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A MODIFICATION OF HYPERELASTIC INCOMPRESSIBLE CONSTITUTIVE MODELS TO INCLUDE NON-CONSERVATIVE EFFECTS

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Abstract. *The study and modeling of hysteresis effects in hyperelastic materials is usually accomplished with complex modeling techniques rooted in the laws of continuum mechanics. Nonetheless, many originally non-hysteretic hyperelastic/viscoelastic materials can benefit from a posteriori inclusion of these effects. This paper presents a study on the improvement of a constitutive model for hyperelastic incompressible materials in order to take hysteresis into account. This was accomplished by adding to the hyperelastic potential a function which reduces the value of the strain energy function during the different steps of the unloading process. Therefore, it is not the objective of this work to develop a new hyperelastic model, but to adapt existing one to consider hysteresis in its formulation, since the idea can be extended to virtually any constitutive model. Two materials were analyzed, polypropylene and latex rubber, since they typically present small and large elongations, respectively. The so modified models has three more constants than the original ones and showed good performance in capturing the hysteresis effects.*

Keywords: *hysteresis, hyperelatic material, constitutive model, damage model.*

1. INTRODUCTION

Hyperelastic materials are those which can suffer a large elastic deformation. The study of these materials through a traditional constitutive model is usually developed without taking in account its hysteretic behavior. That means that during the unloading process the material behave the same way as during the loading path. The goal of this paper is to propose a new method to improve traditional models to take in account this phenomenon through a softening model that appears only during the unloading part of the loading process.

The method proposed herein can be used to improve any traditional model, provided it has a strain energy function. Its implementation is based on the original Dorfmann's model (2007). Evidently the fewer the constants necessary to produce the material's softening, the better to avoid excessive computational effort during the fitting of the model against experimental data.

Two sets of experimental data were used to verify the consistency of the modified model. The modification proposed was tested on two hyperelastic models originally without any hysteretic capability, namely polypropylene and natural rubber, and were the HMHSI/HMLSI models (Hoss 2009, Hoss 2010)

The main objective is to improve and generalize Dorfmann's proposal (2007). It is important to emphasize that no new hyperelastic model has been created, but a function to adapt an existing model.

2. THEORETICAL REVIEW

The models used were the Hoss and Marczak models for low and high deformations (HMLSI and HMHSI) proposed by Hoss (2009). Hoss (2009) considered low deformations those smaller than 100%, and these models were choose because they have a good behavior during the fitting.

2.1 Hyperelastic models

Both models are hiperelastic and their use is restricted for incompressible materials only. The stress-strain relation is based on the differentiation of the strain energy function to the stress invariants. The strain energy function is commonly based on the principal stretches (λ_1, λ_2, λ_3) or the stress invariants (I_1, I_2, I_3).This models are based on a power-law dependency on the strain invariants, that can describe well the deformation globally and an exponential term that helps within low deformations (Hoss, 2009). The energy equation for the low deformation (HMLSI) has the form:

$$W = \frac{\alpha}{\beta}\left(1 - e^{-\beta(I_1-3)}\right) + \frac{\mu}{2b}\left(\left(1 + \frac{b(I_1-3)}{n}\right)^n - 1\right) \quad (1)$$

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where α, β, μ, b and n are constants of the material, I_1 is the first stress invariant. For high stress deformations (HMHSI) a logarithm term is included as follows:

$$W = \frac{\alpha}{\beta}\left(1 - e^{-\beta(I_1-3)}\right) + \frac{\mu}{2b}\left(\left(1 + \frac{b(I_1-3)}{n}\right)^n - 1\right) + C_2 \ln\left(\frac{1}{3} I_2\right) \tag{2}$$

where α, β, μ, b, n and C_2 are the material constants, I_1 is the first strain invariant and I_2 the second invariant.

