



## A NUMERICAL STUDY ABOUT THE INFLUENCE OF THE INACCURACY IN HYDRAULIC DIAMETER OF MICROCHANNELS IN SINGLE-PHASE LAMINAR FLOW

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**Abstract.** *In the past decades, several experimental investigations performed to the hydrodynamics and heat transfer in microscale flow showed divergences to the Darcy friction factor and the Poiseuille and Nusselt numbers, when these were compared to the results predicted by the classic theory. There are reports of deviations to the Darcy friction factor and the Poiseuille and Nusselt numbers attributed to inaccuracy in the measurement of hydraulic diameter in microchannels. The aim of this numerical study is to analyze how the hydrodynamic and heat transfer characteristics in single-phase laminar flow, of a fluid with constant thermophysical properties, can be affected by inaccuracy in the hydraulic diameter of the microchannels. The results obtained for single-phase laminar flow of water in microchannels with inaccuracy in the hydraulic diameter are compared to the ones obtained for a microchannel with perfect hydraulic diameter through Poiseuille and local Nusselt numbers. Deviations at Poiseuille and local Nusselt numbers, due to inaccuracy in the hydraulic diameter of the microchannels, were verified. The results showed that deviations at Poiseuille number in microchannels with hydraulic diameter below nominal value are higher compared to the ones obtained for microchannels with hydraulic diameter above nominal value. The deviations found for the local Nusselt number were proportional to the magnitude of the inaccuracy in the hydraulic diameter of the microchannels. The results of this study were determined through mass conservation, Navier-Stokes and energy equations, by computational fluid dynamics (CFD).*

**Keywords:** *microchannels, Poiseuille, Nusselt, CFD.*

### 1. INTRODUCTION

In recent years, the reduction of electronic devices in several application fields, such as biomedicine, chemistry and electronics has been providing high efficiency related to the space in equipments. At the same time, this reduction in physical space is counterweighted by the high performance required at the refrigeration systems in such equipments. Therefore, thermal control is one of the most critical areas for the development of modern microelectronic devices (Celata *et al.*, 2004, 2006a; Tabeling, 2005; Rosa *et al.*, 2009; Chiu *et al.*, 2011).

A lot of experimental studies (Wu and Little, 1984; Pfahler *et al.*, 1990, 1991; Choi *et al.*, 1991; Yu *et al.*, 1995; Pfund *et al.*, 2000; Mala and Li, 1999; Xu *et al.*, 2000; Li *et al.*, 2000; Tunc and Bayazitoglu, 2001; Judy *et al.*, 2002; Wu and Cheng, 2003; Celata *et al.*, 2002, 2004, 2006a, 2006b, 2006c; Steinke and Kandlikar, 2006; Chiu *et al.*, 2011), besides theoretical (Mala *et al.*, 1997; Yang *et al.*, 1998; Tso and Mahulikar, 1998) and numerical (Toh *et al.*, 2002; Koo and Kleinstreuer, 2004, 2005; Gamrat *et al.*, 2005; Chiu *et al.*, 2011) studies, have been carried out, in the past decades, seeking to investigate the hydrodynamics and heat transfer characteristics in microscale. The results obtained in several of these studies show diversions among themselves and, also, with the conventional theory. In general, the reported divergences can be viewed through analysis of the Darcy friction factor or the Poiseuille and Nusselt numbers, when the results obtained for them are compared to those provided by conventional theory (Celata *et al.*, 2004, 2006a, 2006b; Steinke and Kandlikar, 2006).

