression component.

Adiabatic compression process:

$$\frac{p_3}{p_0} = \left[ \frac{\Psi}{\Psi(1-\eta_c) + \eta_c} \right] \quad (17)$$

#### 4.2 Constant Area Combustion Component

Conservation of momentum:

$$V_4 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (18)$$

where:

$$a = 1 - \frac{R}{2c_{pb}}$$

$$b = -\frac{V_3}{1+f} \left[ \left( 1 + \frac{RT_3}{V_3^2} \right) + \frac{fV_{fx}}{V_3} - \frac{c_f A_w}{2A_3} \right]$$

$$c = \frac{RT_3}{1+f} \left\{ 1 + \frac{1}{c_{pb}T_3} \left[ \eta_b fh_{PR} + fh_f + fc_{pb}T^0 + \left( 1 + f \frac{V_f^2}{V_3^2} \right) \frac{V_3^2}{2} \right] \right\}$$

where:  $c_{pb}$ ,  $\frac{V_{fx}}{V_3}$ ,  $\frac{V_f}{V_3}$ ,  $c_f \frac{A_w}{A_3}$ ,  $T^0$  are respectively the heat at constant pressure for the combustor, the ratio of fuel injection axial velocity to  $V_3$ , the ratio of fuel injection total velocity to  $V_3$ , burner effective drag coefficient, enthalpy of fuel entering combustor and reference temperature to estimate the absolute static enthalpy  $h = c_{pb}(T - T^0)$

Conservation of momentum and energy:

$$T_4 = \frac{c}{R} - \frac{V_4^2}{2c_{pb}} \quad (19)$$

Conservation of mass:

$$\frac{p_4}{p_0} = (1+f) \frac{p_3}{p_0} \frac{T_4}{T_3} \frac{V_3}{V_4} \quad (20)$$

### 4.3 Expansion Component

Adiabatic Expansion Process:

$$T_{10} = T_4 \left\{ 1 - \eta_e \left[ 1 - \left( \frac{p_{10}}{p_0} \frac{p_0}{p_4} \right)^{\frac{R}{c_{pe}}} \right] \right\} \quad (21)$$

Conservation of energy:

$$V_{10} = \sqrt{V_4^2 + 2c_{pe}(T_4 - T_{10})} \quad (22)$$

Conservation of mass:

$$\frac{A_{10}}{A_0} = (1 + f) \frac{p_0}{p_{10}} \frac{T_{10}}{T_0} \frac{V_0}{V_{10}} \quad (23)$$

where  $\frac{p_{10}}{p_0}$  is a independent parameter provided as a input.

## 5. RESULTS AND DISCUSSIONS

The results for the calculations discussed in the sections 4 and 5 are presented in Tab. 1. Note that for the Stream Thrust Analysis the stations “1”, “a” and “4” is not determined because this method is not based in the specific geometry of the vehicle, but into account the effects of the geometry by means of independent parameters  $\Psi = T_3/T_0$  and  $P_{10}/P_0$ . Also is important observe that, although the station 4 corresponds to the final of the real combustor, this problem was modeled considering that the addition of heat occurs in the section of constant area that ends in station b.

In Tab. 1, the conditions along the 14-XB for each methodology and air model adopted are presented. It is important remember that the amount of heat added in each was 90% of the maximum amount that can be added without generate normal shock waves for each case, therefore, different amounts of heat. For variable specific heat ratio along 14-XB (Tab. 1A), we can see that  $\gamma$  decreases slowly in the begin of the compressor, decrease more rapidly after the third wave shock and along the heat addition and back to increase until a value lower than the initial value in the exit. This shows that the combustion chamber is a critical component for analysis and requires a suitable model choice. In the case of frozen flow along the chamber, the specific ratio was constant and equal the value in the inlet of the combustor,  $\gamma = 1.336$ . The average value for  $\gamma$  in the expansion component was, then, larger than the case with equilibrium flow along the chamber. For Stream Thrust Analysis (Tab.1B), each component was considered with constant value of specific ratio, although different of each other. In both methodologies (variable specific ratio and STA), the exit temperature is larger in the case of equilibrium than frozen flow model. Although seems contradictory, since equilibrium flow contain endothermic processes, we must remember that  $q_{\text{máx,eq}} > q_{\text{máx,fr}}$ , and more amount of heat was added in the equilibrium flow case in the chamber. This choice of added different quantities of heat was made because the aim is a preliminary calculation of the optimized performance of the 14-XB.