For simplicity the present study considers the uniaxial tensile stress case only (Fig. 1). Therefore, we presented the strain invariants for this case only, others can be found in Marczak (2006). Considering an incompressible sheet, where stress in the directions two and three are nul (σ_{22}=σ_{33}=0), we have:

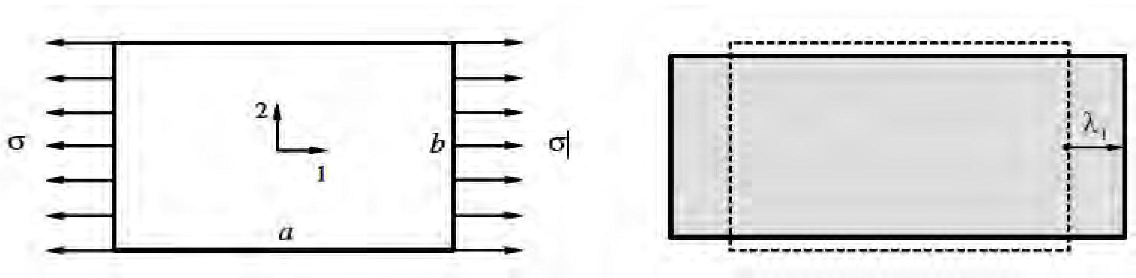

Figure 1. Axial tension of an incompressible sheet

$$\lambda_1 = \lambda \qquad \lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}} = \frac{1}{\sqrt{\lambda}} \tag{3}$$

Using the definition of the right Cauchy-Green deformation tensor, $C = F^T F$:

$$I_1 = tr(C) = \lambda^2 + \frac{2}{\lambda}$$
$$I_2 = \frac{1}{2}\left((tr\, C)^2 - (tr\, C^2)\right) = 2\lambda + \frac{1}{\lambda^2} \tag{4}$$

where λ is the stress, tr is the trace, F is the deformation gradient, and C is the right Cauchy-Green tensor. In order to hold the equilibrium, the hydrostatic pressure must be constant (Atikn Fox 2005), in the uniaxial case it is given by:

$$p_0 = 2\frac{1}{\lambda}\frac{\partial W}{\partial I_1} - 2\lambda\frac{\partial W}{\partial I_2} \tag{5}$$

Finally the Cauchy stress is given by:

$$\sigma = \sigma_{11} = 2\left(\lambda^2 + \frac{1}{\lambda}\right)\left(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W}{\partial I_2}\right) \tag{6}$$

Equation (6) describes the behavior of the material in uniaxial tensile stress. There are similar expressions for biaxial stress and pure shear (Marczak 2006, Hoss 2009). Eq. (6) was deduced from the energy form for this specific case, taking incompressibility into account.

2.2 Dorfmann's model

Dorfmann proposed a method to simulate the hysteresis effect in an arthropod muscle, but it applies to other hyperelastic materials and models as well. He developed the model for small deformation cases.

The model is based on the inclusion of two additional terms to simulate the softening of the material. The resulting equations can be easily applied to more general cases. The general equation of Dorfmann's model is given by:

$$\widehat{W}(I,\eta) = \eta\, W(I) + \phi(\eta) \tag{7}$$

where I represents the strain invariants, η is the function that represents the material softening, which is stored in ϕ.

When we define the new stress state of the material using Eq. (7) we find the stress depending only on the stress from the original equation and from η. This is due to the fact that ϕ does not depend on the strain invariants. The new equation for the stress is given by:

$$\hat{\sigma} = \eta \frac{\partial \widehat{W}}{\partial I} = \eta\, \sigma \tag{8}$$

which is expressed in terms of the original stress and the function η. In Eq. (8), σ is the stress of the original model and $\hat{\sigma}$ is the stress considering the new form of the energy potential, Eq. (7).

Finally, it is necessary to define the function η. It is important to note that η has to have a unitary value when we are loading the material and a value between zero and one during the unloading process. The Dorfmann's proposal is based on the following equation, which has the necessary characteristics:

$$\eta = 1 - \frac{1}{r} \tanh\left(\frac{\widehat{W}_m - \widehat{W}_0(\lambda)}{m}\right) \tag{9}$$

where r and m are the material constants to be determined, $\widehat{W}_m$ is the maximum deformation energy which is imposed to the material and $\widehat{W}_0(\lambda)$ is the energy calculated using the original equation from energy. In this way, when we are loading the material these two terms has the same value and η assumes 1. During the unloading process the maximum energy ($\widehat{W}_m$) remains at a constant value while the current value of $\widehat{W}_0(\lambda)$ is reduced. With this process it is possible to represent the material's softening in a very simple manner.