There are reports of friction factors and Poiseuille numbers either above (Wu and Little, 1984; Yang *et al.*, 1998; Mala and Li, 1999; Koo and Kleinstreuer, 2004; Celata *et al.*, 2006a) or below (Pfahler *et al.*, 1990, 1991; Choi *et al.*, 1991; Yu *et al.*, 1995; Judy *et al.*, 2002) of what is predicted by the classical theory, as well as in good agreement with it (Pfund *et al.*, 2000; Mala and Li, 1999; Xu *et al.*, 2000; Li *et al.*, 2000; Judy *et al.*, 2002; Celata *et al.*, 2002, 2006a). Some researchers attribute the deviations found for the friction factor or Poiseuille number to variations of the cross-section of microchannels due to the surface roughness (Wu and Little, 1984; Mala and Li, 1999; Li *et al.*, 2000; Wu and Cheng, 2003). Other researchers attribute these deviations to deformations existing in the cross-section of the microchannels (Celata *et al.*, 2006a; Steinke and Kandlikar, 2006), to aspect ratio of channels (Pfahler *et al.*, 1991; Wu and Cheng, 2003; Celata *et al.*, 2006a; Chiu *et al.*, 2011) and, also, to scaling effects, such as viscous dissipation (Koo and Kleinstreuer, 2004; Celata *et al.*, 2006b) and electrokinetic effect (Mala *et al.*, 1997; Yang *et al.*, 1998), for example. Uncertainty analysis carried out in several experimental studies attribute to the inaccuracy in the measurement of hydraulic diameter of the microchannels as one of the main reasons of errors in determining the friction factor and Poiseuille number in microscale flow (Judy *et al.*, 2002; Celata *et al.*, 2004, 2006b, 2006c; Steinke and Kandlikar,

2006). Additionally, there are reports (Pfahler *et al.*, 1991; Celata *et al.*, 2002) of deviations to the friction factor that showed dependence on the Reynolds number.

In relation to microscale heat transfer, some researchers have reported results obtained for the Nusselt number in good agreement with the classical theory (Lee *et al.*, 2005; Lin and Yang, 2005; Yang and Lin, 2007; Lin *et al.*, 2009; Yang *et al.*, 2012). However, other researchers have indicated that differences in rates and coefficients of heat transfer, as well as in Nusselt number, can be related to the flow velocity and fluid temperature (Peng and Wang, 1993; Wang and Peng, 1994; Peng *et al.*, 1995; Judy *et al.*, 2002), the Reynolds number (Choi *et al.*, 1991), the heat transfer conjugated (Maranzana *et al.*, 2004; Tiselj *et al.*, 2004; Celata *et al.*, 2006a, 2007), the viscous dissipation (Koo and Kleinstreuer, 2004; Celata *et al.*, 2006c; Lelea and Cioabla, 2010; Judy *et al.*, 2002; Morini, 2005), the surface roughness (Young and Kandlikar, 2008), the aspect ratio of microchannels (Chen, 2007; Chiu *et al.*, 2011) and the conductivity of the material that compose them (Maranzana *et al.*, 2004; Tiselj *et al.*, 2004; Celata *et al.*, 2006a, 2007), besides experimental uncertainties (Lin *et al.*, 2009; Hetsroni *et al.*, 2005; Rosa *et al.*, 2009; Lin and Yang, 2005; Yang and Lin, 2007; Yang *et al.*, 2012). Some numerical studies (Croce and D'agaro, 2004; Croce *et al.*, 2007) showed that the Nusselt number is more sensitive to the shape of the cross-section of the microchannels in comparison to other parameters, such as surface roughness, for example. Furthermore, there are numerical and theoretical studies that consider simplifications which differ a lot from what actually should occur experimentally as, for example, the negligence of viscous dissipation in numerical model (Gamrat *et al.*, 2005; Lee and Garimella, 2006), the true boundary condition at the limits (Hetsroni *et al.*, 2005) and the consideration of the fluid with constant thermophysical properties, in general.

Thereby, geometrical parameters of the microchannels, experimental uncertainties and the presence of several possible scaling effects, at once, complicate the identification of probable error sources in experimental studies in this application area. Therefore, the use of numerical techniques can be advantageous in the study of flow in microscale, since significant effects in this field, such as surface roughness, viscous dissipation and the inaccuracy in the hydraulic diameter of the microchannels, for example, can be considered separately in numerical model.

The aim of this numerical study is to analyze how the hydrodynamic and heat transfer characteristics, in single-phase laminar flow of a fluid with constant thermophysical properties, can be affected by inaccuracy in the hydraulic diameter of the microchannels. Other scaling effects, such as viscous dissipation and electrokinetic effect, for example, are not considered in the numerical model. The results obtained for single-phase laminar flow of water in microchannels with inaccuracy in the hydraulic diameter are compared to the ones obtained for a geometrically perfect microchannel through Poiseuille and Nusselt numbers. The results of this study were determined through mass conservation, Navier-Stokes and energy equations, by computational fluid dynamics (CFD).