The Tab. 2 presents the performance parameters calculated with the two methodologies considering equilibrium or frozen air model in the chamber, and equilibrium in the compression and expansion components. Although the overall efficiency is less that 50% for all the calculations, the lowest specific impulse calculated,  $I_{sp} = 1638s$ , is much greater than typical values obtained with a  $H_2-O_2$  rocket engine, (400s to 500s). In both methodologies (variable heat ratio and STA) we can observe that the maximum amount of heat before normal shock being generated is larger for equilibrium than frozen flow, according with the fact that equilibrium flow take account endothermic processes. Also the overall efficiency  $\eta_0$  and the specific thrust (ST) were larger for the equilibrium case in the chamber. Although this similar trends, the quantitative values presents considerable differences.

Table 1: Thermodynamic properties at the VHA 14-X B lower surfaces considering (A) variable heat ratio along 14-X B and (B) Stream Thrust Analysis, for frozen or equilibrium average properties in the combustor.

A - RESULTS CONSIDERING VARIABLE SPECIFIC HEAT RATIO ALONG VHA 14-X B										
					frozen combustion			equilibrium combustion		
	0	1	a	3	b	4	10	b	4	10
$\gamma$	1.398	1.398	1.379	1.336	1.336	1.291	1.332	1.215	1.282	1.325
M	-	6.022	4.120	2.758	1.256	2.510	3.801	1.204	2.474	3.745
T (K)	226.50	296.39	561.55	979.69	2618.01	1713.99	1026.23	2804.94	1869.42	1131.60
P (Pa)	1197	2875	16685	87822	314625	40171	4645	361312	45084	5240
$\rho$ (kg/m <sup>3</sup> )	0.018	0.034	0.103	0.311	0.417	0.081	0.016	0.449	0.084	0.016
V (m/s)	2111.90	2081.26	1946.15	1693.75	1263.64	2004.75	2385.82	1173.87	2055.97	2461.43
B - RESULTS FOR STREAM THRUST ANALYSIS										
					frozen combustion			equilibrium combustion		
	0	1	a	3	b	4	10	b	4	10
$\gamma$	1.378	1.378	1.378	1.378	1.313	1.391	1.3191	1.285	1.313	1.313
M	-	-	-	2.777	1.260	-	3.520	1.267	-	3.38
T (K)	226.5	-	-	979.69	2552.16	-	1067.03	2569.34	-	1193.13
P (Pa)	1197	-	-	87538	342629	-	4647	350948	-	5240
$\rho$ (kg/m <sup>3</sup> )	0.01841	-	-	0.311	0.466	-	0.015	0.474	-	0.015
V (m/s)	2111.9	-	-	1698.06	1246.96	-	2253.98	1229.16	-	2276.96

The specific impulse, on the other hand, presents similar values for all cases except for variable specific ratio with frozen flow in the chamber, where the specific fuel consumption was bigger.

Dividing the length of the constant section chamber by the maximum and minimum velocities calculated in Tab. 1 we can estimate that the residence time of the flow in the chamber is the order of  $10^{-5}$  s. Considering yet that the flow entry in the combustor right after passing a third shock wave is reasonable to assume that the model considering frozen flow in the chamber must be more adequate. However, for a more exactly conclusion we have to estimate the time for vibrational relaxation process in this case.

Table 2: Performance parameters and  $q_{\max}$  considering variable specific ratio along VHA 14-X B and using Stream Thrust Analysis, for frozen or equilibrium combustion.

	Variable Specific Ratio along VHA 14-X B		Stream Thrust Analysis	
	frozen combustion	equilibrium combustion	frozen combustion	equilibrium combustion
$q_{\max}$ (kJ/kg)	1378	1955	1317	1667
ST (N.s/Kg)	364.24	451.75	268.82	314.38
$\eta_0$	31.44%	48.78%	43.10%	39.82%
S (kg/N.s)	$6.22 \times 10^{-5}$	$4.01 \times 10^{-5}$	$4.08 \times 10^{-5}$	$4.42 \times 10^{-5}$
$I_{sp}$ (s)	1638.44	2542.09	2495.88	2306.02
f	0.023	0.018	0.011	0.014

## 6. CONCLUSION

As can be seen, the model flow choice adopted for calculations of the conditions along 14-XB, the performance parameters and the maximum amount of heat that can be added may result in considerably different values and a suitable model is very important for an adequate estimative of this quantities. A wrong estimative of the maximum amount of heat, for example, can result in normal shock waves in the combustion chamber (if the value is overestimated) or lower values for performance parameters if the opposite occurs.

Although some tendencies has been similar, the use of the Stream Thrust Analysis may not give an adequate quantitative estimative. However, this method, that is based in closed equations, can be useful in the design phase and

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may facilitate the analysis of tendencies as the influence of altitude and initial Mach  $M_0$ , momentum and kinetic energy fluxes contributed by the fuel and changes in geometry in the performance parameters.

For future works the behavior of the performance parameters can be analyzed considering changes in the altitude, entry Mach number and angle of attack and the influence of the momentum and kinetic energy fluxes contributed by the fuel can also be studied.

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