The function ϕ assumes the form of the following equation in case it is necessary to calculate it for the unloading process:

$$\phi(\eta) = -m(\eta - 1)\tanh^{-1}\big(r(\eta - 1)\big) - \widehat{W}_m(\eta - 1) - \frac{m}{2r}\log(1 - r^2(\eta - 1)^2) \tag{10}$$

3. METHODOLOGY

The adopted solution for the problem was mainly based on the Dorfmann's model due to its easily implementation and good results it gives. However, to understand how this model works it was necessary first to analyze if its functions are able to represent the materials behavior. Afterwards it is possible to propose an improvement to the present model and test its results. It is also necessary to keep always in mind that this methodology remains applicable to all hyperelastic traditional models. Later we present some proposals to modify the parameter η and give more flexibility to the original proposal presented by Dorfmman.

3.1 The materials studied

The chosen materials were Latex (natural rubber) and polypropylene reinforced with carbon fibers (PPC7712) (Niemczura 2011, Tomita 2008 and Vandenbroucke 2010). These two materials were chosen due to their difference on the elastic behavior, the first one can support large deformation and the second is usually used in small deformation ranges. The stress-strain curves of the materials analyzed here are shown in Fig. (2).

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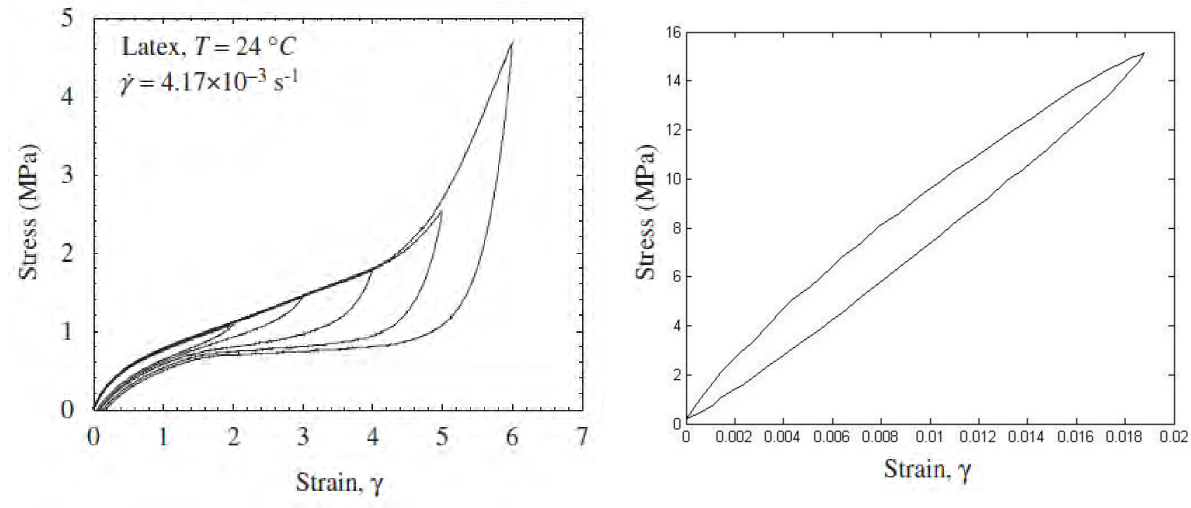
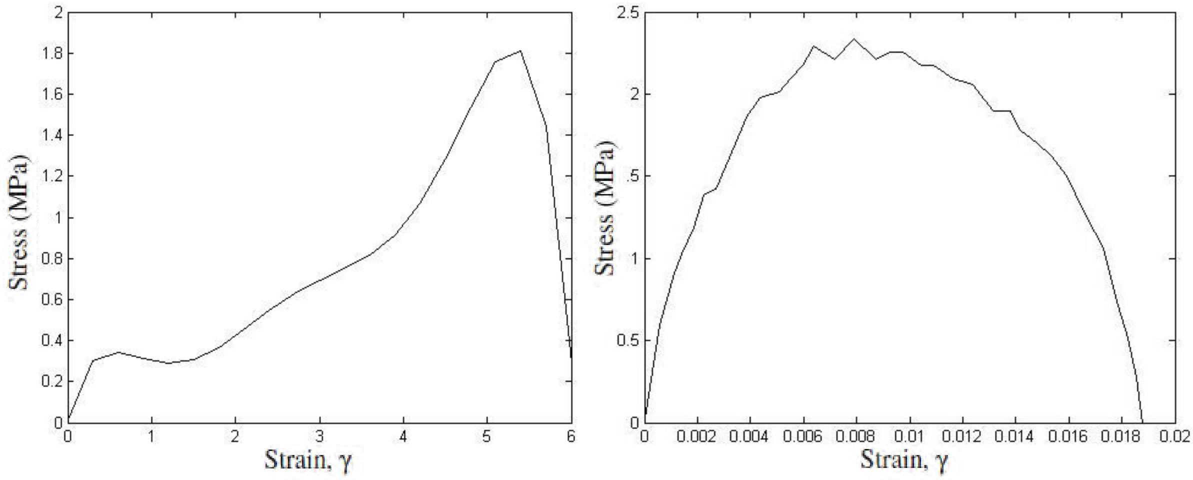