## 2. COMPUTATIONAL MODEL

Figure 1 shows the computational domain used in this study. It consists of a microtube<sup>1</sup> with circular cross-section with diameter  $D_h$ . It's considered as inaccuracy in the hydraulic diameter of the imperfect microtubes of this study small variations ( ) for them, in relation to ideal hydraulic diameter  $D_h$  of the correspondent perfect microtube. Thus, the  $D_h$  of the imperfect microtubes is within a range that varies from a minimum value ( $D_{h,min}$ ) to a maximum value ( $D_{h,max}$ ).

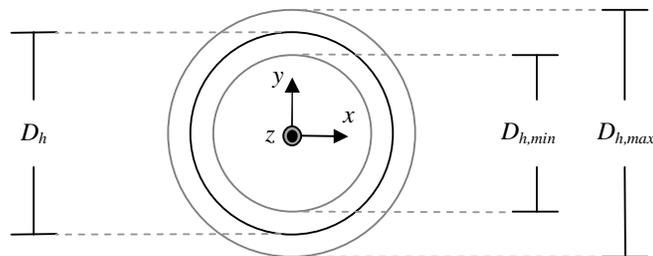


Figure 1. Scheme adopted for the computational model.

Based on the works of Celata *et al.* (2002, 2004), an ideal microtube with diameter  $D_h$  of 130  $\mu\text{m}$  was selected. The length  $L$  used was 0.13 m, to ensure hydrodynamically and thermally developed flow conditions at the tube exit (since  $L/D_h = 1,000$ ). Six cases of inaccuracy ( ) for the hydraulic diameter of the perfect microtube were considered:  $\pm 1\%$ ,  $\pm 2\%$  and  $\pm 3\%$ . Table 1 shows the hydraulic diameter  $D_h$  of the microtubes of this study and their corresponding deviations (inaccuracy) .

<sup>1</sup> A microtube consists of a microchannel with circular cross-section.

Table 1. Hydraulic diameter of the microtubes of this study and their corresponding deviations .

$D_h$ [ $\mu\text{m}$ ]	126.1	127.4	128.7	130.0	131.3	132.6	133.9
[%]	-3	-2	-1	0	+1	+2	+3

In all simulations, the working fluid chosen was water and the Reynolds numbers ( $Re$ ) considered were 200, 400, 600 and 800, according to the work by Steinke and Kandlikar (2006). To transfer heat to fluid, it was considered a constant heat flux ( $q_s''$ ) of 2 kW/m<sup>2</sup> applied on the surface of the microtubes. The entrance region of the microtubes was considered with simultaneous development of hydrodynamic and thermal boundary layers.

### 3. MATHEMATICAL MODEL

The fluid used is incompressible and with constant properties. The flow regime is laminar and permanent. Neither the viscous dissipation is considered, nor the gravitational effects. Based on these considerations, the equations of mass conservation, Eq. (1), Navier-Stokes, Eqs. (2-4), and energy, Eq. (5), in rectangular coordinates, are presented in the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \quad (3)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (5)$$

where  $u$ ,  $v$  and  $w$  are the velocity components of the fluid in the  $x$ ,  $y$  and  $z$  directions, respectively,  $\rho$ ,  $\nu$ ,  $\alpha$ ,  $p$  and  $T$  are the specific mass, the kinematic viscosity, the thermal diffusivity, the pressure and the temperature of the fluid, respectively.

The hydraulic diameter  $D_h$  is defined by

$$D_h = \frac{4A_c}{Per}, \quad (6)$$

where  $A_c$  is the cross-section area of the channel and  $Per$  is the perimeter of this one.