(a) (b)

Figure 2. Stress-strain curves: (a) Natural rubber (Niemczura 2011); (b) PPC7712 (Zrida 2009)

Our first attempt to detect the energy dissipation is examining the difference between the loading and unloading path. These graphs were made by plotting the difference between both curves during the loading and unloading process (Fig. 3).



(a) (b)

Figure 3. Dissipated energy between loading and unloading: (a) Natural rubber loaded to the maximum stress curve of Fig. 1; (b) PPC7712

3.2 Improoving Dorfmann's model

The Dorfmann's model (2007) it is a strategy predicting the material's softening during the unloading part of one cycle; it is of no practical use for more than one cycle. This model can be improved, though, especially whenever we have large deformations, since the original paper presented results within small deformations only. In order to improve this model, several attempts were made by the authors, and in the next session these attempts are presented. The main objective is to include a power-law term in the existing constitutive model. Therefore, it was necessary to incorporate a new material constant in the expression for η (Eq. 9). This new material constant is presented on the next session in two different forms.

The main objective of this paper is to propose a new expression to η that can adjust better the theoretical curves against experimental data than the original one.

4. RESULTS AND DISCUSSIONS

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4.1 Dorfmann's original model

Figure 4 presents the performance of Dorfmann's original model for both materials samples, to be used as a reference with the results obtained with the modified model's later on.

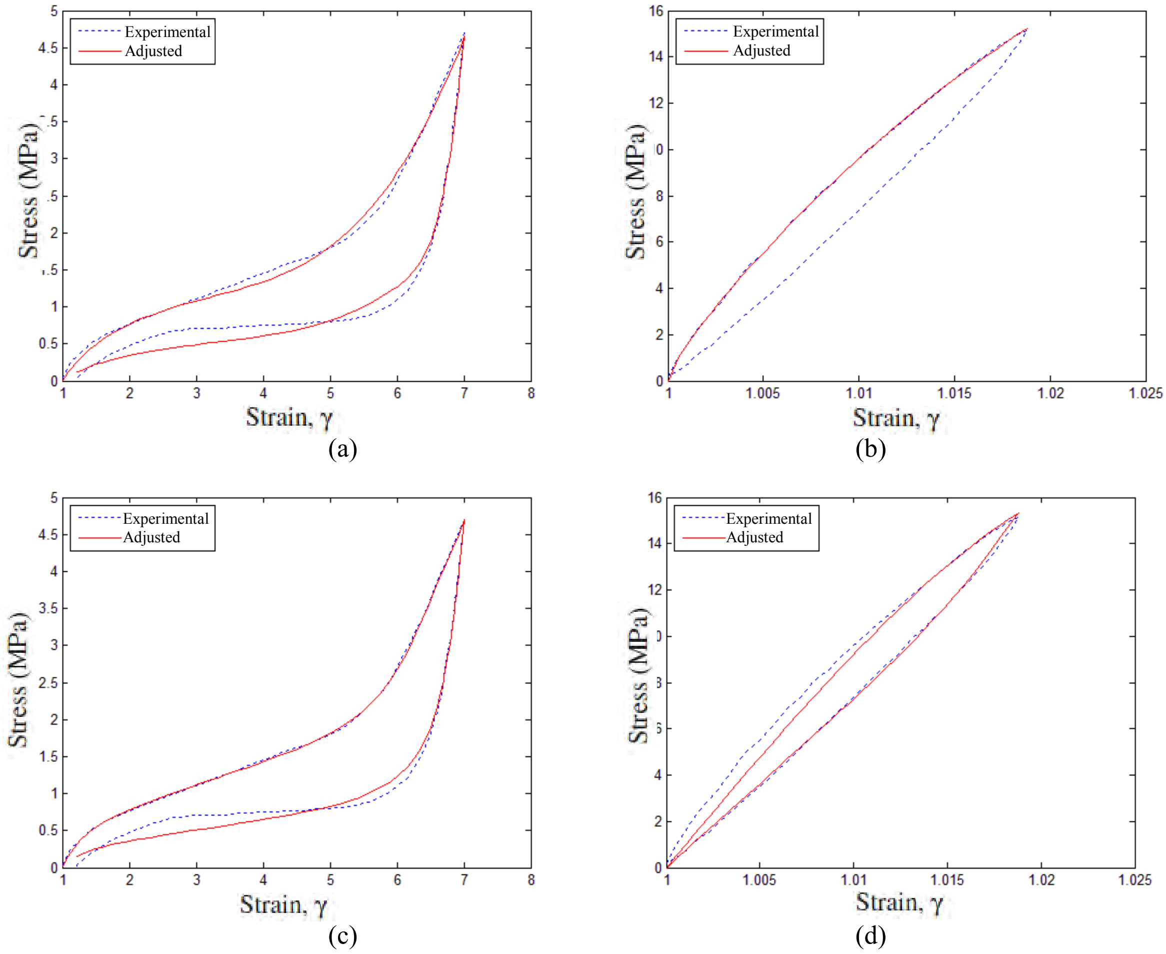

Figure 4. Adjusted data for models HMLSI and HMHSI using Eq. (9): (a) Latex using HMLSI (b) PPC7712 using HMLSI (c) Latex using HMHSI (d) PPC7712 using HMHSI

In a first analysis of this model, it can be seen that the results can still be improved. When analyzing Fig. 4.a and 4.c, it is easy to visualize that during the unloading part of the process that the fitting could be improved. This is the part we dedicate this study.

4.2 Dorfmann's modified model

As mentioned above, the main goal of the present work is to modify the softening effect (Eq. 9) to fit better the stress-stain curves during the loading cycle. So we tried to find equations that maintained the requirements needed (Section 2.2) and at the same time behave better than the original one. The first modified implementation proposed herein needs three additional constitutive constants (and not two as Dorfmman's one, Eq. 9), and is given by:

$$\eta = 1 - \left(\frac{1}{r}\right)^{z} \tanh\left(\frac{\widehat{W}_m - \widehat{W}_0}{m}\right) \tag{11}$$

Although Eq. (11) does not require much computational effort, it still is more computer intensive than the original Dorfmann's model. The role of the additional constant *z* is to allow a better adjustment of the curves during the unloading path.

The graphs with the results of the fitting based on Eq. (11) are shown, for both models, in the Fig. 5. The modification on the softening parameter (η) was not good enough to justify its use. As it can be seen in the unloading

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path in Fig. 4.a and 4.c behaves the almost the same as before. To justify the use of one additional material constant the results should be clearly better.