The Darcy friction factor  $f$ , the Poiseuille number  $Po$  and the Reynolds number  $Re$  are defined, respectively, by

$$f = \frac{2 D_h A_c^2 \Delta p}{\dot{m}^2 L}, \quad (7)$$

$$Po = f Re \quad (8)$$

and

$$Re = \frac{w_m D_h}{\nu} = \frac{\dot{m} D_h}{A_c \nu}, \quad (9)$$

where  $\Delta p$  is the drop pressure,  $\dot{m}$  is the mass flow rate,  $w_m$  is the mean velocity of flow and  $\mu$  is the dynamic viscosity of the fluid.

The shear stress on the walls of channels, in the region of hydrodynamically developed laminar flow, is related to the pressure gradient  $dp/dz$  and the hydraulic diameter  $D_h$  of channels by

$$= \frac{1}{4} D_h \left( \frac{dp}{dz} \right). \quad (10)$$

The hydrodynamic entrance length  $L_{he}$  is defined by

$$L_{he} = 0.05 Re D_h. \quad (11)$$

The local Nusselt number  $Nu$  is defined by

$$Nu = \frac{h D_h}{k}, \quad (12)$$

where  $k$  is the thermal conductivity of fluid and  $h$  is the local heat transfer coefficient, which is determined by

$$h = \frac{q_s''}{T_s - T_m}, \quad (13)$$

where  $T_s$  is the surface temperature of tubes and  $T_m$  is the mean temperature of fluid in cross-section, which is defined by

$$T_m = \frac{A_c \int w c_v T dA_c}{\dot{m} c_v}, \quad (14)$$

where  $c_v$  is the specific heat of fluid at constant volume.

The thermal entrance length  $L_{te}$  is defined by

$$L_{te} = L_{he} Pr, \quad (15)$$

where  $Pr$  is the Prandtl number.

The magnitude of deviation of a generic variable  $\Phi$ , in relation to the theoretical value expected for it,  $\Phi_t$ , is determined by

$$\phi = \frac{|\Phi - \Phi_t|}{\Phi_t} \times 100\%, \quad (16)$$

whereas the deviation on the hydraulic diameter of the imperfect channels, related to the hydraulic diameter of the ideal channel, is particularly indicated by (so that  $\phi = \pm D_h$ ).

#### 4. NUMERICAL SOLUTION AND MESH INDEPENDENCE TEST

In the commercial software Ansys CFX-12, which was used to analyze the problem, the differential equations, Eqs. (1-5), were discretized and numerically solved, for each point at the computational domain.

As a boundary condition to hydrodynamic problem, at the exits of all tubes a static pressure of 0 Pa was taken. At the entry of tubes, a temperature of 293.15 K was taken and the velocity was computed according to the value of Reynolds number  $Re$  used in simulation. The temperature and the entry velocity were used as their initial field. The boundary condition on the walls is with no slip and with a constant heat flux. The mesh used was hexahedral, with refinement next to the walls and also in the inlet and outlet sections of tubes. The error stability criterion used, for which the solution is taken as convergent, was  $1 \times 10^{-6}$ .

The determination of the number of elements for the meshes was made by analysis of the Poiseuille  $Po$  and local Nusselt  $Nu$  numbers. This analysis was performed for the case of flow with  $Re = 800$ , which provides the greatest thermal entrance length.

Table 2 shows the results of analysis for  $Po$  and  $Nu$ , and their respective numerical errors, for different meshes, for the ideal microtube.

Table 2. Number of elements,  $Po$ ,  $Nu$  and their respective numerical errors, for the ideal microtube.

Mesh	Number of Elements	$Po$ [-]	$\epsilon_{Po}$ [%]	$Nu$ [-]	$\epsilon_{Nu}$ [%]
1	23,490	65.322983	2.067161	4.211707	3.401216
2	174,440	64.357538	0.558653	4.373103	0.300528
3	429,590	64.108599	0.169685	4.370965	0.251491
4	520,030	64.104147	0.162730	4.370206	0.234083
5	1,001,280	64.006248	0.009763	4.367390	0.169495
6	1,505,390	63.949702	0.078590	4.366810	0.156193