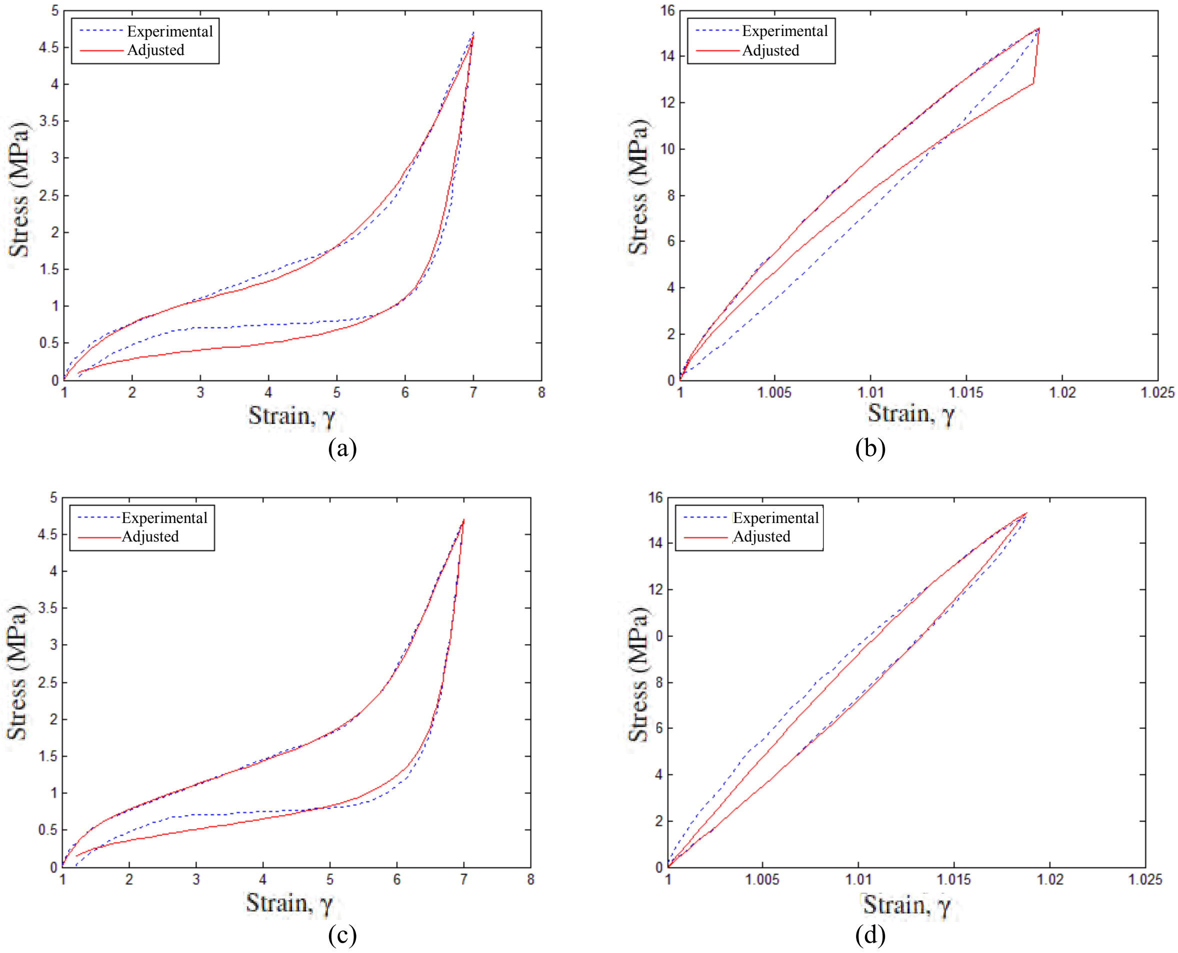

Figure 5. Adjusted data for models HMLSI and HMHSI using Eq. (11): (a) Latex using HMLSI. (b) PPC7712 using HMLSI. (c) Latex using HMHSI. (d) PPC7712 using HMHSI.

In order to have a better view of what is happening in the function η, this function is shown in Fig. (6-a). It can be seen how it changes its value in very few "time steps", stabilizing the function in the very beginning of the unloading process. There is no dependency on the time in these equations; we refer to time step as the evolution of the variable value during the process, which would be the increment of deformation step.

So to improve the model, we introduced an energy term outside the hyperbolic tangent. The value of hyperbolic tangent remains zero for the loading process and η assumes the unitary value, so it is consistent with Dorfmann's original proposal. Besides that, it will change the value of η just on the unloading process, awarding a not stabilized value for η during the unloading path.

$$\eta = 1 - \left(\frac{1}{r}\right)^{z(\widehat{W}_m - \widehat{W}_0)} \tanh\left(\frac{\widehat{W}_m - \widehat{W}_0}{m}\right) \tag{12}$$

where z is the material constant that have been included to the original model, in which the power law depends on the materials softening properties.

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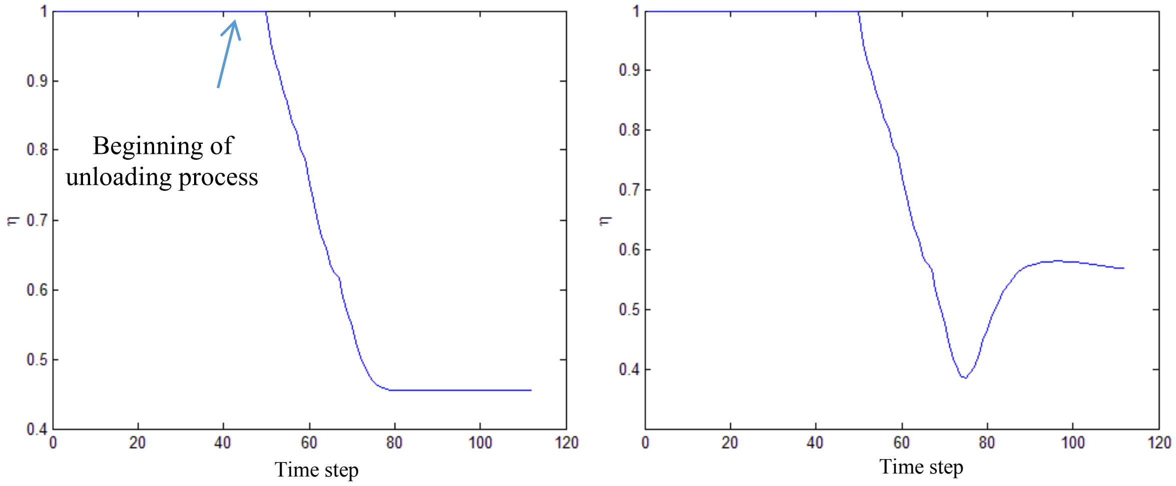

(a) (b)

Figure 6. Variation of η in function of the time step for Latex using HMHSI. (a) results for Dorfmann's model, (b) results for the proposed model from Eq. (4.6)

The changes that the model produced in the curves of the material can be seen in Fig. 7. Comparing Fig. 7.c with Fig. 4.c and 5.c, it becomes evidently that now the curve has fitted better than before.