According with Table 2, the error analysis for Poiseuille number  $Po$  shows that the result obtained by meshes 5 and 6 presented the best accordance with the theoretically provided value,  $Po_t = 64$  (Shah and London, 1978), for tubes of circular cross-section, in relation to the ones obtained by meshes from 1 to 4. The error analysis for the local Nusselt number  $Nu$  shows that the result obtained by mesh 6 didn't show very significant change related to the one obtained by mesh 5, which are quite close to the expected theoretical value,  $Nu_t = 4.36$  (Shah and London, 1978), for tubes with circular cross-section. However, based on the behavior of  $Po$  and  $Nu$  observed for the meshes 5 and 6, it can't state that the results obtained for  $Po$  and  $Nu$  from these meshes are independent of the refinement's degree applied. Though, it can be stated that very significant changes to  $Po$  and  $Nu$  wouldn't be observed by increasing the refinement's degree applied from mesh 6. Thus, the refinement applied to mesh 5 was considered appropriate for this study (with  $Po \cong 64.00$  and  $Nu = 4.37$ ). The same also was applied to the imperfect microtubes.

Figure 2 shows an aspect of cross-section of the mesh selected to represent the microtubes of this study.

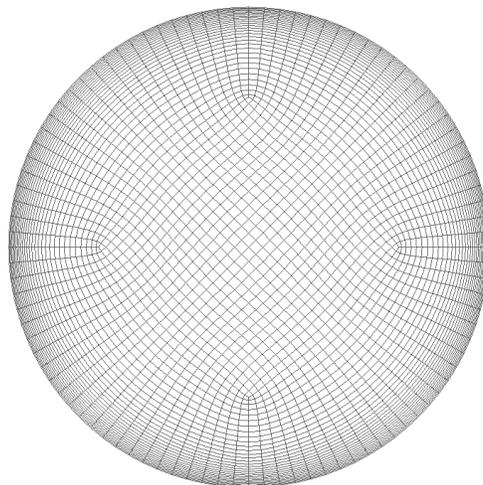


Figure 2. Cross-section of the mesh selected to represent the microtubes of this study.

Figure 2 shows the cross-section of mesh selected to represent the microtubes of this study. In this mesh, it can be noticed the high level of refinement applied next to the walls of the microtubes, to ensure the correct development of the hydrodynamic and thermal boundary layers of flow.

## 5. PROCEDURE ADOPTED

First, the thermophysical properties of water ( $\rho$ ,  $\mu$ ,  $k$ ,  $c_p$  and  $Pr$ ) were based on the mean temperature of entrance of water ( $T_{m,i}$ ) defined in numerical model, for the Reynolds number  $Re$  considered. Afterwards, the mean temperature of water in the outlet ( $T_{m,o}$ ) of ideal microtube was determined by the mean temperature difference  $\Delta T_m$  between the mean temperatures in the ends of this tube, which was obtained by solving the Eq. (5). Thereby, all thermophysical properties of water were evaluated based on the mean temperature of reference  $\bar{T}$  given by arithmetic average between

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the mean temperatures of water at inlet ( $T_{m,i}$ ) and at outlet ( $T_{m,o}$ ) of the tube. This procedure was performed iteratively until  $\bar{T}$  didn't vary more significantly, assuming a convergence criterion of 0.1%. Then, the thermophysical properties of the fluid were considered constant and configured in the simulations, for both the ideal microtube as the imperfect microtubes.

The determination of Darcy friction factor  $f$  in region of hydrodynamically developed laminar flow, for the microtubes, considered the drop pressure in this region, the length of this region ( $L - L_{de}$ ) and the mass flow rate obtained from the simulations. The hydraulic diameter  $D_h$  and the cross-section area  $A_c$  used were the correspondent to the ideal microtube. Thus, according to Eq. (7), for the imperfect microtubes, the deviations to Darcy friction factor  $f$  are due to changes in the drop pressure and mass flow rate, because of the inaccuracy in the hydraulic diameter and in the cross-section area of the same ones. Therefore, the Poiseuille number  $Po$  is determined by Eq. (8) based on Darcy friction factor  $f$  obtained by Eq. (7).

The determination of local Nusselt number  $Nu$  in the region of thermally developed laminar flow ( $L - L_{te}$ ), for all microtubes, according to Eq. (12), considered their local heat transfer coefficient  $h$  of the same ones and the hydraulic diameter  $D_h$  of ideal microtube. In the case of  $h$ , which is provided by Eq. (13), the surface temperature of microchannels  $T_s$  corresponds to average of the surface temperatures along the perimeter of the cross-section of the channels, at the axial position  $z$  considered. Therefore, the deviations for  $Nu$  at the imperfect microtubes are due to changes in  $h$  through the mean temperature  $T_m$  of the fluid in the cross-section and in the surface temperature  $T_s$  of these tubes, due to inaccuracy in the hydraulic diameter of the same ones.

## 6. RESULTS AND DISCUSSIONS

This section shows the results for Poiseuille  $Po$  and local Nusselt  $Nu$  numbers for the microtubes considered.

### 6.1 Poiseuille Number

Figure 3 shows the results of  $Po$  according to  $Re$  for the microtubes considered.

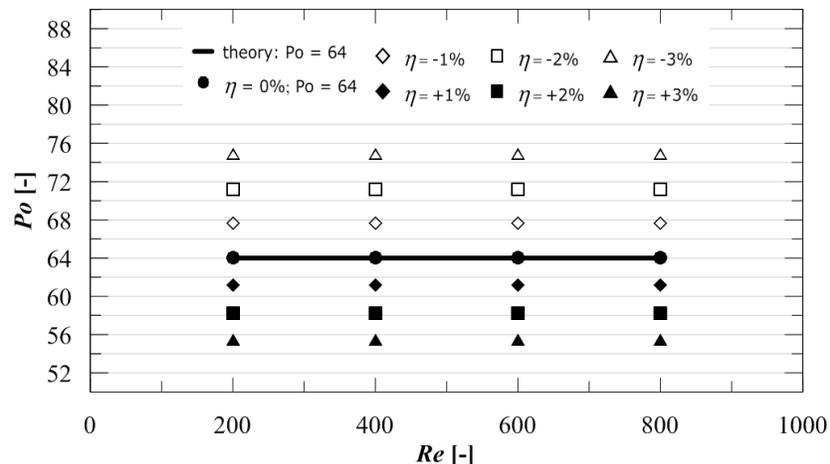


Figure 3. Poiseuille number vs. Reynolds number.

According to Figure 3,  $Po$  is higher than theoretical value  $Po_t$  in microtubes with  $D_h$  below the nominal value ( $\eta < 0$ ), while the opposite occurs for microtubes with  $D_h$  above the nominal value ( $\eta > 0$ ).

For microtubes with  $D_h$  below the nominal value ( $\eta < 0$ ), their  $A_c$  is lower than in the ideal microtube. In this case, the shear stress on the wall in these microtubes is increased, as well as the mean velocity of flow  $w_m$ , the drop pressure  $\Delta p$ , the Darcy friction factor  $f$  and  $Po$ . The opposite occurs for microtubes with  $D_h$  above the nominal value ( $\eta > 0$ ). It's according to the experimental reports by Celata *et al.* (2006b), in which Poiseuille numbers above the theoretical value were observed for microtubes that showed contraction at the cross-section.

Figure 4 shows the results of the deviation in Poiseuille number  $Po_p$  according to  $Re$  for the microtubes considered.

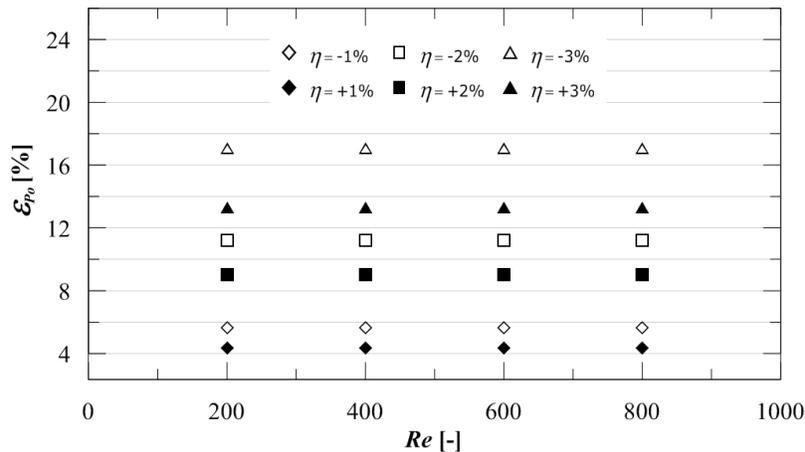


Figure 4. Deviation of Poiseuille number vs. Reynolds number.