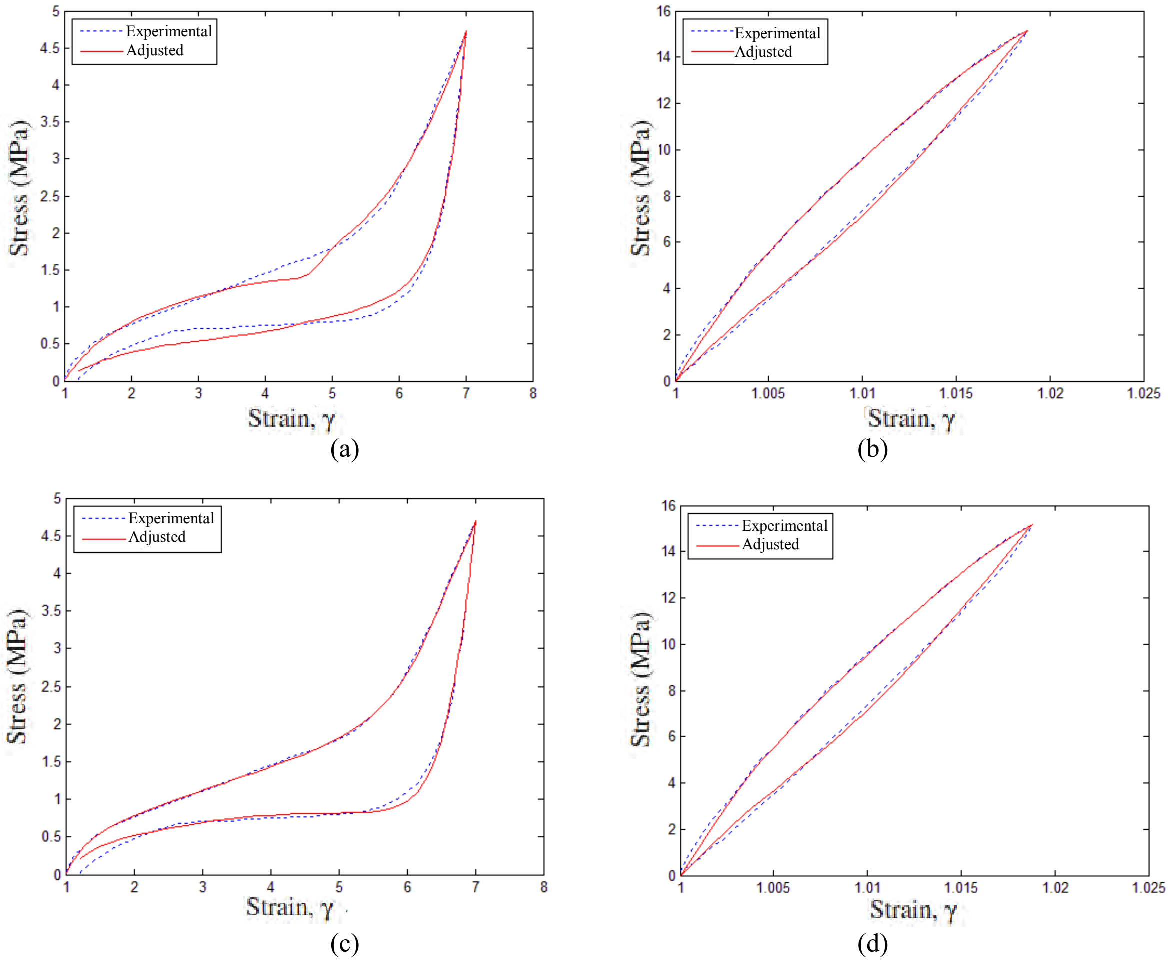

Figure 7. Adjusted data for models HMLSI and HMHSI using Eq. (12): (a) Latex using HMLSI. (b) PPC7712 using HMLSI. (c) Latex using HMHSI. (d) PPC7712 using HMHSI.

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The main change occurred for natural rubber, but the results for polypropylene are also improved. So with the use of Eq. (12) on Eq. (8) we can clearly achieve a better result than the one using Dorfmman's equation (Eq. 9).

4.3 Calculated constants

This section presents the constants calculated with Dorfmann's model and the two propositions presented on this paper.

Table 1. Calculated constitutive constants

	HMHSI - PP	HMLSI-PP	HMHSI - Látex	HMLSI - Látex
	Dorfmann			
α	2.376E+02	8.406E+01	8.064E-02	2.385E-01
β	2.247E+01	1.534E+02	-3.264E-02	5.249E-02
b	2.962E-02	1.989E+07	-7.495E-02	1.698E+04
n	-2.176E-04	8.319E-01	1.612E+00	2.442E+00
μ	2.005E+02	-7.238E+02	-1.536E-01	-7.093E-09
C2	-7.116E+01		-5.389E-01	
r	3.593E+00	2.343E+10	1.838E+00	1.820E+00
m	2.456E-01	-5.581E+09	9.967E-01	1.290E-01
	(1/r)^z			
α	2.376E+02	8.406E+01	8.064E-02	2.385E-01
β	2.247E+01	1.534E+02	-3.264E-02	5.249E-02
b	2.962E-02	1.989E+07	-7.495E-02	1.698E+04
n	-2.176E-04	8.319E-01	1.612E+00	2.442E+00
μ	2.005E+02	-7.238E+02	-1.536E-01	-7.093E-09
C2	-7.116E+01		-5.389E-01	
r	0.000E+00	3.586E-04	7.252E-02	0.000E+00
m	4.619E-01	3.566E-01	9.966E-01	1.921E-01
z	-1.135E-03	-2.392E-01	-2.321E-01	-6.606E-04
	z(Wm-W0)			
α	2.376E+02	8.406E+01	8.064E-02	2.385E-01
β	2.247E+01	1.534E+02	-3.264E-02	5.249E-02
b	2.962E-02	1.989E+07	-7.495E-02	1.698E+04
n	-2.176E-04	8.319E-01	1.612E+00	2.442E+00
μ	2.005E+02	-7.238E+02	-1.536E-01	-7.093E-09
C2	-7.116E+01		-5.389E-01	
r	1.000E+00	2.624E-01	1.026E+00	8.846E-01
m	9.814E+01	4.685E-01	3.613E+00	2.909E+00
z	1.589E+02	6.724E-01	3.601E+00	-9.449E-01

5. CONCLUSIONS

This paper presented the inclusion of non-conservative effects in hyperelastic incompressible constitutive models by modifying already existing energy potentials using a softening function based on the proposed model of Dorfmann (2007), but with better fitting than the original.

After testing the hyperelastic models for two different materials, the main conclusion is that the HMHSI presented better results in almost every situation tested, and so should be the chosen, in lieu of HMLSI.

Finally, choosing between the two modifications proposed here to capture the softening effect, the second attempt presented better results. It can be easily incorporated to any other traditional constitutive equations (that means, to already existing hyperelastic models), as it depends only on the stress or the stress invariants. The total computational cost necessary to incorporate this capability to other hyperelastic incompressible constitutive models under this approach will need the evaluation of three additional material constants, those of the softening model. Future works can further test the proposed functions for other constitutive models and test materials.

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