As shown in Figure 4, the  $\epsilon_{p0}$  showed magnitude higher than the imprecision for  $D_h$  of the imperfect microtubes. They become larger as  $Re$  increases. Also, it's noticed that the deviations are higher for microtubes with  $D_h$  below the nominal value ( $\eta < 0$ ), while the opposite occurs for microtubes with  $D_h$  above the nominal value ( $\eta > 0$ ).

The  $\epsilon_{p0}$  are higher in microtubes with  $D_h$  below the nominal value ( $\eta < 0$ ) because the shear stress next to the wall in these microtubes is higher and, also, more intense than in microtubes with  $D_h$  above the nominal value ( $\eta > 0$ ). As the  $A_c$  of the microtubes with  $\eta < 0$  is smaller in relation to the one in ideal microtube, the fluid must rub more intensively next to their wall to flow, increasing the shear stress intensity. The opposite occurs for microtubes with  $\eta > 0$ . For example, in the case of flow with  $Re = 200$ , the increase on the shear stress for microtube with  $\eta = -3\%$  has an intensity of  $(98.42-92.31) \text{ Pa} = 6.11 \text{ Pa}$  compared to the one in the ideal microtube, while the reduction on the shear stress for the microtube with  $\eta = +3\%$  has an intensity of  $(92.31-87.31) \text{ Pa} = 5.00 \text{ Pa}$  compared to the one in ideal microtube.

## 6.2 Nusselt Number

Figure 5 shows the results of  $Nu$  according to  $Re$  for the tubes considered.

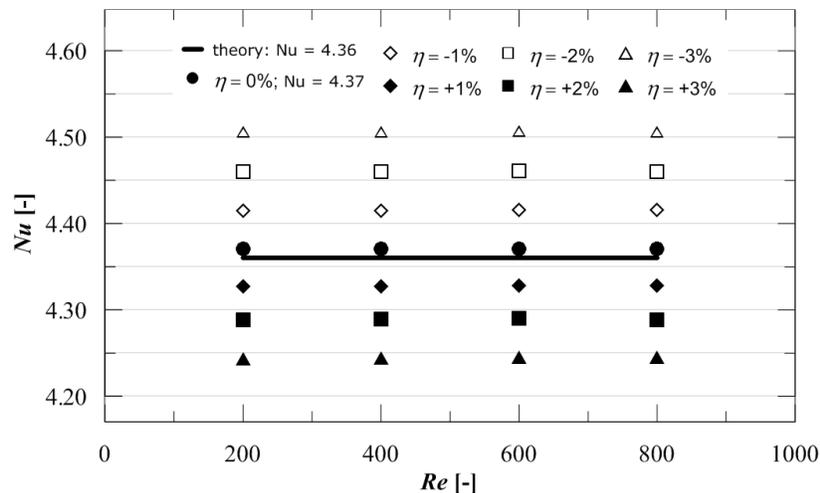


Figure 5. Nusselt number vs. Reynolds number.

According to Figure 5, for microtubes with  $D_h$  below the nominal value ( $\eta < 0$ ),  $Nu$  is higher than theoretical value  $Nu_t$ , while the opposite occurs for microtubes with  $D_h$  above the nominal value ( $\eta > 0$ ).

As  $w_m$  is higher for microtubes with  $D_h$  below the nominal value ( $\eta < 0$ ), in relation to the one in the ideal microtube, these microtubes should present higher  $h$ . Therefore, the same should occur to  $Nu$  in these microtubes with  $\eta < 0$ . The opposite occurs for the microtubes with  $D_h$  above the nominal value ( $\eta > 0$ ).

According to Table 2, the  $Nu$  obtained numerically for the ideal microtube (of 4.37) is higher than  $Nu_t$  (of 4.36). Therefore, it's expected that deviations for local Nusselt number  $Nu_t$ , in relation to the theoretical value  $Nu_t$ , according to Eq. (16), are higher for microtubes with  $D_h$  below the nominal value ( $< 0$ ).

Thus, Figure 6 shows the results of deviation in local Nusselt number  $Nu_t$ , related to the numerical value considered appropriate for this study (of 4.37), according to Eq. (16), as a function of  $Re$  for the microtubes considered.

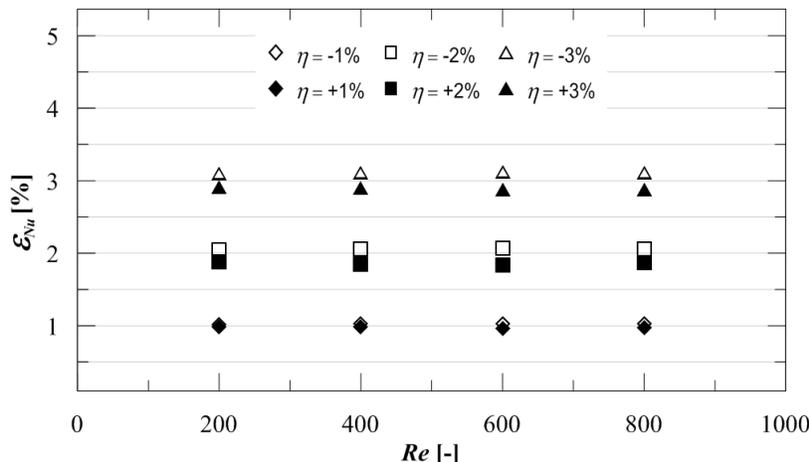


Figure 6. Deviation of local Nusselt number,  $\epsilon_{Nu} = (|Nu - 4.37|/4.37) \times 100\%$ , vs. Reynolds number.

According to Figure 6, the  $\epsilon_{Nu}$  are proportional to the magnitude of inaccuracy (deviation) at  $D_h$  of the microtubes considered.

According to Eq. (9),  $w_m$  is inversely proportional to  $D_h$  of the microtubes. Thus, microtubes with imperfect  $D_h$  of a certain percentage below the nominal value ( $< 0$ ) should present  $w_m$  above the ideal value with the same percentage. Therefore, the same should occur for  $h$  and, hence, for  $Nu$ , in these microtubes with  $< 0$ . The opposite occurs for the imperfect microtubes with  $D_h$  of a certain percentage above the nominal value ( $> 0$ ).

## 7. CONCLUSIONS

The aim of this numerical study was to analyze how the hydrodynamic and heat transfer characteristics for the single-phase laminar flow in microscale, of a fluid with constant thermophysical properties, can be influenced by inaccuracy in the hydraulic diameter of the microchannels. For this purpose, all other scaling effects, such as surface roughness and viscous dissipation, for example, were omitted in the numerical model. It seeks to isolate and highlight the effects related only to the inaccuracy in the hydraulic diameter of the microchannels on the hydrodynamic and thermal parameters of flow, as it is the case for Poiseuille and local Nusselt numbers, respectively. Although the simplifications considered in numerical model may overestimate the results founded, this study provided some general indications about the relative impacts to the inaccuracy in the hydraulic diameter of the microchannels on Poiseuille and Nusselt numbers.

This study showed that the deviations for the Poiseuille number in microchannels with hydraulic diameter below the nominal value are higher compared to those obtained for the microchannels with hydraulic diameter above the nominal value, which is in agreement with some results observed in experimental studies (Celata *et al.*, 2006b). Furthermore, the results of this study indicate that deviations found for the local Nusselt number were proportional to the magnitude of inaccuracy in the hydraulic diameter of microchannels considered.

However, direct comparisons among experimental results and numerical results obtained in this study are not adequate. This is due to the experimental uncertainty present in the experimental data and the natural superposition of several different physical effects (of scaling), which were omitted in numerical model considered for this study, as well as other simplifications adopted for the same one.

Thus, more systematic numerical studies may be developed. Therefore, other models of microchannels, with cross-sections of different geometric shapes, should be considered.

## 8. ACKNOWLEDGEMENTS

The authors acknowledge the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Universidade do Vale do Rio dos Sinos (UNISINOS) for financial support for this work.

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